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Dualist tonal space and transformation in nineteenth-century musical thought

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Introduction: tonal systems without scales

Nineteenth-century music theory in German-speaking countries divides reasonably into two main traditions: thorough-bass styles of music theory and harmonic dualism. The approaches are usually thought of nowadays as scale-degree theory and functionalism, respectively; since the emphasis in the account here is on chord structure and chordal relations as expressions of such structure, the traditions are characterized so as to foreground these particular aspects in their approach.

Interestingly, by the last half of the nineteenth century, the two traditions had become connected to different geo-political formations in Central Europe, such that we may properly speak of thorough-bass theory as Viennese (or more generally, Austrian) and harmonic dualism as Prussian, in the sense that these approaches were developed or extended within the context and dynamic of relevant educational institutions and their corresponding research *ethoi* in those two areas.[1] A third major tradition, the fundamental-bass theory emanating from the work of Rameau, was more international in scope and influence. In spite of obvious dissimilarities, it was considered by harmonic dualists (in particular, Riemann) to form an important early articulation of a number of theoretical concepts basic to their own approach, a judgment shared less positively by Heinrich Schenker, who saw Riemann's approach to tonality as little more than warmed-over Rameau.[2] This particular alignment of approaches seems based entirely on whether one held that the structure-forming relations within chords could withstand registral rearrangement (as both Riemann and Rameau did) or not (as asserted by thorough-bass theorists).

The thorough-bass tradition of music theory has its institutional origins in the late feudal/early modern institution of the Kapellmeister system of central Europe and extends as an identifiable theoretical movement roughly from the work of Heinichen to that of Sechter and late nineteenth-century Viennese theory in general, including

1 A particularly useful examination (in English) of German universities in the nineteenth century is McClelland, *State, Society and University in Germany*. 2 **See Chapter 26, pp. 832–33.**

Schenker's.[3] The basic tenets of the approach remained generally stable throughout this period, although there were important attempts to update the tradition in the second half of the century, none of which gained even local or partial acceptance. A particularly interesting example is the work of Heinrich Joseph Winzenhörlein (1819–1901), who, under the pen name Heinrich Joseph Vincent, complained about thorough-bass theory's failure to accommodate chromatic music and to take into account the phenomenon of the tonal center's absolute dominance in music. Yet even more importantly for him, the tradition was hopelessly entangled with primitive keyboard temperament schemes. His principal work, *Die Einheit in der Tonwelt*, advances a detailed revision – although Vincent himself saw it as a repudiation – of Sechter's version of thorough-bass theory, a revision that assumes equal temperament and, accordingly, twelve chromatic scales. Vincent furthermore proposes that all figures be calculated from the contextual tonic rather than from the bass pitch of each chord. The former figures represent what he calls "absolute intervals"; the latter, traditional figures are in his view merely "incidental intervals."[4]

As an approach – and this is as true of Vincent as it is of Heinichen or Sechter – thorough-bass theory might reasonably be characterized as principally scale-based, in the sense that it begins by taking as a *donnée* the concept of scale – conceived as a collection of pitch-classes with a corresponding scheme of structural differentiation among its members – and developing from it all other pitch elements, particularly chords, their internal structure, and their interrelations. In effect the scale represents the originary, imaginary topography within which tonal music is to be conceived. The topography or space projected by scales, though unidimensional, is quite clearly derived from the material space of instrumental construction (itself emerging from modal conceptions of melodic systems as well as from acoustic properties of air flowing through metal or wooden pipes).

The second music-theoretical tradition, harmonic dualism, is the starting point for the present chapter. Unlike thorough-bass theorists, almost all of those belonging to this tradition took seriously the Prussian physicist Hermann von Helmholtz's materialist and empiricist research project – established not only in his well-known *Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik* of 1863 (translated as *On the Sensations of Tone as a Physiological Basis for the Theory of Music*) but also in his work on optics and color theory – which involved using physiology (studied according to the research protocols of physics) as a point of departure. If scales appear in the writings of these theorists, they do so not as a foundational concept, but rather as a product of other procedures. Accordingly, the class of theoretical topoi within

3 Discussed in Wason, *Viennese Theory*. Also see **Chapter 25, pp. 788–94.**

4 Wason's "Progressive Harmonic Theory" represents, as far as I can tell, the first treatment of Vincent in English. For more on Vincent, see **Chapter 10, p. 286.**

which such approaches originate is markedly different from the unidimensional locus of pitch(-classes) projected in scale-based theory.

The first major articulation of harmonic dualism as a full-blown theory of music was put forward by yet another Prussian-based physicist, Arthur von Oettingen,[5] who took the results of Helmholtz's work on the physiology and acoustics relevant to music, and synthesized it with features found in the more traditionally articulated harmonic-theoretical work of Moritz Hauptmann;[6] the second major impulse is the more influential work of Hugo Riemann, who repackaged the work of Oettingen for use in the recently established professional programs in conservatories and universities, and whose approach – or at least, aspects of it – dominated continental music theory well into the twentieth century.

