Fine-Tuning a Global History of Music Theory: Divergences, Zhu Zaiyu, and Music-Theoretical Instruments

ALEXANDER REHDING

The issue of equal temperament offers an object lesson in the challenges of the new global history of music theory: Twelve-tone equal temperament was mathematically formulated at almost the same time in Ming-dynasty China and sixteenth-century Europe. While the old debate got stuck on questions of dates and cultural rivalries, recent work in comparative humanities, especially Kuriyama (2002), opens up new avenues. His concept of "divergence" is applied to the specific "music-theoretical instruments" in which Chinese and European theories of tuning manifested themselves in sound. Zhu Zaiyu's pathbreaking 1584 theory is reexamined specifically from this angle: He credits the *qin* (zither) for holding knowledge that the *12 lü*, the traditional Chinese pitch-pipes, could not convey. Zhu's example—and the concept of "divergence"—offers ways forward for a new, materially oriented, global history of music theory.

Keywords: Chinese music theory, cultural transfer, divergence, equal temperament, global music theory, Shigehisa Kuriyama, music-theoretical instruments, pitch pipes, *12 lü* [shi-er-lü], Zhu Zaiyu.

For better or worse, the online encyclopedia Wikipedia has become a major repository of worldwide knowledge. Wikipedia famously relies on crowdsourcing, but turning over the task of generating content to unspecified masses rarely makes for a straightforward process of knowledge acquisition. Some entries are distinctly contentious and undergo multiple back-and-forth edits. At the time I first started working on this essay, in 2014, the most edited Wikipedia pages included the entries for George W. Bush (45,273), List of WWE personnel (38,158), United States (32,571), Michael Jackson (27,050), and Jesus Christ (26,580).[1] Many of these edits, admittedly, are motivated less by broad societal concerns than by the self-selected group of people who habitually edit Wikipedia pages. Yet it is easy to glean how these entries represent a mix of topical public interest and personal passion, which often elicit extreme views that are then countered with equal vehemence. In these cases, the quest for knowledge turns into a tug-of-war.

Music theory rarely arouses the passions in a similar way. However, anyone who witnessed the ferocious edits and additions to which the music-theoretical term "Equal Temperament" was subjected would think that this entry should also occupy one of those top ranks.[2] Statistically this is not quite borne out. This is a shame—after all, when do you get a chance to make the boastful claim that music theory is "bigger than Jesus"? But certainly within the music-theoretical realm the entry became the target of a strikingly large number of edits. This is certainly surprising: One would have thought that a fairly basic term of music theory like equal temperament had long been settled.

The center of the controversy concerns the question of where twelve-tone equal temperament was discovered *first*. Histories of tuning will often draw attention to Vincenzo Galilei's (1520–1591) approximate solution proposed in 1581 and Marin Mersenne's (1588–1648) calculations.[3] This chronology is not strictly true, or rather, it is only true in a very limited sense: The European efforts were preceded by the Ming-dynasty scholar and aristocrat Zhu Zaiyu 朱載堉 (1536–1611).[4] Chinese music theory can legitimately claim to have formulated twelve-tone equal temperament as early as 1584, the year of Zhu Zaiyu's treatise *Lü xue xin shuo* 律學新說 (A New Explanation of Musical Temperament).[5]

Example 1 shows a timeline of the key names that are mentioned in these competing chronologies. The best effort to come up with the earliest calculation of equal temperament in

1 Chalabi (2014).

2 The edit history can be viewed on https://en.wikipedia.org/w/index.php?title=Equal_temperament&action=history. There are a few spikes of activity, and most of the changes that I am interested here go back to additions made in 2011, which was around the time that I first became aware of this phenomenon. As expected, a small number of very active editors were involved. On 6 April 2020, a separate entry on "12 Equal Temperament" was established, which now contains (and partly reduplicates) the earlier discussion and has its own editing history.

3 For instance, see Lindley (1984).

4 Western reference texts on tuning and temperament, Barbour (1951) and Lindley (1984), do not discuss Zhu Zaiyu. A revised German version of Lindley's work on tuning includes a brief discussion of Zhu's work (Ertelt and Zaminer 1987, 180–81). A popular science book on the topic (Isacoff 2003, 158–70) includes a chapter on Zhu. A new study in English dedicated to Zhu is Huynh (2012). The literature in Chinese on Zhu is extensive, but falls outside the scope of this essay (on the bibliographic challenges of this project see further note 10 below).

5 Some commentators push this date a bit earlier, to 1581 or 1580. While Zhu's main work on equal temperament dates from 1584, his earlier works contain references to his calculations.

the West leads us to the Dutch mathematician Simon Stevin, dating from the years around 1600. The situation is complicated by the fact that Stevin's important manuscript, *Vande Spiegeling der Singkonst*, remained unpublished and was not edited and printed until the nineteenth century.[6] In the heat of the argument, adding the ancient Greek music theorist Aristoxenus, who flourished around 335 B.C.E., to the Wikipedia entry may seem like a trump card in the favor of the West.[7] (And just as quickly, Aristoxenus is countered by the other side with another Chinese precursor, He Chengtian [ca. 370–447 C.E.], an important mathematician from the Northern and Southern dynasties.) But at closer inspection, it stretches the substance of Western theories to fashion Aristoxenus into the originator of twelve-tone equal temperament. Aristoxenus' formulation of the principle that interval sizes—and not the underlying ratios, *contra* the Pythagoreans—should be in simple relationships to make them interchangeable across the tetrachord and beyond, is not the same as the calculation of the irrational numbers that would form the mathematical basis of this claim. It glosses over precisely that problem. Meanwhile, what the inclusion of Aristoxenus, as a last-ditch effort, as it were, *does* highlight quite clearly is how high the cultural stakes are.[8]

This intercultural online controversy between supporters of China and the West may seem like a niche interest. But it is symptomatic of some wider trends. In the age of global media, the blinders of national traditions have become a serious problem for historiography, which includes even a somewhat arcane and rarefied subject such as the history of music theory. We can take a page out of the emerging world music theory, as promoted by ethnomusicologist Michael Tenzer and others. The principal purpose of such a theory, Tenzer writes, is to make sense of our musical selves in an increasingly intertwined world. The idea of "bi-musicality" is no longer sufficient, Tenzer contends, in a world dominated by iTunes, Pandora, and Spotify compressing impossibly vast swathes of musical styles, periods, and provenances. What we need instead is a "multi-musicality" or even a "pan-musicality," a response to the sweeping cultural and economic transformations that allow, encourage, or even compel us to approach any kind of music from an integrative perspective.[9]

There are, of course, important differences. Tenzer and his colleagues are talking about a primarily analytical and pedagogical approach for world music. Abstract questions concerning the theory of equal temperament are rarely found on Spotify preference settings. Nor, for that matter, is Wikipedia generally the most useful forum for involved scholarly debates. But it is easy to see that the pugnacious back-and-forth edits in the digital world are not a million miles from the overall question: The controversy about Chinese versus European equal temperament is the more aggressive and more speculative Janus face of a global music theory. The question of "who got there first" may be of burning interest to those for whom cultural stakes are bound up with questions of precedence—and ultimately, one suspects, with national pride and cultural supremacy. And this leads us back to that mix of passion and public interest that seems to be the driving force behind the most fervent Wikipedia edits, which are closer in spirit perhaps to Worldwide Wrestling Entertainment than to sober scholarly pursuits.

How is an integrative multicultural approach to music theory possible? Such an undertaking is always massive, and all that can be accomplished here is to provide some pointers toward a more global view of music theory.[10] There is in fact a whole long tradition, going back to the early modern period, of examining the parallelisms between Chinese and ancient Greek music theories.[11] The admiration for ancient Chinese culture is no coincidence: Enlightenment philosophers from Leibniz to Voltaire would habitually hold up China as an example of an ancient culture that matched European culture in every respect or even surpassed it, but that did not, in marked contrast to the political reality constituted by the European monarchies, depend on an aristocratic order.[12] What made China such an enticing model for the European philosophers was the fact that the birthright of the European aristocracy was replaced in China with what they saw as the meritocracy of the statewide bureaucratic system—in other words, a society that specifically rewarded learning. No wonder intellectuals were enthralled.

6 See Stevin (1884), Fokker (1944), Rasch (2008). Stevin refers to music theory in his *Mathematical Memoirs*, vol. 1 (1605), so it is likely that *Singkonst* was written before that time. See Cohen (1984, 48). Some suggest that Stevin's earliest calculations may have been made around 1585, see Rasch (2008, 255 and 269).

7 The sources listed in this paragraph are quite familiar. They feature in the Wikipedia discussion, and can also be found on the first few pages of Kenneth Robinson's dissertation, originally written in 1951 (Robinson 1980). To be sure, the incorporation of ancient Greece into a seamless history of Western progress is problematic, as postcolonial critics such as Appiah (2016) have pointed out.

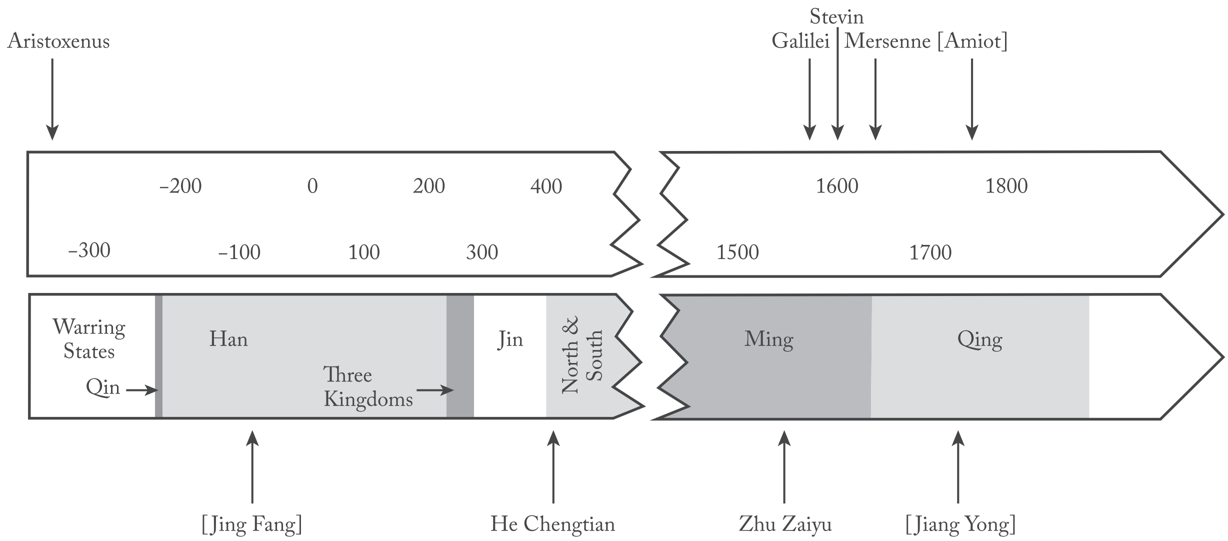
8 The most important music-theoretical fragments by Aristoxenus are included in Barker (1989).