This chapter examines the theoretical approaches developed by Hauptmann, Oettingen, and Riemann, with a particular emphasis on the issues of chord structure and chord relations or transformations. In doing so I shall give a sympathetic account of harmonic dualism as a structural premise and as a historical development. An examination of associated topographies of chords, topographies whose dimensions are articulated by transformations, follows.

Klangs: monism and dualism

Almost all tonal theorists have proposed that triadic structure arises from a fundamental, conceptually anterior, constituent pitch – such as *radix*, *son fondamental*, *Grundton*, *Hauptton* – that exerts unity on the collection by means of an array of intervallic relationships sanctioned by Nature (through, say, various properties of string vibrations or harmonic overtones) or, less commonly, by convention or practice, that is, history. (See **Chapter 3, pp. 85–91** for further discussion of this question.) Theorists have disagreed, however, on the factors that could determine the dominant pitch in triads, the intervallic relationships that ought to be privileged, and the manner in which these considerations are deployed in triadic structure.

In classifying this kind of theoretical work, it has become commonplace to established a primary opposition between Rameauian – that is, pertaining to Rameau of the *Traité de l'harmonie* – fundamental-bass procedures and the operations of figured-bass

5 Earlier attempts by Rameau (*Génération harmonique*, 1737) and Goethe in his *Tonlehre* (1815) seem not to have made much impression on their contemporaries or immediate followers. For more on Rameau's proto-functional theories and their progeny in the eighteenth century, see **Chapter 24, pp. 768–70, 774**.
6 It needs to be remembered that however inspirational Oettingen (and Riemann) found Hauptmann's work to be in connection with Helmholtz's research, Hauptmann himself was dismissive of Helmholtz's writing on music, claiming that since Helmholtz failed to account for the role of psychology in the structuration of musical events and musical systems his work did not achieve the status of a proper music theory. Hauptmann's remarks are contained in the form of a letter to Otto Jahn, later published in 1863 ("Ein Brief M. Hauptmann's").

theorists, corresponding conceptually to an opposition between "harmony" and "counterpoint," which are construed in this context more as theoretical *ethoi* rather than properly structural categories. Under this view, the corresponding music-theoretical work of writers such as Oettingen, Hauptmann, and Riemann – all considered harmonic dualists to some degree – constitutes an unsuccessful peripheral tradition. It is safe to say that this view or some reliable variant of it serves as the dominant approach in Anglo-American theoretical circles. In other words, contemporary music-theoretical debate about triadic structure (to the extent that it actually takes place) is framed by a common acceptance – or better, the naturalization – of some variety of harmonic monism. The degree to which figured-bass and fundamental-bass protocols, all of which depend on scales as a point of departure, have been hypostatized by theorists is easily measured by the degree to which active discussion of premises – whether presented in cognitive or in structural categories – are either thoroughly mystified (ironically, by appeals to empiricist themes) or simply avoided altogether. Correspondingly, critiques of harmonic dualism are generally empty of content, and rely either on similar enactments of mystification or on sheer invocation of disciplinary sanction in order to reinforce the predominant orthodoxy of harmonic monism.

Before progressing any further, it is worthwhile clarifying the use of certain terminology. I take "harmonic monism" to represent categories of music-theoretical work that assume the abstract primacy of the major triad, which finds its concrete form in the acoustic structure of the overtone series or in the properties of certain advantaged integer ratios applied to string division; accordingly, the minor triad appears in such theories as a derivative, produced by History, or in the case of Schenker, by the true Subject of History, the Artist. I take "harmonic dualism" to represent categories of music-theoretical work that accept the absolute structural equality of major and minor triads as objects derived from a single, unitary process that structurally contains the potential for twofold, or binary, articulation. There are, of course, other procedures for formalizing a distinction between monist (of some kind) and dualist (of some kind) theories of triadic structure, but they do not engage the particular issues I am concerned with here.

Hauptmann. Moritz Hauptmann (1792–1868) published his most important work, *The Nature of Harmony and Metre*, in 1853. The commonplace characterization of his work as Hegelian and idealist is rather unhelpful, since it encourages an easy dismissal of Hauptmann's significant theoretical insights, in turn distorting a proper understanding of technical development within nineteenth-century North German music theory. Furthermore, singling Hauptmann out as an idealist distracts us from the styles of idealism underlying most approaches to music theory even in its current forms. And while Hauptmann himself regarded his work as Hegelian-dialectical in character, it is difficult to see the relations between his Categories as instantiations of properly dialectical progression, despite the stream of Hegelian code words.

Nevertheless, *The Nature of Harmony and Metre* sought to provide for the first time a natural rather than aesthetic basis for the foundational harmonic and metrical structural categories of music in both their subjective and objective extensions. On Riemann's view, Hauptmann thereby formulated music theory's dominated research project.[7]

In Hauptmann's dualistic model, there are three "functions" assigned to pitches that constitute major and minor triads (or as we will call them, following Hauptmann, "klangs"): unity (*Einheit*); duality or opposition (*Zweiheit*); union (*Verbindung*).[8] The functions or "Moments" (as Hauptmann prefers to call them) are respectively associated with the octave, the perfect fifth, and the major third, whose primacy he derives from string division. Labeling the three functions respectively I, II, and III for reference, Hauptmann assigns them to triad members according to two rules:

1. I and II form a perfect fifth (mod 8ve)
2. I and III form a major third (mod 8ve)[9]

The rules stipulate that only the pitch that acts as I or the *Einheit* participates in both the perfect fifth (mod 8ve) and the major third (mod 8ve) relationships. The octave relation regulates the two structural intervals by allowing them to appear modulo the octave, as inversions or compounds. In turn, the structural assignment of I, II, and III withstands registral rearrangement of triadic members.

Figure 14.1 demonstrates how Hauptmann, following these formulations, distributes the three symbols I, II, and III among the pitches that form a major triad. Figure 14.2 carries out on a minor triad the procedures for assigning the functions I, II, and III. Comparing the assignment of function labels in minor triads and major triads, Hauptmann analyzes the constitutive perfect fifths and major thirds as intervals directed upwards: in major klangs the two intervals extend respectively from I to II and from I to III; in minor klangs the two intervals extend respectively from II to I and from III to I. Furthermore, Hauptmann writes,

> [t]he determination of triadic intervals is . . . taken to proceed from a positive unity, from a fundamental tone, to which the fifth and third relate. They may be considered as opposed. If we express one by saying that a tone *has* a perfect fifth and major third, then we can express the other in the opposite sense that a tone *is* a perfect fifth and major third. Having is an active condition; being, passive. Both determinations in their two meanings relate to Unity which is subject, on one hand, to Having (*Haben*) in the first determination, and, on the other hand, to Being Had (*Gehabt-werden*) in the second. The first corresponds to the major triad; the second, the minor triad.[10]

7 Harrison, *Harmonic Function*, pp. 218–21. Harrison's work is an extremely interesting and thorough recounting of harmonic dualism beginning with an examination of the theorists discussed here, although with a somewhat different focus. For a discussion of Hauptmann's theories on meter and rhythm, see **Chapter 21, pp. 677–82**.

8 Hauptmann's remarks on chord structure appear in *Harmonik und Metrik*, pp. 25–35. *Klang* is technically the German word for "resonance" or "sound," although in this context it refers specifically to the ontological entities of major and minor triads, whether generated acoustically or logically.

9 This discussion is expanded in Klumpenhouwer, "Riemann Transformations," paragraph 9.

10 Hauptmann, *Harmonik und Metrik*, p. 32. My translation.

F	II
D	III
B♭	I

Figure 14.1 Hauptmann's pitch functions assigned to members of a major triad

F	I
D♭	III
B♭	II

Figure 14.2 Hauptmann's pitch functions assigned to members of a minor triad

To foreground these chord-structural issues, we shall represent dualist klangs here as ordered pairs. The first element defines Hauptmann's I-function or *Einheit*. The second element defines the klang's modality: the symbol ↑ (replacing Riemann's and Oettingen's "+") represents a major ("over" or "super") klang, or a "positive" *Einheit* as Hauptmann calls it; the symbol ↓ (replacing Riemann's and Oettingen's "○") represents a minor (or under, or sub) klang, or "negative" *Einheit*. Hence, the klangs in Figures 14.1 and 14.2 are respectively represented as B♭↑ and F↓.

Under Hauptmann's explanation, a dualist model organizes aural sensations in roughly the following way: when listening to a triad, pick out a major third or its inversion, and pick out a perfect fifth or its inversion; when you do, you will become aware that one pitch in the triad is involved in both relationships and thereby seems more prominent than the others. By way of contrast, a fundamental-bass model organizes sensations in roughly the following way: when listening to a triad, reorganize it so that it takes up the smallest registral space and so that only thirds and fifths are formed; assign prominence to the lowest pitch and take note of the quality of the third between that pitch and the next highest. And a figured-bass model organizes aural sensations in roughly the following way: When listening to a triad, concentrate on the lowest-sounding pitch, and assign it prominence; imagine a third and a fifth above the lowest pitch (the qualities of which are determined by a contextual diatonic collection); pitches that do not lie a diatonic third or fifth above the prominent pitch are momentarily displacing the pitches that do.