9 Tenzer (2006, 33–34). See also Hijleh (2012, 1).

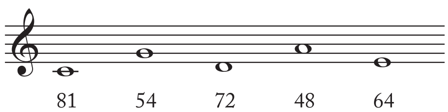
10 See also Raz et al. (2019). This is as good a place as any to insert a word on the linguistic challenges involved in a music theory that tries to go beyond its traditional European borders. As I am embarking on a comparative exploration, as a non-Chinese speaker, I am grateful for the extraordinary help and support I received in this project. Lester Zhuqing Hu has dispensed expert advice on countless questions of translation, interpretation, and nuance. I further wish to acknowledge the two anonymous reviewers, Rujing Huang, who offered indispensable advice, and Lingwei Qiu, on whose bibliographic support I relied. On one level, this article is an attempt to adapt the model practiced by Global historians such as Jürgen Osterhammel for music theory—a model that, for better or worse, ultimately relies on the traditional ideal of single authorship. Osterhammel ([2009] 2015, 1–113) offers a robust defense of this model, its strengths, and its challenges.

11 See, for instance, Rehding (2014) and Irvine (2020).

12 The group-curated exhibition *Sounding China: Transmission/Transformation* (2012) explored these themes; see https://soundingchina.fas.harvard.edu/.



EXAMPLE 1. *A timeline of Chinese and European music theorists mentioned in this discussion. The names in square brackets are not mentioned in the Wikipedia entry, but have been included here as they will be mentioned later*



Ratio	Product	Chinese name	Western pitch
1	81	黃鐘 Huang Zhong (= Do)	C
2:3	54	林鐘 Lin Zhong (= Sol)	G
8:9 = (2:3) × (4:3)	72	太簇 Tai Cu (= Re)	D
16:27 = (8:9) × (2:3)	48	南呂Nan Lü (= La)	A
64:81 = (16:27) × (4:3)	64	姑洗 Gu Xian (= Mi)	E
etc.			

EXAMPLE 2. *The traditional Chinese up-and-down method of scale generation*

UNEQUAL TEMPERAMENTS

The heated public fight on the global arena of Wikipedia is paralleled by a long, drawn-out scholarly debate about the same question. To review the basics of Chinese musical tradition, Example 2 summarizes how traditional music theory derives the pentatonic scale from an up-and-down principle (*sanfen sunyi* 三分損益) that alternates a perfect fourth down (4:3) and a perfect fifth up (2:3).[13] A keynote pitch, the Yellow Bell (*huang zhong* 黃鐘), is the starting point for a musical procedure that can be described in fairly straightforward mathematical terms.[14]

For simplicity's sake, I will designate the starting pitch as C in this example, as is common in the Western tradition—though this is not quite accurate, as we shall see later. To demonstrate the underlying math at work, we can further assign the numerical value 81 to this pitch, a number chosen for heuristic reasons, which has the benefit of working nicely, as a multiple of 3, with all the following pitches. What matters here is that the ratios derived in the up-and-down principle of Chinese scale generation would apply to any fundamental

13 This principle is first recorded in the *Guanzi* 管子, an encyclopedic treatise dating from the Spring and Autumn Period, in the seventh century B.C.E. See, for instance, Chen (1996, 49–51). For a recent exploration of the up-and-down method, see Li (2017).

14 The term may be confusing, since the pitch designation has nothing to do with bells per se, though there are historical resonances, see notes 65, 71, and 77 below.

pitch. Five consecutive pitches in this series are then reordered in ascending order, to form the pentatonic scale seen here. While the pitch material used in the pentatonic scale ends after five pitches, Chinese music theory typically continues to calculate the ratios for twelve scale degrees.[15] These other scale degrees, though not used within one pentatonic scale, can be drawn on for the purpose of extending the tonal range, equivalent to modulation.

To be sure, the precise mechanism of calculating these twelve scale degrees is slightly different from the circle of fifths in the West: The Chinese up-and-down method means that the intervals stay within the range of one octave-and-a-fourth. But adapting the calculations to the traditional Pythagorean intervals of 2:3 to form a circle of fifths is a mathematically trivial undertaking. As a consequence, Chinese music theory runs up against exactly the same problem as does Western music theory: the twelfth fifth is not exactly identical to a pure octave. This small difference is known in the West as the Pythagorean comma; Chinese music theorists were aware of the same discrepancy.[16] The Jesuit missionary Père Amiot reported this transcultural convergence in his *Mémoire sur la musique des Chinois* (1779) in colorful terms: "Pythagoras himself had done no more than to apply to strings what the Chinese had said before him in speaking of pipes."[17] Amiot was being less metaphorical here than it may appear: The parallels between the foundations of Chinese and Western music were so striking that the only way eighteenth-century theorists could explain the similarities was by arguing that Pythagoras must have gotten them directly from China.[18]

Anyone trained in Western music in the modern age is familiar with the solution to this problem: If the discrepancy could be split equally among the intervals, there would not be one large noticeable gap but eleven tiny ones that would be all but imperceptible. The difficult part was to formulate the appropriate calculation. The mathematically precise solution, as expressed in Western terms, holds that each semitone is separated by a factor of $\sqrt[12]{2}$, an irrational number, from its neighbor. One hindrance in the West was the powerful Pythagorean tradition with its abhorrence of irrational numbers. It was not until the mid-sixteenth century that European mathematicians made any headway on this fundamental question—and interestingly, as Peter Pesic has shown, questions of musical intervals were a driving force in the conceptual development of irrational numbers.[19]

Not content with stating the chronological precedence of Chinese music theory in establishing equal temperament, a number of scholars have felt the need to spin the argument further by speculating how this newly found knowledge would have spread from China to Europe, effectively extending Forkel's eighteenth-century narrative into the present. Joseph Needham and Kenneth Robinson, writing in Needham's monumental study *Science and Civilisation in China*, conjure up the image of a traveler to China who first hears about the new method to calculate equal-tempered intervals there. Needham and Robinson explain:

> It is particularly important to note how little had to be memorised by any traveller in touch with Chinese ideas for him to be able to transmit the idea of the mathematicians and musicians of Europe. Such a traveller would only have to say: "I understand that the Chinese temper their viols with great accuracy. They simply divide the length of their first string by $\sqrt[12]{2}$ to get the length of the string for the second note, and then they do the same again for the third note, and so on, till they reach the 13th which is a perfect octave." Not a book but a sentence only was required for the diffusion of this great idea.[20]

They concede that it "cannot be proved that a copy of [Zhu Zaiyu's treatise *Lü xue*][21] made its way to Europe and was there acted upon," but speculate that there was "ample opportunity" for this to happen.[22] They draw attention to the important Jesuit missionary and scholar Matteo Ricci, conjecturing that learned Europeans in China such as Ricci could "scarcely have avoided hearing of [Zhu]'s books so recently published."[23] Ricci (1552–1610) had studied mathematics in Europe, and upon his arrival in China he immersed himself in Chinese culture and language. He adopted the Chinese name Xitai 西泰, ascended to imperial circles, and gained the Wanli Emperor's special favor.[24] He also had some musical interests—he took keyboard instruments with him to China, and it seems that he knew how to play them—but we have no music-theoretical writings by him.

15 As the Book of Han *Hanshu* 漢書 (111 C.E.) records, the Han-dynasty music theorist and mathematician Jing Fang 京房 (78–37 B.C.E.) even extended the fifths all the way to sixty (McClain and Hung 1979).

16 The discrepancy—not yet called "Pythagorean comma"—was first calculated in the Euclidian *Sectio canonis* (Barker 1989, 199), dating from the third century B.C.E., there formulated as what corresponds to $(9:8)^6$ in modern mathematical terms. In the Chinese tradition, the discrepancy is first mentioned in the important Confucian and Daoist treatise *Huainanzi* 淮南子(122 B.C.E.). Jing Fang's more detailed calculations (see previous note) are an attempt to solve this problem.

17 Amiot (1779, 9); also see Robinson (1980, 39).

18 Forkel (1783, 237). As Irvine (2020, 160) details, this story is complicated by the fact that Forkel's influential report, which calls Pythagoras's ideas a "theft from the Chinese," is in turn plagiarized from Jean Benjamin de la Borde's review of Amiot's work.

19 Pesic (2010).

20 Needham and Robinson (1962, 224).

21 To facilitate continuity, and to avoid multiple spellings of Zhu's name and works, I have replaced all transliterations with pinyin.

22 Needham and Robinson (1962, 226).

23 Needham and Robinson (1962, 227). The Wikipedia entry on "Equal Temperament" currently claims: "Matteo Ricci, a Jesuit in China[,] recorded his [Zhu's] work in his personal journal and very likely brought it back to the West." This claim is wildly inaccurate. Curiously, Robinson's dissertation, completed in 1951 on the same topic, still made the case for parallel, independent discoveries. See Robinson (1980).

24 On Matteo Ricci in China, see particularly Spence (1984) and Hsia (2016).

How would this knowledge have come back to Europe? As historian Jonathan Spence describes, travel between China and Europe at that time was long and dangerous—the voyage took a whole year.[25] In this very limited cultural exchange, where every visitor from China to Europe caused a sensation, we have a fairly good picture of the few musicians who traveled from China to the West.[26] Ricci died in China in 1610 and did not return to Europe, so it is not immediately clear how Zhu's theories would have spread there from the Middle Kingdom.