These scripts for generating monist and dualist structure respectively for major and minor triads from simple elements of structure and the sensations that correspond to them are especially suggestive of Zarlino's well-known discussion of triads in his *Le istitutioni harmoniche*.[11] There, he gauges the character of the third (or tenth) that extends above the lowest-sounding pitch in a triad: "Either this is minor and the resulting harmony is ordered by, or resembles, the arithmetic proportion or mean, or it is

11 Riemann famously mistranslates the passage which enables him to promote Zarlino as a harmonic dualist, work debunked later by others, notably Dahlhaus in "War Zarlino Dualist?"

major and the harmony is ordered by, or resembles, the harmonic."[12] Yet, scarcely a paragraph later he writes:

> But since the extremes of the fifth are invariable and always placed subject to the same proportion, apart from certain cases that are used imperfectly [i.e. "only two parts are heard singing together"], the extremes of the thirds are given different positions. *I do not say different in proportion; I say different in position*. I say different in position for when . . . the major third is placed below, the harmony is made joyful and when it is placed above, the harmony is made mournful. Thus from the different positions of the thirds which are placed in counterpoint between the extremes of the fifth or above the octave, the variety of harmony arises.[13] [Italics mine]

Zarlino's two explanations are particularly striking in the context of the monist and dualist schemes for organizing triadic intervals presented earlier. In fact, using Zarlino's categories, it is possible to characterize monist theory as the view that arises from "listening across position" over against dualist theories that arise from "listening across proportion." Fixing the boundaries of the fifth and concentrating on the major third compels the organization of aural sensations described above with respect to Hauptmann's model, though admittedly it does not address in any way the procedures under which triadic structure is generated. *Those* rather are suggested most strongly by Zarlino's derivations of major and minor triads from harmonic and arithmetic means of the fifth, respectively, since the harmonic mean is obtainable by taking the reciprocals of the terms of an arithmetic series. The point here is not primarily to salvage Riemann's frequently discredited characterization of Zarlino as a dualist, though that issue is a potentially engaging and fruitful enterprise, but rather to foreground in Zarlino's account the possibility of embracing both models as equally conditional and serviceable "modes" of conceptualizing the structure of major and minor triads and their relation to one another, by using relative registral position and diatonic interval size as variables.

Oettingen. The physicist Arthur Joachim von Oettingen (1836–1920) can be seen as the true heir of Hauptmann's dualism, having developed and pursued most rigorously in his *Harmoniesystem in dualer Entwicklung* of 1866 the dualistic framework laid out philosophically by the Leipzig Kantor. But Oettingen could not simply appropriate Hauptmann's thesis uncritically, for an important work had appeared in the years immediately following the publication of Hauptmann's principal treatise that cast considerable doubts upon its dualistic foundation: Hermann von Helmholtz's *Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik*. In this critical work (discussed in more detail in **Chapter 9, pp. 259–62**), Helmholtz had disputed Hauptmann's dualism by showing how the minor harmony was really an inferior and "corrupted" (*getrübt*) form of the major triad by virtue of its having

12 Quoted in *SR*, p. 448. 13 Ibid., **p. 449**. See also the excerpt quoted in **Chapter 24, p. 754**.

$5^m 3^n$

m \ n	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8
2	c	g	d	a	e	b	f♯	c♯	g♯	d♯	a♯	e♯	b♯	f𝄪	c𝄪	g𝄪	d𝄪
1	a♭	e♭	b♭	f	c	g	d	a	e	b	f♯	c♯	g♯	d♯	a♯	e♯	b♯
0	f♭	c♭	g♭	d♭	a♭	e♭	b♭	f	c	g	d	a	e	b	f♯	c♯	g♯
-1	d♭♭	a♭♭	e♭♭	b♭♭	f♭	c♭	g♭	d♭	a♭	e♭	b♭	f	c	g	d	a	e
-2	b♭♭♭	f♭♭	c♭♭	g♭♭	d♭♭	a♭♭	e♭♭	b♭♭	f♭	c♭	g♭	d♭	a♭	e♭	b♭	f	c

Figure 14.3 Oettingen's diagram of tonal space (from *Harmoniesystem*, p. 15)

greater interference among its constituent upper partials. Oettingen attempted to salvage the equal ontological status of the minor triad by showing its generation to be oppositional (*gegensätzlich*) to that of the major triad. If Hauptmann had claimed that the minor triad carries "passive" characteristics because its tones are themselves overtones of various fundamentals, while the tones of a major triad share a common fundamental (*sein* vs. *haben*), for Oettingen, the opposite was true. That is, the notes of a minor triad actively *have* differing fundamentals, while tones in a major triad are passively *being* overtones of the same fundamental. This is the basis of his distinction between *phonicity* and *tonicity* (discussed further below). The point is that Oettingen attempted to reconcile Hauptmann's logical arguments with Helmholtz's acoustical and physiological arguments. The result was the most thorough-going and undiluted doctrine of harmonic dualism articulated in the nineteenth century.

Oettingen takes as a starting point a notion of individual pitches (under just intonation) defined as frequencies expressible as $5^m 3^n 2^p$, where m, n, and p are integers. According to Oettingen, no matter how much the integers m, n, and p vary, one can never express one pitch in more than one combination, since every number may be analyzed into prime factors in only one way. (This is, of course, untrue if one takes the fifths and major thirds involved to be those developed by equal temperament – measuring 700 and 400 cents, respectively – rather than the pure fifths and major thirds of just intonation, measuring 702 and 386 cents, respectively.)