More recently, another scholar, Gene Cho, continued in the same vein and added more detail to the Ricci hypothesis. He fleshed out the Jesuit network suspecting a link to the important French scholar Marin Mersenne, who was among the first in the West to write about equal temperament. (Mersenne was in fact not a Jesuit himself, but he was an active correspondent who was linked to intellectuals throughout Europe, from Athanasius Kircher to Thomas Hobbes.) The missing link between Ricci and Mersenne would be the Jesuit missionary Nicolas Trigault, who was sent out, as Cho argues, upon the news of Ricci's death to put his earthly possessions in order, and who spent the years 1610–13 in China.[27] This hypothesis seems a little overdetermined. Trigault, after all, set sail for China before Ricci's death, arriving in Macau in 1610 and in Nanjing in 1611, so it is difficult to make the case that Trigault was sent out at Rome's behest as Ricci's executor. The long journey makes it unlikely that Rome was even aware of Ricci's demise then.[28] But it is certainly true that, once in China, Trigault was entrusted with the translation of Ricci's journals into Latin and returned to Europe with them in 1613, where they were published. It contains several short mentions of music, but nothing that bears on tuning.[29]

Cho continues his speculation by ramping up Needham and Robinson's rhetorical questions:

> Given these facts, it would seem impossible that Trigault would have been wholly unaware of Zhu's fame and his life's works. [⋯] Would it not be highly likely that Trigault [⋯] had passed on to Mersenne at least the news of Zhu's theory? More easily, they could have simply mentioned only the mathematical equation (the twelfth root of two) or only its product (1.059463⋯), even without citing Zhu as the original author of this number.[30]

Every good conspiracy theory, they say, needs a motivation. As to the question of why the Chinese origin of this theory would have been suppressed in the West, particularly at the hands of Mersenne (or Stevin, for that matter), Cho suggests that relations between the Vatican and the Jesuit order were strained, and that any reference to pagan knowledge would have been impolitic.[31]

As is so often the case, it is less productive to guesstimate whether or not these chains of oral communication are true than it is to probe the underlying assumptions. What is noticeable is that, despite the voluminous correspondence between the Jesuit missions in China and Rome in the seventeenth century, no written trace of Zhu's music theory has been identified. The earliest documentation of European awareness of Zhu's work on music appears to be in Amiot's *Mémoire* (1779) mentioned earlier. The absence of evidence, as they say, is not the evidence of absence. But the complex speculative networks that Cho, Needham, and Robinson conjure up is built on a rather optimistic concept of scientific dissemination. There are a number of assumptions at play that seem difficult to maintain when regarded at close range.

Let's begin by reminding ourselves that twelve-tone equal temperament is not the Higgs Boson. In other words, it is hardly the case that scholars around the world were waiting for someone to come up with the calculation for twelve-tone equal temperament. The assumption underlying Needham and Robinson's and Cho's speculations that the sheer brilliance of Zhu Zaiyu's solution would be immediately obvious to European readers, who would then somehow tacitly communicate the concept back to Europe without writing it down (or suppressing any reference to precursors), is a stretch.

We should be mindful of the specific status and the conditions of this piece of knowledge. First of all, the necessity for temperament was recognized in European music-theoretical circles as a problem in the wake of the redefinition of major and minor thirds as consonances—a case made most forcefully by Zarlino.[32] But twelve-tone equal temperament was only *one* solution among numerous competing proposals. We must be wary of a teleological fallacy here: In musical practice, it was not until the nineteenth century that twelve-tone equal temperament gained the preeminence that it holds in current musical thought.[33] From the perspective of the seventeenth century there was nothing inevitable about equal temperament. Rather, in an intellectual climate that still worked within a (slowly waning) Pythagorean tradition, twelve-tone equal temperament was a radical outlier among the many possible solutions to the problem of intonation.[34]

It is worth remembering, second, that the musical situation in China was quite different from European music. Temperament gains in urgency in a musical situation that tries to align harmonic and melodic intervals covering large parts of the circle of fifths.[35] This was the case in European polyphonic

25 Spence (1984).
26 It is rare that new information comes up, as in the case of the Chinese visitor to London (see Clarke 2010).
27 Cho (2003, 253–54).
28 See also Brockey (2007).
29 Trigault (1953).
30 Cho (2003, 254).
31 See Cho (2003, 231).
32 See Zarlino (1558).
33 See Weber (1958); also see Parkhurst and Hammel (2020).
34 See, for instance, Duffin (2006).
35 Chua (2001) shows, using the example of Vincenzo Galilei, how enharmonic distinctions could lead to situations that worked melodically but not harmonically. The case specifically for equal temperament was less strong than Chua makes it out to be; nonetheless, this is an excellent

music, but much less so in Chinese music, which had a predominantly monophonic (or heterophonic) tradition.[36] Temperament in Europe was an urgent practical problem, whereas in Chinese music it was predominantly a speculative question, with only limited practical application in musical structures.[37] There are two situations in the structure of Chinese music in which temperament becomes relevant: Either in music with a very wide modulatory range or in music that modulated "flatwards."[38] After all, going back to the chart in Example 2, in the up-and-down method, the pitch corresponding to our F (really E♯) was the twelfth generated pitch and it had accumulated the maximum mistuning.

Third, it is quite a stretch from Trigault's putative awareness "of Zhu's fame and his life's works" to a practical understanding of twelve-tone equal temperament. Quite specific knowledge in *both* music theory *and* advanced arithmetic would have been required to grasp the nature of the problem and its proposed solution. As far as is known, Trigault, who became the procurist of the Jesuit Mission in China—a kind of PR person in the early modern age—had neither.[39] And fourth, expanding on this point, even mathematicians schooled in Europe would not have been familiar with the calculation of higher roots, which were not conceptually available then, but would only be developed by Simon Stevin ca. 1600. The assurances that "only little would have to be memorized" and that "they simply divide the length of their first string by $\sqrt[12]{2}$," as Needham, Robinson, and Cho suggest, were perhaps not quite as straightforward in the sixteenth century as it seems from today's vantage point. The same is apparently also true on the Chinese side: The historian of mathematics Xu Fei suggests that Zhu's work was likely not understood by anyone until Jiang Yong 江永 (1681–1762), a scholar during the Qing dynasty.[40]

In other words, the speculation falls short because of what historians call "backshadowing,"[41] and because of an overly simple idea of knowledge transfer based on personal contact. In this model, it seems, knowledge is conceived of as a series of "inventions," to use Robinson's preferred term, or "discoveries," in Cho's preferred term—as ideas that are not only self-evidently true but also exist free from any preconditions: Once they are in the world, it appears, they can be simply communicated from person to person and can always be reproduced as if they had always existed. In fact, the abstract concept of equal temperament had long existed in the West, but the broad musical consensus for the longest time, well into the eighteenth century, was that the resulting sounds were mistuned and undesirable. Even in the late eighteenth century, Amiot commented specifically on equal temperament in his *Mémoire*—and he thought it was a bad idea.[42] In other words, if equal temperament was secret knowledge that was clandestinely circulated among Jesuits, they did a fantastic job hiding it.

All in all, knowledge transfer between these two cultures in the early modern period was perhaps not quite as straightforward as has been assumed. These obstacles are greatly compounded when we take Fritz Kuttner's skeptical position into account.[43] In a trenchant critique of Needham and Robinson, Kuttner points out that the notions of "invention" (and "discovery") misrepresent the situation: The problem of temperament had been known in China at least since the important Daoist treatise *Huainanzi* 淮南子 (122 B.C.E.), in which pure fifths (2:3) were modified somewhat, just as the general problematic of tuning systems in general terms can rightly be traced back to Aristoxenus in ancient Greece in the third century B.C.E. What Zhu Zaiyu offered is best understood as a technology to solve the problem more accurately. Kuttner specifies: "Just as Stevin in Europe, Prince [Zhu] was the [· · ·] (calculator) of this temperament, not the inventor."[44]

Kuttner's point may seem pedantic, but it highlights a bigger issue: There is not *one* way of calculating equal temperament. It is easy to forget that the numerological traditions found in the Confucian Classic Yi Jing 易經 and the apocrypha formed a foundation for mathematical procedures in traditional Chinese thought. As Xu Fei reconstructs, Zhu Zaiyu availed himself of specific Daoist mathematical technologies to arrive at his calculation.[45] Zhu developed an enormous 81-bead abacus, and used the Yellow River map 河圖 and the Luo Shu Square 洛書, esoteric mathematical diagrams derived from interpretations of the Confucian Classics, which led him to the calculation of the nine-digit product 749,153,538. This is a remarkably precise operator for the purpose of calculating twelve-tone equal temperament, though, crucially, it is not derived by means of extracting higher roots but by culturally specific mathematical technologies that would not have been available outside of the Confucian tradition.[46] Kuttner chides Needham and Robinson for assuming that identical results mean identical mathematical procedures, and for using tendentious translations that suggest the extraction of roots where

example of the specific problem sixteenth-century music theory in Europe was dealing with.

36 One of Ricci's observations on music was that the Chinese did not have keyboards (Trigault 1953).

37 Even though the practice of Chinese music was predominantly monophonic, a conceptual or speculative "harmony" existed in the music-theoretical realm of tuning. Zhu Zaiyu explains, in a key passage in Chapter 7 新舊律試驗 from his *Lülü jingyi* 律呂精義 (1596), that one should take the *huangzhong* (C) and the *linzhong* (G) pipes, the G and D pipes, the D and A pipes etc. and make sure that each pair "accord to one another" (*xie* 協). Thanks go to Lester Hu for this point.

38 In addition, it is of course of immediate relevance to performers. The *qin*, prized by the literati, for instance, had to be tuned before each playing.

39 The classic biography of Trigault is Dehaisnes (1864).

40 Fei (2008, 289).

41 Bernstein (1994).

42 Amiot (1779, 116n and 202n).

43 Kuttner (1975).

44 Kuttner (1975, 173).

45 Fei (2008, 275–90).

46 See Nielsen (2003). Thanks to Joys Cheung for enlightening conversations about the Confucian classics.

the Chinese makes no such implications: "The trouble is clearly," Kuttner sighs, "that the Western mind is too prejudiced to imagine another method of equal temperament calculation."[47]

The case for two independent derivations seems convincing; certainly, as far as the question of knowledge transfer is concerned.[48] If there is no compelling reason to believe that the method of extracting higher roots was used in China, then there really is very little to transfer. This does not take away from Zhu's outstanding achievement, of course, but it casts doubt on the specific claims that have been made on behalf of Chinese music theory. If there was in fact no mathematical knowledge—especially nothing as concise as "$\sqrt[12]{2}$"—to bring back to Europe, what do we make of this case of early knowledge transfer?