Such premises lead very naturally to diagrams like the one presented in Figure 14.3, reproduced from Oettingen's *Harmoniesystem in dualer Entwicklung* (1866). Rows are measured in perfect fifths, columns in major thirds. All columns and rows are understood to extend infinitely beyond the limits shown in the diagram. The single and double over- and underlines remind us of the distinctions between pitches of the same letter name but which correspond to different frequency measurements under just intonation.

The diagram aids in calculating the relationship or interval from one pitch to another as powers of major thirds (5/4) and perfect fifths (3/2), as $(5/4)^{m*}(3/2)^n$. As the row headings suggest, moves upwards within columns – that is, moves by increments of a major third – are measured by positive integers, and moves downwards by negative.

And as the column headings suggest, moves to the right within a row – that is moves by increments of a perfect fifth – are measured by positive integers, and moves to the left by negative integers. So, for example, the interval from c^1 in the center of the diagram and b^1 to its upper right corresponds to (5/4)*(3/2), or (15/8).

Oettingen's notion of chord structure may be reasonably described as the application of Helmholtz's discussion of harmonic overtones and undertones to certain broad features of Hauptmann's notion of triadic construction, using non-scale-generated intervals, so that distinctions between, say, major and minor thirds (whose common designation as "thirds" is possible only with reference to the idea of a diatonic scale) are strictly observed.

In *On the Sensations of Tone*, Helmholtz describes the phenomenon of overtones and its corollary concept of undertones.[14] The latter does not – as is often assumed – involve the notion of a series of harmonic partials emitted or extended "downwards" from a fundamental as a direct parallel to the series of harmonic partials emitted or extended upwards from a fundamental.[15] Overtones are an easily observable acoustic feature of tones in general; undertones are not. By overtones, Helmholtz means just that pattern of partials associated with the acoustic *donnée*; by undertones, he means just the patterns of fundamentals associated with a particular partial. The notion can be engaged acoustically, following Helmholtz, by way of a resonator, a hollow sphere of glass with two openings of different sizes, the smaller of which may be sealed with wax and placed in one's ear. If the "proper tone" of the resonator is, say, c^3, that pitch will sound when a nearby musical instrument plays c^2, or f^1, c^1, $a\flat$, f, d, c, and so on. (One could of course repeat the results by silently depressing c^3 on a piano and playing c^2, or f^1, c^1, $a\flat$, f, d, c, and so on.) Accordingly, the concept of undertones is an assertion of no acoustic or psychological phenomenon other than the phenomenon of a tone comprising a fundamental and an associated series of partials. (See also the discussion in **Chapter 9, pp. 251–54**.)

It is in this connection that Oettingen develops his well-known twin constructs of "tonicity" (*Tonicität*) and "phonicity" (*Phonicität*). Tonicity corresponds to the property of an interval or chord to be grasped as a partial of a fundamental (p. 31). Accordingly under this conception the "tonic" fundamental of the interval c^1–g^1 is c since the pitches that constitute the interval may be understood as partials of c. Phonicity, on the other hand, corresponds to the property of the pitches that constitute an interval or chord to possess common partials. The lowest of all such common partials is called the phonic overtone. Consequently, the phonic overtone of the interval c^1–g^1 is g^2. Under Oettingen's conception, then, each interval or chord possesses both properties and accordingly has both a tonic fundamental and a phonic overtone.

14 Helmholtz's discussion of undertones begins on p. 33 of the English translation.

15 This would be similar to Rameau's "resonance" theory of the minor triad articulated in his *Génération harmonique*, but abandoned soon thereafter (see **Chapter 24, p. 771**). However, Riemann himself quite clearly attempted to strengthen the concept of undertones along precisely these lines.

So, given the major triad c^2, e^2, g^2 and the minor triad c^2, $e\flat^2$, g^2, Oettingen says the first chord has a tonic fundamental of c and a phonic overtone of b^6, while the second triad has a tonic fundamental of $a\flat_1$ and the phonic overtone of g^4. Furthermore, the tonic fundamental of the major triad is the structural parallel of the phonic overtone of the minor triad: in each case these tones are consonant with their respective chord. On the other hand, the phonic overtone of the major triad and the tonic fundamental of the minor triad are dissonant with their respective chord.