MODELS OF COMPARISON FROM CULTURAL TRANSFER TO DIVERGENCE

Case closed? Certainly, if the question that we are interested in is one of precedence. The case that Zhu Zaiyu was the first to calculate equal temperament is unassailable, based on what we know at present. Meanwhile, the claim that the Chinese mathematician therefore has ownership over the calculation of equal temperament and that Europeans are in breach of some intellectual copyright seems quite thin, whereas the case that European music theorists came up with their own calculations independently and slightly later is quite strong. But framing this question as an issue of ownership and intellectual property is not even particularly interesting. A more enlightening angle would try to go deeper into the roots of this parallelism: Why then? And how? What needed to be in place to get to a mathematical calculation? Here Zhu actually gives us a clue:

> By day and by night I searched in my mind and studied exhaustively the principle [of temperament]; one dawn I suddenly had a perfect understanding of it, and for the first time I realized that the ancient sorts of pitch-pipes all gave mere approximations to the [true] notes. Moreover, this was something pitch-pipe exponents had not been conscious of for a period of two-thousand years. Only the makers of the seven-stringed zither [*qin*] in their method of placing the markers at three quarters or two thirds [of the length of the strings] had as common artisans transmitted by word of mouth [the way of making the instrument] from an unknown source. I think that the men of old probably handed down the system in this way, only it is not recorded in literary works.[49]

The moment of epiphany for Zhu Zaiyu came when he considered his music-theoretical instruments. Music-theoretical instruments are defined as those devices that music theorists use in order to substantiate their claims. They are instruments in both scientific and musical senses of the word that, in producing sounds, simultaneously highlight important features about music and generate music-theoretical knowledge.[50] Different eras have relied on various music-theoretical instruments: In our time, in the Western world, the piano is the instrument of choice in the theory classroom, whereas the monochord traditionally served this function in the European West from antiquity into the early modern period. Traditional Chinese music theory had relied on pitch pipes or the *12 lü* 十二律. Reproduced in Example 3, from Zhu's treatise, these are bamboo pipes of specific length that encode the up-and-down method of scale generation.

From the perspective of music-theoretical instruments, Zhu's observation can also be read as an exhortation to music theorists. When he refers to "pitch-pipe exponents," he includes a subtle double-entendre: *lülü* 律吕 (pitch pipes) was used as a metonym for music theory and tuning. Its exponents are therefore both people who are preoccupied only with pitch pipes, and more broadly, music theorists in general. Zhu is making simultaneously an epistemological and organological argument: Music theorists have failed to perceive what the musical practitioners have known by heart for generations, because music theorists—"pitch pipe exponents"—only care about, well, pitch pipes.[51]

Before taking a closer look at the *12 lü*, let's freeze the image of Zhu realizing that stringed instruments have different epistemic qualities than blown pipes. I want to tarry over this moment for a methodological reflection, to consider some recent approaches on the broader questions of cultural comparisons.[52] Outside of music, there is in fact a sizeable literature on the comparison of cultures, often focusing on ancient Greece and ancient China, starting with the eminent historian of science G. E. R. Lloyd, who for all intents and purposes founded the modern version of this field of study in the late 1980s. Much of Lloyd's work is concerned with methodology, probably in reflection of the sizeable groundwork that needed to be covered. Lloyd particularly opposed the concept of different *mentalités* of *Annales* school fame, which enjoyed great popularity in the 1970s and 1980, and argued that the concept does not illuminate so much as describe a situation which then requires further explanation.[53] Lloyd tends to juxtapose Greek

47 Kuttner (1975, 183). It is worth noting that Kuttner himself came under some criticism for representing Zhu Zaiyu's calculations inaccurately (Dai 2011, 259–63), but his critique of Needham and Robinson stands unaffected. Thanks go to Shingkwan Woo for directing me to this critique.

48 Woo (2018) has also recently made an impassioned case for Zhu's and Stevin's independent calculations.

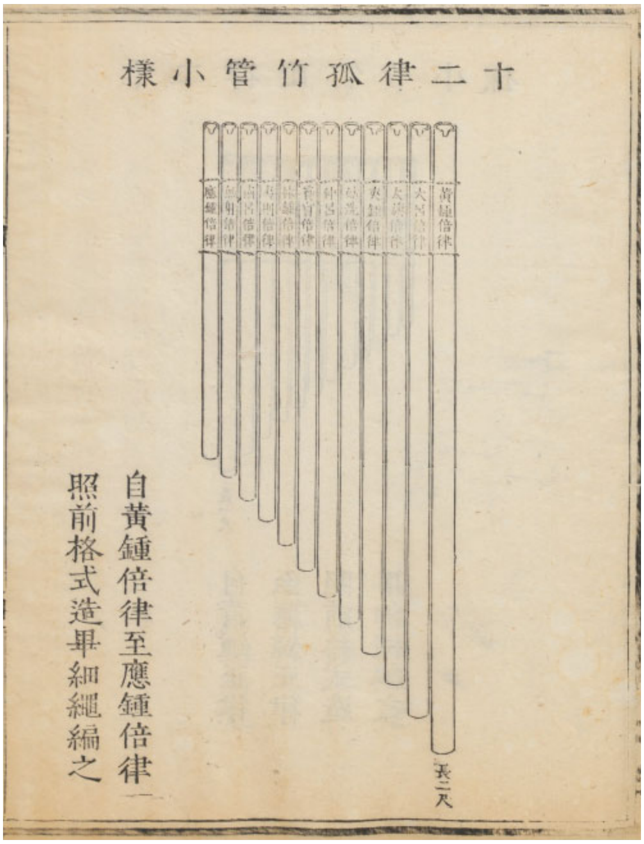
49 Translated in Robinson (1980, 12–13). Editorial insertions are in the original text, except for my use of pinyin, as above. The translation is from *Lü xue xin shuo*, ch. 1, fol. 5a.

50 See Rehding (2016a).

51 I am indebted, again, to Lester Hu for this point. See also note 67 below.

52 It may seem unusual that I am leaving out approaches within musicology here. This is because I am specifically interested here in organology (material culture) on the one hand and in the comparison of cultures on the other—a musical version of what Siegfried Zielinski (2005) has called variantology. Gabriela Currie (2019) on the circulation of music and James Millward (2018) on stringed instruments along the Silk Road come perhaps closest here to what I have in mind.

53 See especially Lloyd (1990). For an example of *mentalités* in the musical



EXAMPLE 3. *The* 12 lü *(pitch pipes). From Zhu Zaiyu* 朱載堉*. Yue lü quan shu* 樂律全書*. c. 1596, vol. 7, seq. 669. Harvard Yenching Library, Rare Book T 6730 2944. (Printed between 1736 and 1861.) https://iiif.lib.harvard.edu/manifests/view/drs:15498530$669i. In this representation the twelve pitch pipes are tightly bound; they are more commonly kept as individual pipes*

and Chinese responses to similar scientific and philosophical issues, asserting that while a complete grasp must probably remain elusive, it is certainly possible to gain *some* degree of understanding across cultures.[54]

Much of Lloyd's approach intersects well with Needham's Chinese history of science. But another branch of the global history of ideas, specifically in the literary field, also has important methodological tools to offer for the comparative history of music theory. This branch tends to be indebted either to the structural comparison of myths in the wake of Claude Lévi-Strauss, or to *Begriffsgeschichte*, the history of concepts, which explores the specific meaning of ideas in historical contexts.[55] Lisa Raphals's *Knowing Words* is a sophisticated and challenging example of this model—an application of the Greek concept of *mētis* (cunning, intelligence) in the context of Chinese intellectual history.[56] Raphals's work sensitively brings concepts and contexts to bear on each other across cultures, even when there are no direct means of transmission. Raphals's work also owes a debt to the field of comparative mythology, especially in her comparison of the *Odyssey*—whose title character epitomizes the concept of *mētis* or cunning—with Chinese novels from the Ming period. It is particularly the appeal to *Begriffsgeschichte*, which has long been an important tool in the history of music theory, which opens up some intriguing possibilities for a comparative approach.[57]

The epistemological differences between these two broad approaches, put simply, can be located in their conception of reality. The history of science is often configured around a kind of reality—usually furnished with the moniker "objective"[58]—that appears to reside outside of scientific theories, and that determines points of cultural comparison. It is less clear that the same is true for the study of literature, which is typically thought to operate in a culturally contingent framework. But both approaches grapple for a joint element, something that can be usefully compared, a common core—however we wish to define it.[59]

Shigehisa Kuriyama, a historian of medicine, takes up elements of both the scientific and discursive traditions in his provocative comparative investigation of the human body from the dual perspective of ancient medical traditions in China and in ancient Greece, but employs them to very different ends. Where most cultural comparisons aim to find points of convergence, Kuriyama goes in the opposite direction by tackling this question from the perspective of "divergences." Rather than looking for a common core, a kernel of truth (again, however we wish to define it), emerging from the comparison, he is interested in diverging trains of thought emanating from a joint starting point. Issues of "truth," "correctness," or "objectivity" play no part here.

In his extraordinary book *The Expressiveness of the Body*, Kuriyama explores his concept of divergences, starting with an analysis of pulse-taking.[60] It is hard to imagine a more basic aspect, more fundamental to life, than this bodily

realm, ironically, see Needham and Robinson (1962). Despite the impressive ambition and the sheer scope of Needham's undertaking, a number of conclusions rely on a notion of differing *mentalités*. But the methodological differences between Needham and Lloyd should not be overstated: Lloyd is scholar-in-residence at the Needham Institute in Cambridge.

54 Lloyd (2004).

55 See for instance, Müller (2014); in a global context also Moyn and Sartori (2015).

56 Raphals (1992).

57 This tradition is less explicitly appealed to in Anglo-American scholarship than in its continental counterpart. See especially the German *Handwörterbuch der musikalischen Terminologie* (Eggebrecht 2012) or the Italian *Storia dei concetti musicali* (Borio 2007).

58 See Daston and Galison (2007).

59 The much-maligned early-twentieth-century discipline of comparative musicology was founded on such a belief. This negative example also explains why the comparative approach is now treated with considerable skepticism: a cultural comparison turns all too easy into a demonstration of one's own cultural superiority.

60 Kuriyama (2002).

representation of our heartbeat. Despite appearances, however, pulse taking is anything but universal: Whereas Western doctors were quick to latch onto the regular palpitations of the pulse with its systolic ups and diastolic downs, medicine in the Middle Kingdom went a very different way. The Chinese medical tradition knows six pressure points on each arm with each signifying a different part of the body, and each feels the *mai* 脈 (transliterated by Kuriyama, archaically, as *mo*) which can be translated as "blood vessel" or "pulse," but which signifies more specifically the flow of blood coursing through the body. What Chinese and European doctors each sense when they feel the pulse leads them into very different directions: The connection to music, especially rhythmic theory,[61] that the West made early on in its history is not at all part of the Chinese discourse. This is what Kuriyama means by divergence—a natural phenomenon, often one that has the appearance of universality, is interpreted in different cultures in fundamentally divergent ways. The question of which interpretation is the correct one does not come up in Kuriyama's approach, as it is unanswerable and uninteresting. He deliberately avoids any hypothesizing about links between these two ancient cultures and allows himself and us to understand how such different perceptions can emerge of something supposedly as universal as the human body under the medical gaze. There is no direct comparison between them; what he offers is a reconstruction of different discourses erected around a thing, each with its own system of signification. What matters from the perspective of divergence is where these discourses and interpretations take us: Sensing rhythm in the pulse allows Western medicine to open up the discursive field toward music and rhythmic temporality, whereas sensing the flow in multiple pressure points offers a haptic mapping of the body to Chinese medicine that is foreclosed in the West.

From a similar vantage point, the divergence in the approach to sound between different traditions can be productively analyzed. It offers a framework in which a responsible and meaningful cultural comparison can be carried out. Like the different haptic techniques that Chinese and European doctors employ in taking the pulse of the living body (or the different mathematical technologies employed by Zhu Zaiyu and Simon Stevin, for that matter), the devices and technologies used to create and measure music-theoretically sanctioned sounds lead into very different discourses.

The term discourse may appear bound up with a text-critical approach, but it is important to remember that the material basis in which the arguments are rooted are central here.[62] An approach that explores divergence takes part in the ontological turn that was spearheaded in music theory in recent years by such initiatives as Critical Organology.[63] What we are dealing with here is, as they say, not so much a difference in world views but a difference in worlds. As we turn to a closer re-investigation of Zhu's work on equal temperament, it turns out that the divergent categories are bound up less with geographical or cultural differences (Europe versus China) than with differences between music-theoretical instruments and the sounding realities that they each produce.[64] The old geographies of difference recede and new knowledge empires of sound emerge—less fixed geographically, but of no lesser reach.

THE *12 lü* AND *qin* AS MUSIC-THEORETICAL INSTRUMENTS

Now it's time to unfreeze the image that we left behind to embark on this short methodological excursion. Our situation is interesting: Zhu Zaiyu's discovery that the relations he had been trying to calculate can be demonstrated using a different music-theoretical device. As the quotation with which we stopped indicated, these relations became a music-theoretical reality precisely when music-theoretical thinking switched away from the *12 lü* to the *qin* 琴 (now *guqin* 古琴), which had not been traditionally in use as a music-theoretical instrument.[65] In other words, the perspective of the material traditions of Chinese musical thought highlights one aspect that is otherwise easily overlooked: Zhu Zaiyu introduced a paradigm shift, a switch to a different music-theoretical instrument that allowed equal temperament to be not merely an arithmetic speculation but a sounding reality.

This, then, is a more interesting question than the tedious brinkmanship of who-got-there-first. What kinds of insights and connections did the *12 lü* allow, as compared with the music-theoretical instrument of choice in the West, the monochord? What are the cultural divergences of sound? A full answer would have to take into account the cultural and material history of the Ming Dynasty and the intellectual heritage of Confucianism, which is beyond the scope of this article.[66] In this context, all I can hope to do is to sketch out some possibilities in hopes that others will provide a fuller picture.

Let's start with the material properties of the *12 lü*. Given the prominence accorded to *qi* 氣 in traditional Chinese

61 Wellmann ([2010] 2017).

62 Any associations with the Power–Knowledge nexus that is a central part of the Foucauldian concept of discourse are entirely welcome here. But it is worth remembering that critics in the post-Foucauldian era have moved his work to a more strongly material basis, most relevantly Ernst (2000).

63 Tresch and Dolan (2013).

64 A common pitfall in comparative work is to forget that cultures themselves are constantly in flux. The contrast between Chinese ideas and European ones should ideally be imagined within a geographic-temporal matrix: Zhu's (and Ricci's) Ming-dynasty China in the seventeenth century was a rather different culture than the Qing-dynasty China that Amiot described in the eighteenth century—just as the same was true for European culture.

65 The other music-theoretical instrument of choice in ancient China, beside the *12 lü*, were bronze bells (see Falkenstein 1994). The *qin* with its silk strings was known for its refined sound and contemplative qualities, and it enjoyed great popularity among the scholars and literati. Playing the *qin* was one of the traditional "four arts" that all scholar-gentlemen should master.

66 See, for instance, Lam (1998); Elman (2005); Schäfer (2011).

thought, usually translated as breath, air, or vital energy, it is perhaps not surprising that pitch pipes should play such a central role. One of the meanings of *lü* 律 is principle, order, rule, or law—the sounding manifestation of an ordering principle—not dissimilar to *kanōn* in the West, the alternative Greek term for the monochord, a "measuring rod," which can also mean "rule" or "standard."[67]

The material conditions here could not be more different from those of the monochord. The *12 lü* were made out of bamboo cane, which meant that there actually needed to be twelve individual pipes, all at perfect lengths. Once they were cut, there was no changing them (certainly no lengthening them). Unlike the monochord, they could not be divided into various parts, as each pipe produces one sound.[68] Comparison between *lü* was a fixed affair: The length of the first pitch pipe determined the others in fixed rotation, without any flexibility.

Because of their rigid material and the necessity to produce a different pipe for each part of the sounding sequence, the pitch pipes only worked at one specific pitch level at any given time. That means the intervals that are being theorized here are all tied to particular frequencies, as articulated by the specific pipe lengths (and diameters). For a monochord, by comparison, it doesn't matter whether the string is long or short; string divisions work either way, at any pitch. The monochord was used primarily to demonstrate intervals as ratios, so the pitch level of both constituent tones was immaterial—what counted was their proportion to one another.[69] As a consequence, pitch names in the West were only associated with specific frequencies relatively late; identifying the relationship between them was considered a major scientific problem that was not solved until Robert Hooke presented his wheel to the Royal Society in 1681.[70] In China, by contrast, setting an absolute standard was fundamental to any act of theorizing. This absolute pitch standard was the Yellow Bell or *huang zhong* 黃鐘 that we encountered earlier. And this is, finally, the reason it was not quite correct to identify the Yellow Bell with C, as I did in Example 2 above, at least not in the sense of a frequency associated with C4, at 262 Hz (contingent on A4 440 Hz).[71]

To make matters more complicated, the specific acoustics of bamboo pipes is more complex than most physics textbooks care to admit: The standing wave that is set up inside the pipe when air is blown over it is not identical in length with the bamboo pipe itself. Its final node rests outside the pipe, making the standing wave slightly longer than the pipe itself. As a consequence, dividing the pitch pipes into 4:3 and 2:3 of each predecessor cannot yield accurate results. Add to that the annoying complicating factor that most bamboo pipes are not perfectly straight, and you get a sense of the practical problems with the *12 lü*. Later generations found a practical way of solving this physical problem by slightly modifying the bore of the pitch pipe, which made the (outer) proportional lengths of the pipes commensurate with the intervals they sounded.[72]

The system of *12 lü* may sound complicated and rigid. Why would anyone follow such an impractical system? Why would anyone theorize around an absolute pitch standard? Example 4 outlines the principal differences between the ways in which the *12 lü* and the monochord operate. As the Sinologist Erica Fox Brindley countenances, "the study of the pitch-standards is more than the study of disparate tones issued forth by pitch-bearing instruments."[73] The wider relevance of the pitch pipes has to be seen in the cultural context in which they operated. From that perspective, bamboo pipes—one of the classic materials that constituted the "eight sounds" (*ba yin* 八音) of traditional Chinese instrumental classification—also had certain material advantages that made themselves felt in the political organization of the Chinese Empire.

An absolute frequency standard, after all, was not just a complication, but also an opportunity: Setting the pitch of the Yellow Bell was no less than an imperial privilege, going back over many centuries. The Han-dynasty historian Sima Qian 司馬遷 (ca. 145–86 B.C.E.) wrote in his treatise on the pitch standards: "When kings govern affairs and establish laws, things attain their measure through rules and regulations and are uniformly endowed by the six pitch-standards. The six pitch-standards are the roots of the myriad affairs of the world."[74] Needless to say, a new emperor would never say that they altered the standard, or set it arbitrarily, but rather that they *corrected* the standard in order to come closer to the heavenly harmony. The Yellow Bell sets the standard at one

67 More specifically, in *lülü* 律呂 (pitch pipes/music theory), *lü* 律 refers to the odd-numbered pipes, whereas *lü* 呂 refers to the even-numbered pipes, in accordance with the yin-yang dualism of Daoist–Confucian philosophy.

68 The study of musical measuring devices was proposed in Hornbostel (1928), who was hoping to place it at the basis of cultural and comparative history. Hornbostel was firmly associated with the comparative musicology against which modern ethnomusicology defined itself, see note 59.