Relating triadic structure to the diagram in Figure 14.3, Oettingen provides a topographic version of major–minor opposition. He writes that "all pure consonant triads stand in the form of right triangles, whose hypotenuses all form a diagonal minor third. In the major klang, the right angle is oriented to the top (of the diagram); in the minor klang, the right angle is oriented to the bottom."[16]

These notions provide Riemann with his theoretical point of departure, and although later on he extends his research agenda to cover an extremely wide array of activities, from phrasing to keyboard technique to more properly music-historical topics, he retains the basic outlines of Oettingen's conception of chord structure and chord relationship, along with the deployment of those structural elements in imaginary topographies. Indeed, it may be appropriate to characterize as Oettingen-Riemannian a theory that involves certain of Oettingen's fundamental conceptions and Riemann's later revision of its details, carried out to integrate the approach more readily with established conservatory theoretical practices.[17] It only remains to say here that Riemann's argumentation on behalf of the undertone series led to any number of unfruitful byways and expended much wasted energy on his part. It was obviously with some regret – but probably also considerable relief – that at the end of his life, he finally abandoned the search for an acoustical proof for the series and instead posited a psychological grounding.[18] It should be emphasized, however, that the heuristic value of Riemann's ontological dualism is by no means dependent upon any natural justification of the undertone series. Its ultimate vindication comes in the logical and revealing network of chord relationships that a dualist perspective affords.

Schritte, Wechsel and topographies

These relationships emerge from the intervals of perfect fifth and major third, the intervallic relations that constitute triads. Moreover, they arise from implementing

16 Oettingen, *Harmoniesystem in dualer Entwicklung*, p. 17. Compare also the related *Tonnetz* by Hostinský, **Plate 23.1, p. 737**.

17 Riemann's own views on conservatory-style education are particularly interesting in this regard. These views are stated most forcefully in an article entitled "Unsere Konservatorien," published just after he had left the Leipzig Conservatory of Music for a position at The University of Leipzig.

18 Riemann, "Ideen zu einer 'Lehre von den Tonvorstellungen.'" See also the helpful discussion in Harrison, *Harmonic Function*, pp. 261–65.

the dualism immanent in the interaction between the notion of dyadic interval – measuring magnitude alone, as in "major third" or "perfect fifth" – and the notion of directed interval – measuring both magnitude and direction, as in "major third up" or "perfect fifth down": the former defines triads as triads; the latter forms the basis for the distinction between major and minor triads. Since the definition of specific chord relationships in Riemann and Oettingen involves the notion of directed intervals, they possess essential structural features of mathematical "transformations," principally that such relationships are one-to-one: they relate one pitch to only one other pitch. (By contrast, the dyadic notion of interval – a more commonly employed conception – relates one pitch to two others.) This feature is particularly important for us since it serves as the basis of contemporary interest in Riemann's work in contemporary American theoretical circles. In the account of such chord relationships, I shall concentrate on Riemann's simplified version of those first defined by Oettingen.

Riemann establishes two classes of chord relationships or transformations. One, whose elements all have the suffix *Schritt* (step), is analogous but not identical to pitch-class transposition, and preserves the polarity of the klangs to which they are applied. Hence *Schritte* map major klangs onto major klangs, and minor klangs onto minor klangs. Such relationships are termed "homonomic" by Oettingen. The second class of transformations, whose elements all have the suffix *Wechsel* (exchange), is analogous but not identical to pitch-class inversion, and reverses the polarity of the klangs to which they are applied. Hence, *Wechsel* map major klangs onto minor klangs, and minor klangs onto major klangs. Such relationships are termed "antinomic" by Oettingen.

Riemann's catalogue of *Schritte* and *Wechsel* varies considerably from his first "practical" harmony text, *Skizze einer neuen Methode der Harmonielehre* (1880) to its later reworking as *Handbuch der Harmonielehre* (1887), his popular handbooks such as *Handbuch der Harmonie- und Modulationslehre* (1890), and his mature exposition of functional harmony, *Vereinfachte Harmonielehre* (1893). Ultimately, Riemann's purpose is to provide a thorough enough lexicon of relations so that any two klangs could find a relevant transformation within the system, a notion taken up most strikingly by his student Max Reger.[19]

Riemann's interest in these transformations appears within the context of his topographical conception of tonality, which in turn arises from Oettingen's topographical conception of pitch relations regulating the design given earlier in Figure 14.3. Troping Oettingen's diagram, Riemann replaces pitches with klangs, and pitch intervals with klang transformations. Figures 14.4 and 14.5 provide maps of Riemann's