69 See Creese (2010).

70 See Jackson (2011).

71 It may be confusing that a pitch called "bell" might be determined by a bamboo pipe. The tradition is complicated and murky, but it goes back to a foundation myth that has a similar significance as the story of Pythagoras in the smithy in the West, described in the *Lüshi chunqiu* 呂氏春秋 (ca. 239 B.C.E.). Ling Lun 泠倫, the legendary founder of music, was tasked by the Yellow Emperor to bring order to music. Ling Lun went West to cut bamboo at the right length. Out in the field he heard the cry of a phoenix and captured the sound in bamboo flutes. He listened to the singing of the male and female phoenix and divided the twelve pitch-pipes into two halves, six female, six male, in accordance with the Daoist principles of *yin* and *yang*. For those who took the story literally, this act of foundation happened in the year 2698 B.C.E., in the prehistoric time associated with the mythical Yellow Emperor.

72 See Fei (2008).

73 Brindley (2012, 65).

74 Brindley (2012, 78). On the number six for pitch standards see note 67 above. It is noteworthy that the equivalent of the Do–Re–Mi system in the Chinese pentatonic scale is explicit about its political nature: *gong* (ruler), *shang* (minister), *zhi* (intendant), *yu* (wings), *jue* (horn), see Major (1994, 122).

Feature	12 lü	Monochord
Physical model	Air column (closed tube)	Vibrating string
Material	Bamboo pipes	Silk string (qin) or gut string (traditional monochord)
Operating principle	Fixed lengths, individual pipes for each sound	Variable length, one string divided
Measurement principle	Operates with fixed pitches	Measures only intervals between pitches
How to read	Lengths not exactly proportional to sound (requires bore adjustment)†	Lengths proportional to sound
Prerequisites	Unified standard pitch	No pitch standard
	Measure of length standardized	Length irrelevant

† To be clear, in many theories this discrepancy goes unacknowledged. They simply took the 2:3 and 4:3 aggregates and prescribed them as the ratios that corresponded to the pipes, without being concerned about the adjustments that needed to be made in practice. Zhu Zaiyu complained in Chapter 5 of his *Lülü jingyi* 律呂精義 that previous scholars had failed to pick up on this: "The diameters of the pitch pipes are not all the same" 不取圍徑皆同. (Thanks go, again, to Lester Hu.)

example 4. *Properties of pipe- and string-based music-theoretical instruments*

particular sound, a specific frequency. The advantage of this manifest yardstick and its absolute pitch levels—which also meant absolute length—was that it could, indeed that it had to, be unified in all parts of the Middle Kingdom. This was certainly one way in which "harmony" could be created in a very literal way across the vast geographical distances of the empire. To rule the Yellow Bell meant no less than to rule over time and space. It was truly an imperial mandate.

It would be wrong, however, to think of this only from the narrow perspective of music theory. In all likelihood, most emperors were not terribly interested in the specific acoustical qualities of the Yellow Bell. The pitch standard was part of a broader metrological project.[75] By setting the pitch standard the same all over the empire, they set the Yellow Bell as a measure of length. The Yellow Bell was traditionally defined as 9 cun in length, 3 fen in inner diameter, and 9 fen in outer circumference.[76] Moreover, the Yellow Bell could also serve as a measure of volume, fitting 1,200 grains of millet inside the tube. In other words, the pitch pipe system was predestined to work within an imperial political system in which it was highly desirable to unify all measurements. The whole empire resonated at the same wavelength, at the sound of the Yellow Bell.[77]

A monochord, the traditional music-theoretical instrument of choice in the West, by contrast, has no way of fixing its pitch. Its strength, by contrast, is in making manifest the abstract relations between pitches. The geometric principles of dividing the string, to which it gives concrete shape in sound, are particularly powerful because of the flexibility they afford. Where each pitch pipe was one specific, unalterable length—which is why a set of all *12 lü* is needed—the monochord could be shown to contain a number of different pitches within it; alterations at the microtonal level were easy to demonstrate on the monochord with its moveable bridges.

This is what Zhu Zaiyu realized.[78] There was a certain advantage, for this particular situation, in the abstraction offered by the *qin*, which was, for all intents and purposes, nothing other than a multi-stringed monochord. In the initial report on Zhu's work in the West, from the *Mémoire de la musique des Chinois*, Amiot does in fact identify Zhu's *qin* as a thirteen-stringed monochord, in which the final string doubles the initial pitch at the octave.[79] Pegs at the appropriate divisions of the thirteen strings marked the pitches for 12-tone equal temperament. It's worth remembering that traditional *qins* had

75 For an important historical precedent of reorganizing the imperial metrology, including music theory, in the service of political unity at the beginning of the Sui dynasty (sixth century C.E.), see Huang (2018).

76 See Lam (1998); Bodde ([1981] 2014); Fei (2008). Careful readers will note that the diameter/circumference ratio gives a very approximate value for π.

77 As seen in note 65 above, there is evidence that the name Yellow Bell is not purely poetic, but that at times actual bronze bells were used to determine the pitch standard. This may have had certain advantages: Bells are less susceptible to variations of embouchure, which was a notorious practical problem of the pitch pipe system. If we are simply interested in pitch questions, musical scales narrowly confined, perhaps expanded into ceremonial and representational issues of statehood, then bells are the way to go. If we are interested, however, in how sound connects with other areas of governance, the practical disadvantages of pitch pipes are greatly outweighed by their advantages.

78 On the historical uses of scientific instrument building in the teaching of mathematics in Ming-dynasty China, as reported by Ricci, see Jami (2009, 63).

79 Amiot (1779, 155n).

seven strings—Zhu's instrument was explicitly designed as a music-theoretical device.

Significantly, Zhu argued not that equal temperament was a form of tampering with pitches, but rather that the old pitch pipes had given faulty values. By contrast, he argued that it was his *qin* that finally revealed the true pitch levels, and that this was in fact ancient knowledge, albeit transmitted orally. The flexibility of strings was precisely of the kind needed for the mathematical approach that Zhu Zaiyu operated with in the realm of numbers. But this flexibility came at a cost; the very worldly order of equal temperament could only be attained on the basis of a shift toward a different mode of thinking, by adopting a different geometry and by using an instrument that could work in infinitesimal gradations.

It's hard to overestimate the iconoclastic spirit behind this shift. Zhu referred to his own approach, consisting of equal temperament, the reconstructed metrology standards, and the "end correction" of the pitch pipes, as the "New Method" 新法. This term is programmatic. His magnum opus *Lülü jingyi* 律呂精義 opened with a series of chapters that amounts to a systematic takedown of every received wisdom of traditional Chinese music theory. Against the weight of tradition, his claim that the 4:3 and 2:3 numbers of the traditional up-and-down method passed down from the ancients through the texts were actually just approximations amounted to an epistemological thunderbolt. In order to restore the perfect tuning of the ancients, Zhu effectively argued, it is necessary to turn away from the texts and scholarly authority toward musical practitioners, specifically "common artisans," *qin* makers, who have transmitted their wisdom "by word of mouth" and "handed down" a system that is "not recorded in literary works" (to summarize the block quotation from the previous section). Put differently, the ratios of the music-theoretical tradition should not be understood as the abstract, cosmic numbers of the neo-Confucian tradition, but as concrete, even embodied sounds. It is surely not wrong to see Zhu Zaiyu's turn to strings—and to embodied processes—as situated within the broader intellectual atmosphere of *shixue* 實學 or "concrete learning" during the late Ming period. *Shi* 實, meaning "concrete," is juxtaposed to the supposed *xu* 虛 ("void" or "hollow") learning of neo-Confucianism.[80]

Robinson and Needham explain in practical terms how Zhu Zaiyu tuned his thirteen-string *qin*, shown in Example 5.[81] The first and last string (no. 13) sound the octave, so the last string length must be precisely half that of string no. 1. The middle string, (7) can be measured at the geometric mean, which marks it at $\sqrt{2}$. The middle strings of each half, that is, strings nos. 4 and 11, are in turn at the $\sqrt{2}$ between middle string and outer strings. The remaining strings, each adjacent to at least one tuned string, could be tuned by approximation so as to fill in the gaps.

These music-theoretical instruments and Kuriyama's relativist concept of "divergences" may help us get out of the rut of the simplistic active/reactive model that characterizes so much of traditional scholarship about cultural transfer, and which takes us back to the internet wars with which I started. The notion of divergence is not interested in discoveries or inventions but in comparing how the same phenomenon can lead in radically different directions when it's approached from different directions, experienced through different technologies, and embedded in different philosophical contexts—or rather, heard through different music-theoretical instruments.

It is worth underlining that the divergence studied here was not one that primarily contrasted European versus Chinese traditions. Instead, it highlighted two different practices—12 *lü* and *qin*—that Zhu both found within Chinese music. Or, even more broadly, the divergence explored the consequences of sound production in tubes and in strings. As Example 4 represented diagrammatically, bamboo pitch pipes are emblematic of one principle, the monochord of the other. That Zhu's switch to the *qin*, away from the *12 lü*, may have also brought him closer to the monochord tradition within which Western music theory typically operated was of no consequence within Zhu's work. He did not have to look to Europe here, he could define his own monochordal tradition by considering the mechanism of the *qin*.

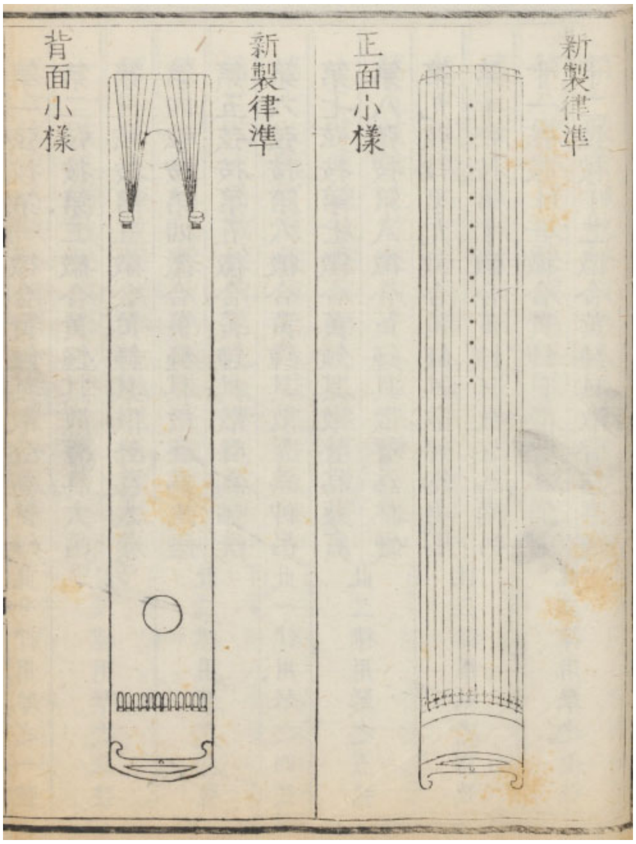
If there is a methodological pitfall of music-theoretical instruments it would be the risk of plunging into technological determinism, which would be the flipside of Robinson's and Cho's extreme reliance on people-to-people contact, and just as fallacious.[82] So it is important to remember that not everything that we *can* do with things will also necessarily be done with these things. Music-theoretical instruments are no more—but also no less—than material devices that open up certain sonic perspectives, while foreclosing certain others. Put simply, the *12 lü* can create a sonic reality in ways that the monochord cannot, and vice versa. Kuriyama's concept of divergence offers us one fruitful model by which to carry out such comparisons. Its exploration of an apparently universal or heuristically natural phenomenon, whether it's the pulse or the wolf-tone problem, both maintains the radical comparability between cultures and allows us to gain a better understanding of the dynamically shifting differences between them. A history of music theory within a global framework doesn't need to be fixated on cultural transfer.

When Amiot earlier explained that all Pythagoras had to do was "to apply to strings what the Chinese had done on pipes," the conceptual basis for Zhu's calculation, crucially,

80 See also Elman (2005, 79–80), on Zhu's *shi* calendar reform. For shifting epistemic authorities during Ming see Vedal (2020). Thanks go, again, to Lester Hu.

81 Needham and Robinson (1962, 223–24).

82 Young (2011, 121) pointed out that the catchphrase has such toxic connotations that accusing someone of "technical determinism" is tantamount to intimating they enjoy strangling puppies in the basement. No further questions are asked, the topic is usually dropped. Peters (2017) bravely offers a cautious rehabilitation of the term.



EXAMPLE 5. *Zhu Zaiyu's new thirteen-string qin, a sketch of back and front of the instrument. From Zhu Zaiyu* 朱載堉. *Yue lü quan shu* 樂律全書. *c. 1596, vol. 1, seq. 34. Harvard Yenching Library, Rare Book T 6730 2944. (Printed between 1736 and 1861.) https://iiif.lib.harvard.edu/manifests/view/drs:15498530$34i*

does the opposite: It abandons the pipes and opts for a string model instead. Only by giving up the "old" model—to use Zhu's terms—of the pitch pipe did he come up with the "new" solution, which he then reapplied to (newly adjusted) pitch pipes. The mechanism of strings provided the answer Zhu needed.

CODA: MIGRATORY THINGS

The American founder of the History of Ideas, Arthur Lovejoy, was convinced that "ideas are the most migratory things in the world."[83] In the case of equal temperament it is not obvious that this is so—unless we take the sentence overly literally. In that case, yes, ideas become migratory *as things*.

Davis Baird offers a useful term, "thing knowledge," to describe how knowledge transfer takes place by means of tinkering, playing around with machines.[84] He is particularly interested in an engineering-like approach to knowledge that adopts mechanisms from one machine to apply it to another machine, even where it does not develop an elaborate theoretical apparatus around it. "Thing knowledge" builds on practical applications, scientific know-how, or *technē*—on a form of implicit knowledge, in other words, that describes Zhu's old *qin* makers and the principles and practices they had passed down the generations, even when they did not explicitly theorize it. This hands-on application of scientific knowledge, from machine to machine, or from instrument to instrument, may have particular resonances in the thornier question of cross-cultural comparisons, where the traditional focus on treatises and its concomitant philological methods will only get us so far.[85]

Thus it is not irrelevant that the same kind of "thing knowledge" was also passed down through the generations among Western lutenists. Gioseffo Zarlino, for one, included a technical description of the mesolabe, an ancient geometric device for determining any number of mean proportions between two given lengths, in his *Istitutioni harmoniche* which could be used to tune a lute in equal temperament.[86] In his later *Sopplimenti musicali*, Zarlino included a detailed geometric demonstration on the equal-tempered tuning of a lute.[87] Nothing would be farther off the mark than to try and construct a direct person-to-person link between these parallel traditions in Ming China and early modern Italy. The critical point is that the knowledge about tuning is intimately linked with the material givens of the object, and that the project of theory-making in this scenario is tantamount to a process of reverse engineering.[88]

The same process of reverse engineering is also the reason, in this History of Ideas-as-things, that the location from which the divergences on an intangible concept such as equal temperament take their starting point are anchored first and foremost in music-theoretical instruments, and from there in practices.[89] It is here that their cultural significance begins to unfold in different contexts across time and space. As the history of music theory grapples with questions of global reach, a context, that is, in which Lovejoy's "migratory things" are of central importance, the study of divergences may shed light on problems where other forms of comparison have fallen short.

WORKS CITED

Amiot, Jean Joseph Marie. 1779. *Mémoire sur la musique des Chinois, tant anciens que modernes*. Ed. Pierre-Joseph Roussier. Paris: Nyon.

83 Lovejoy (1940, 4).
84 Baird (2004).
85 For a different approach to moving the history of music theory beyond the traditional focus on treatises see Christensen (2011).
86 Zarlino (1558, 2: 94–96); also see Lindley (1984, 25–26).
87 Zarlino (1588).
88 This is where the study of music-theoretical instruments converges with approaches in media archaeology; see Ernst (2016) and Rehding (2016b).
89 In a media-theoretical context (see previous note) these kinds of practices are sometimes referred to as "cultural techniques"; see Krämer and Bredekamp (2003), and Siegert (2015).

Appiah, Kwame Anthony. 2016. "There is No Such Thing as Western Civilisation." *The Guardian*, November 9, 2016. https://www.theguardian.com/world/2016/nov/09/western-civilisation-appiah-reith-lecture.

Baird, Davis. 2004. *Thing Knowledge: A Philosophy of Scientific Instruments*. Berkeley: University of California Press.

Barbour, J. Murray. 1951. *Tuning and Temperament: A Historical Survey*. East Lansing: Michigan State University Press.

Barker, Andrew. 1989. *Greek Musical Writings. Vol. 2: Harmonic and Acoustic Theory*. Cambridge: Cambridge University Press.

Bernstein, Michael. 1994. *Foregone Conclusions: Against Apocalyptic History*. Berkeley: University of California Press.

Bodde, Derk. (1981) 2014. "The Chinese Cosmic Magic Known as Watching for the Ethers." In *Essays in Chinese Civilization*. Eds. Charles Le Blanc and Dorothy Borei. 351–72. Princeton: Princeton University Press.

Borio, Gianmario. 2007. *Storia dei concetti musicali*. Rome: Carocci.

Brindley, Erica Fox. 2012. *Music, Cosmology, and the Politics of Harmony in Early China*. Albany: State University of New York Press.

Brockey, Liam M. 2007. *Journey to the East: The Jesuit Mission to China*, 1579–1724. Cambridge, MA: Harvard University Press.

Chalabi, Mona. 2014. "The 100 Most Edited Wikipedia Entries." *FiveThirtyEight*. https://fivethirtyeight.com/features/the-100-most-edited-wikipedia-articles/.

Chen, Joseph C. Y. 1996. *Early Chinese Work in Natural Science*. Hong Kong: Hong Kong University Press.

Cho, Gene. 2003. *The Discovery of Musical Equal Temperament in China and Europe in the Sixteenth Century*. Lewiston, NY: Edwin Mellen Press.

Christensen, Thomas. 2011. "Fragile Texts, Hidden Theory." *Musica Humana* 3 (2): 177–207.

Chua, Daniel K. L. 2001. "Vincenzo Galilei, Modernity and the Division of Nature." In *Music Theory and Natural Order from the Renaissance to the Early Twentieth Century*. Eds. Suzannah Clark and Alexander Rehding. 1–39. Cambridge: Cambridge University Press.

Clarke, David. 2010. "An Encounter with Chinese Music in Mid-18th-Century London." *Early Music* 38 (4): 543–58.

Cohen, H. F. 1984. *Quantifying Music: The Science of Music at the First Stage of the Scientific Revolution*, 1580–1650. Dordrecht: D. Reidel.

Creese, David. 2010. *The Monochord in Ancient Greek Harmonic Science*. Cambridge: Cambridge University Press.

Currie, Gabriela. 2019. "Sonic Entanglements and the Gandhāran Nexus." In *The Music Road: Coherence and Diversity in Music from the Mediterranean to India*. Ed. Reinhard Strohm. 41–70. London: British Academy Publications.

Dai, Nianzu 戴念祖. 2011. 朱载堉 / 明代的科学和艺术巨星 [*Zhu Zaiyu: Scientific and Artistic Superstar of the Ming dynasty*]. Revised edition. Beijing: Renmin Chubanshe.

Daston, Lorraine, and Peter Galison. 2007. *Objectivity*. New York: Zone Books.

Dehaisnes, Chrétien. 1864. *Vie du Père Nicolas Trigault*. Paris: Casterman.

Duffin, Ross. 2006. *How Equal Temperament Ruined Harmony (And Why You Should Care)*. New York: Norton.

Eggebrecht, Hans Heinrich. 2012. *Handwörterbuch der musikalischen Terminologie*. Ed. Markus Bandur. Stuttgart: Franz Steiner.

Elman, Benjamin A. 2005. *On Their Own Terms: Science in China 1550–1900*. Cambridge: Harvard University Press.

Ernst, Wolfgang. 2000. *Medium Foucault: Weimarer Vorlesungen über Archive, Archäologie, Monumente und Medien.* Weimar: Verlag und Datenbank der Geisteswissenschaften.

——— 2016. *Sonic Time Machines, Explicit Sound, Sirenic Voices, and Implicit Sonicity*. Amsterdam: Amsterdam University Press.

Ertelt, Thomas, and Frieder Zaminer, Eds. 1987. *Geschichte der Musiktheorie*, vol. 6: *Rechnen, Messen, Hören in der frühen Neuzeit*. Darmstadt: Wissenschaftliche Buchgesellschaft.