19 In *Harmonic Function* (pp. 296–98), Harrison more thoroughly explores this aspect of Reger's thought.

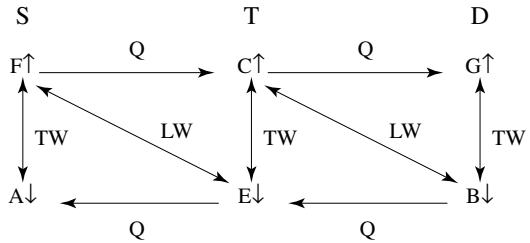

Figure 14.4 A Riemannian map of C major tonality

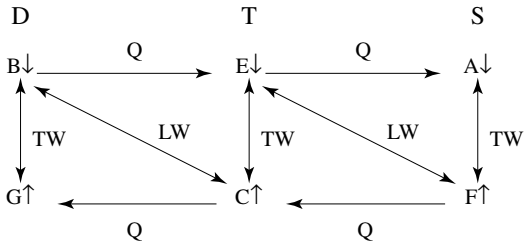

Figure 14.5 A Riemannian map of E minor tonality

major and minor tonalities respectively: the diagrams are developed out of illustrations Riemann presents in *Skizze einer neuen Methode der Harmonielehre* and related diagrams that appear in *Grosse Kompositionslehre*. It is important to stress here that such graphic representations are not simply visual presentations of aspects or features of structure that characterize Riemann transformations. Rather, the topographic models are most fruitfully regarded as representational maps of tonality imagined spatially, and particularly tonality conceived in a space where the distances between the deployed klangs are measured in Riemann's transformational categories. It is only with reference to such maps that Riemannian notions such as chord function and tonality have any concrete relevance.

Each map has two columns of klangs: in place of the perfect fifth, which regulates the horizontal aspect of Oettingen's diagram, Riemann provides *Quintschritt* (abbreviated Q_), a transformation that transposes a klang by the directed interval (mod 8ve) that extends from I to II. In the case of C↑ (C major triad, in standard notation), where C functions as I and G as II, the relevant interval is a perfect fifth up. Accordingly, *Quintschritt* maps C↑ to G↑. In the case of E↓ (A minor in standard notation), where E functions as I and A as II, the relevant interval is a perfect fifth down: hence, *Quintschritt* maps E↓ to A↓ (D minor in standard notation). Alternatively, one could say that

Quintschritt transposes a klang the distance of a *Quint* (perfect fifth) extended in the direction that characterizes the klang in question: up, in the case of major (or over) klangs; down in the case of minor (or under) klangs.

In place of the major third, which regulates the vertical aspect of von Oettingen's diagram, Riemann provides *Terzwechsel* (abbreviated TW), a transformation defined as a composite of *Terzschritt* – which transposes a klang by the interval extending from I to III – followed by *Seitenwechsel* (abbreviated as W), the inversion of a klang around I, which exchanges positive and negative forms of the same *Einheit*, so that it transforms C↑ into C↓, and C↓ into C↑.[20] Taken together, *Terzschritt* and *Seitenwechsel* map C↑ to E↓ via E↑, and E↓ to C↑ via C↓, and are functionally equivalent to what is more commonly called the "relative" relationship.

In each map the top rank of klangs constitute the *Hauptklänge* (primary klangs) of the relevant tonality, the parallel bottom rank the *Nebenklänge* (secondary klangs). The central klang of the primary rank functions as the tonic klang. This function arises from the klang's involvement with both *Quintschritte* in the top rank; accordingly, the function of tonic in this context emerges from the klang's mediation (speaking both visually and dialectically) between the leftmost primary klang (G↑ in Figure 14.4; A↓ in Figure 14.5) and the rightmost primary klang (F↑ in Figure 14.4; B↓ in Figure 14.5). Using Hauptmann's language the tonic both *is* and *has* a *Quintschritt*: its functional centrality is articulated by the two klangs that mark the vertical limits of in each map.

Figures 14.4 and 14.5 include the function labels *S*, *T*, and *D* representing subdominant, tonic, and dominant, respectively. As we have seen, in Riemann's conception of them, these functions have both a dynamic (that is, transformational) and topographical modality. The latter modality on its own is not Riemann's: he himself explicitly traces the origins of this concept of chord function to the work of Fétis.[21] In Riemann's view functions also have a syntactic aspect, since complete harmonic phrases must have the structure **T S T D T**. Moreover, the syntactical functions may be served not only by the primary klangs in a tonality but also by the secondary klangs (as lexical equivalents) that relate to the primary klangs under *Terzwechsel* or *Leittonwechsel* (abbreviated as LW and defined as a composite of *Leittonschritt* [leading-tone step] – itself the composite of *Quint-* and *Terzschritt* – and *Seitenwechsel*).

Before progressing, it is worthwhile to address an aspect of Riemann's dualism and its interaction with his function theory that has often served as a locus from which to discredit his entire approach. This objection, which, as far as I know, was first articulated around the turn of the twentieth century by the Dutch musicologist Ari Balinfante and revived later on by Carl Dahlhaus, runs something like this, using the

20 *Seitenwechsel* appears in Goetschius's work as "stride relation," defined in his context as "a perfect fifth downward from any major keynote, and upward from any minor keynote, with a change in mode." Goetschius, *Tone-Relations*, p. 114. 21 Riemann, *Harmonielehre*, p. 214.