Falkenstein, Lothar von. 1994. *Suspended Music: Chime-Bells in the Culture of Bronze Age China.* Berkeley: University of California Press.

Fei, Xu. 2008. "Exploration and Achievements of the Acoustic of *Yuelü*, the Theory of the Tone System in Ancient China." In *Variantologie* 3. Eds. Siegfried Zielinski and Silvia M. Wagnermeier. 252–92. Wiesbaden: Kunstverlag Kadmos.

Fokker, Adriaan. 1944. *Rekenkundige Bespiegeling der Muziek*. Gorinchem: Noorduijn.

Forkel, Johann Nikolaus. 1783. "Von der Musik der Chineser." In *Musikalischer Almanach für Deutschland auf das Jahr 1784*. 233–74. Leipzig: Schwickert.

Galilei, Vincenzo. (1581) 2003. *Dialogue on Ancient and Modern Music.* Trans. Claude V. Palisca. New Haven and London: Yale University Press.

Hijleh, Mark. 2012. *Towards a Global Music Theory: Practical Concepts and Methods for the Analysis of Music across Human Cultures*. New York: Routledge.

Hornbostel, Erich M. von. 1928. "Die Maßnorm als kulturgeschichtliches Forschungsmittel." In *Festschrift, Publication d'hommage offerte à P. W. Schmidt*. Ed. Wilhelm Koppers. 303–23. Vienna: Mechitaristen-Congregations-Buchdruck.

Hsia, Ronnie Po-chia. 2016. *Matteo Ricci and the Catholic Mission to China: A Short History with Documents*. Indianapolis, IN: Hackett.

Huang, Rujing. 2018. "Storms in Chang-an: On the Debate of Kai-huang Period." *History of Music Theory* (blog). https://historyofmusictheory.wordpress.com/2018/04/11/storms-in-chang-an-on-the-music-debate-of-kai-huang-period/.

Huynh, Dan. 2012. *Musical Equal Temperament and its Inventor Zhu Zaiyu*. Self-published.

Irvine, Thomas. 2020. *Listening to China*. Chicago: University of Chicago Press.

Isacoff, Stuart. 2003. *Temperament: How Music Became a Battleground for the Great Minds of Western Civilization*. New York: Vintage.

Jackson, Myles W. 2011. "From Scientific Instruments to Musical Instruments: The Tuning Fork, Metronome, and Siren." In *The Oxford Handbook of Sound Studies*. Eds. Trevor Pinch and Karin Bijsterveld. 201–22. New York: Oxford University Press.

Jami, Catherine. 2009. "Heavenly Learning, Statecraft, and Scholarship: The Jesuits and their Mathematics in China." In *The Oxford Handbook of the History of Mathematics*. Eds. Eleanor Robson and Jacqueline Stedall. 58–84. New York: Oxford University Press.

Krämer, Sybille, and Horst Bredekamp, Eds. 2003. *Bild—Schrift—Zahl*. Munich: Wilhelm Fink.

Kuriyama, Shigehisa. 2002. *The Expressiveness of the Human Body and the Divergence of Greek and Chinese Medicine*. New York: Zone Books.

Kuttner, Fritz. 1975. "Prince Chu Tsai-Yü's Life and Work: A Re-Evaluation of his Contribution to Equal Temperament Theory." *Ethnomusicology* 19 (2): 163–206.

Lam, Joseph. 1998. *State Sacrifices and Music in Ming China: Orthodoxy, Creativity and Expressiveness*. Albany: State University of New York Press.

Li, Guangmin. 2017. "Chromatic Scale Construction in Ancient China." *History of Music Theory* (blog). https://historyofmusictheory.wordpress.com/2017/04/12/chromatic-scale-construction-in-ancient-china/.

Lindley, Mark. 1984. *Lutes, Viols, and Temperament*. Cambridge: Cambridge University Press.

Lloyd, G. E. R. 1990. *Demystifying Mentalities*. Cambridge: Cambridge University Press.

——— 2004. *Ancient Worlds, Modern Reflections*. Oxford: Oxford University Press.

Lovejoy, Arthur. 1940. "Reflections on the History of Ideas." *Journal of the History of Ideas* 1 (1): 3–23.

Major, John S. 1994. "Celestial Cycles and Mathematical Harmonics in the *Huainanzi*." *Extrême–Orient, Extrême–Occident* 16: 121–34.

McClain Ernest G., Ming Shui Hung. 1979. "Chinese Cyclic Tunings in Late Antiquity." *Ethnomusicology* 23 (2): 205–24.

Mersenne, Marin. 1636–37. *Harmonie universelle*. 2 vols. Paris: S. Cramoisy.

Millward, James. 2018. "The Silk Road and the Sitar: Finding Centuries of Socio-cultural Exchange in the History of an Instrument." *The Journal of Social History* 52 (2): 206–33.

Moyn, Samuel, and Andrew Sartori. 2015. *Global Intellectual History*. New York: Columbia University Press.

Müller, Jan-Werner. 2014. "On Conceptual History." In *Rethinking Modern European Intellectual History*. Eds. Darrin M. McMahon and Samuel Moyn. 74–93. Oxford: Oxford University Press.

Needham, Joseph, and Kenneth Robinson. 1962. "Sound (Acoustics)." In *Science and Civilisation in China*. Ed. Joseph Needham. Vol. IV: *Physics and Physical Technology, Part 1: Physics*. 126–228. Cambridge: Cambridge University Press.

Nielsen, Bent. 2003. *A Companion to Yi Jing Numerology and Cosmology*. New York: Routledge.

Osterhammel, Jürgen. (2009) 2015. *The Transformation of the World*. Princeton: Princeton University Press.

Parkhurst, Bryan, and Stefan Hammel. 2020. "Pitch, Tone, and Note." In *The Oxford Handbook of Critical Concepts in Music Theory*. Eds. Alexander Rehding and Steven Rings. 1–39. New York: Oxford University Press.

Pesic, Peter. 2010. "Hearing the Irrational: Music and the Modern Development of the Concept of Number." *Isis* 101 (3): 501–30.

Peters, John Durham. 2017. "'You Mean My Whole Fallacy Is Wrong': On Technological Determinism." *Representations* 140 (1): 10–26.

Raphals, Lisa. 1992. *Knowing Words: Wisdom and Cunning in the Classical Traditions of China and Greece*. Ithaca, NY: Cornell University Press.

Rasch, Rudolf. 2008. "Simon Stevin and the Calculation of Equal Temperament." In *Music and Mathematics*. Ed. Philippe Vendrix, 253–320. Turnhout: Brepols.

Raz, Carmel, David E. Cohen, Roger Matthew Grant, Nathan John Martin, Caleb Mutch, Melanie Wald-Fuhrmann, Felix Wörner, and Anna Zayaruznaya, 2019. "Going Global, in Theory." *Musicological Brainfood* 3 (1): 3–5.

Rehding, Alexander. 2014. "Music-Historical Egyptomania 1650–1950." *Journal of the History of Ideas* 75 (4): 545–80.

——— 2016a. "Instruments of Music Theory." *Music Theory Online* 22.4. https://mtosmt.org/issues/mto.16.22.4/mto.16.22.4.rehding.html.

——— 2016b. "Three Music-Theory Lessons." *Journal of the Royal Musical Association* 141 (2): 251–82.

Robinson, Kenneth. 1980. *A Critical Study of Chu Tsai-yü's Contribution to the Theory of Equal Temperament in Chinese Music*. Wiesbaden: Franz Steiner.

Schäfer, Dagmar. 2011. *The Crafting of the 10,000 Things*. Chicago: University of Chicago Press.

Siegert, Bernhard. 2015. *Cultural Techniques: Grids, Filters, Doors, and Other Articulations of the Real*. Trans. Geoffrey Winthrop-Young. New York: Fordham University Press.

Spence, Jonathan. 1984. *The Memory Palace of Matteo Ricci*. New York: Viking.

Stevin, Simon 1605. *Wisconstige gedachtenissen*. 2 vols. Leiden: Jan Bouwenszoon.

——— 1884. *Vande spiegeling der singconst and Vande molens*. Ed. D. Bierens de Haan. Amsterdam: D. Bierens de Haan.

Tenzer, Michael, Ed. 2006. *Analytical Studies in World Music*. New York: Oxford University Press.

Tresch, John, and Emily Dolan. 2013. "Toward a New Organology: Instruments of Music and Science." *Osiris* 28 (2): 278–98.

Trigault, Nicolas. 1953. *China in the Sixteenth Century: The Journals of Matthew Ricci, 1583–1610*. Trans. Louis Joseph Gallagher and Richard Cushing. New York: Random House.

Weber, Max. 1958. *The Rational and Social Foundations of Music*. Trans. Don Martindale, Johannes Riedel, and Gertrude Neuwirth. Carbondale: Southern Illinois University Press.

Vedal, Nathan. 2020. "From Tradition to Community: The Rise of Contemporary Knowledge in Late Imperial China." *Journal of Asian Studies* 79 (1): 77–101.

Wellmann, Janina. (2010) 2017. *The Form of Becoming: Embryology and the Epistemology of Rhythm, 1760–1830.* Trans. Kate Sturge. New York: Zone Books.

Woo, Shingkwan. 2018. "The Theory of 12-Tone Equal Tuning: Its Coincidental but Independent Invention in China and Europe." Paper presented at *Music Between China and the West in the Age of Discovery*, 14–16, May, Chinese University of Hong Kong.

Young, Geoffrey Winthrop. 2011. *Kittler and the Media*. Cambridge: Polity Press.

Zarlino, Gioseffo. 1558. *Le istitutioni harmoniche*. Venice: Gioseffo Zarlino.

Zhu, Zaiyu. 朱載堉. (1596) 1983. 律吕精义 Lü lü jing yi. In 樂律全書 *Yue lü quan shu.* In 42 juan. [Taipei]: Taiwan shang wu yin shu guan.

Zielinski, Siegfried, and Silvia Wagnermeier, Eds. 2005. *Variantology: On Deep Time Relations of Arts, Sciences, and Technologies*. Cologne: W. König.

Music Theory Spectrum, Vol. 44, Issue 2, pp. 260–75, ISSN 0195-6167, electronic ISSN 1533-8339.
https://doi.org/10.1093/mts/mtac004