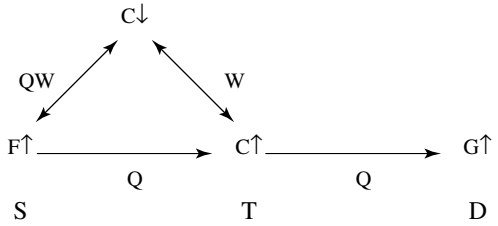

Figure 14.6 A Riemannian map of C major-minor tonality

maps of Figures 14.4 and 14.5 as a context:[22] comparing the two diagrams, one sees that in order to arrange them to match in the terms of harmonic dualism – as the figures certainly do – the deployment of function labels must be reversed. In other words, the argument goes, while Riemann was dualist in chord structure and certain aspects of their interrelations, he was monist in his theory of chord functions. The monist Riemann is the repressed element in this (at least in Balinfante's) account and hence represents Riemann's more basic and fundamental beliefs.

The critique, however, is presumptuous: there is no natural procedure for mapping function assignments onto Riemann's dualist transformations. It still needs to be shown that having function labels and transformation relations line up identically amounts to the proper dualist view. Indeed, it is quite plausible to assert that transformation that maps C↑ onto G↑ – a tonic functioning chord onto dominant functioning chord – *ought* to be the inverse (that is, the structural dual) of the transformation that maps E↓ (as a tonic functioning chord) onto B↓ (a dominant functioning chord) just as the directed interval that extends from I (C) to II (G) in C↑ – namely, a perfect fifth up – is inversely related to the directed interval that extends from I (E) to II (A) in E↓, namely a perfect fifth down. Indeed, such reasoning squares more easily with the dualist klang structure discussed earlier.[23]

Major and minor are just two of the tonal genera defined by Riemann. They may be mixed in systematic ways to produce two further genera, major-minor and minor-major. Figure 14.6 displays the first of these. The primary klangs of the major topography are given and deployed precisely as they are in the (plain) major system. The secondary klangs, which are not provided, are just the secondary klangs of the major

22 Balinfante, "De leer," and Dahlhaus, *Studies*, pp. 51–53. Also see Harrison, *Harmonic Function*, p. 273, n. 37.

23 The Balinfante–Dahlhaus objection to Riemannian dualism interacts suggestively with the more often articulated and less formalized attack generally levelled at Riemann, namely that he sacrificed real musical objects, relations, and experiences in favor of logical consistency. There is a great deal of plain silliness underlying this attack – including anti-intellectualism, and a particularly bone-headed form of empiricism – but in the present context there is an interesting alliance of the concept of "real musical experience" with the concept of function, and coherence and logical consistency with triadic dualism, which are then opposed.

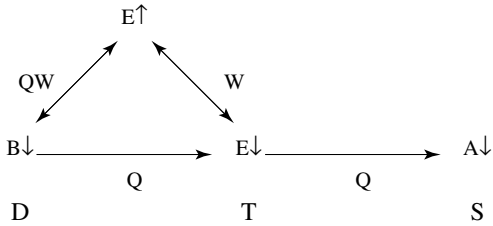

Figure 14.7 A Riemannian map of E minor-major tonality

genus. An additional klang is given in third rank "behind" F↑, namely its *Quintwechsel* relative C↓ (F minor in standard notation), and represents what is commonly called a minor subdominant. The map also measures diagonal distance between C↓ and C↑ as *Seitenwechsel* (W).

Figure 14.7 displays the second genre of Riemann's minor-major tonality. The primary klangs of the minor topography are given and deployed precisely as they are in Figure 14.5. The secondary klangs, which are not provided, are just the secondary klangs of the minor genus. An additional klang is given in third rank "behind" B↓ (E minor in standard notation), namely its *Quintwechsel* relative E↑, and represents what is commonly called harmonic minor. Accordingly, what has changed from major or minor to its relevant mixed genus is the nature of one of the delimiting klangs: F↑ in C major is replaceable by C↓ in C major-minor; B↓ in E minor is replaceable by E↑ in minor-major.

The previous four examples present only a few transformations defined by Riemann. Table 14.1 provides a more complete listing. Since Riemann's own catalogue of transformations changed throughout his publishing career, the table represents a rationalized composite of his various presentations, with an eye to providing enough transformations to map any klang to any other klang.

The top half of the table lists eleven *Schritte*. Each is associated with a particular interval, whose disposition emerges from some internal klang relation or composite of relations. Both major and minor klangs will traverse the same interval under a particular transformation: major klangs will extend that interval upwards, minor klangs downwards. The bottom half of the table lists twelve *Wechsel*. Each is defined as a composite of a *Schritt* defined earlier followed by *Seitenwechsel*. All *Wechsel* are reflexive, which is to say, each serves as its own inverse. Hence, *Quintwechsel* (for instance) maps F↑ to C↓ and C↓ to F↑.

The four tonality maps have two particularly useful and important purposes. First, they each collate the idea of tonal relations as arrangements within imagined geography upon which musical pieces may be seen to traverse. As such, the maps have a direct analytical usefulness when studying pieces with respect to Riemannian transformations. As an example, applying the tonality genus categories to "Im wunderschönen

Table 14.1 *Riemannian Transformations*

I. Schritte

	Transformation	Interval	Klang deployment	Examples
1	*Quintschritt*	P5	I to II	C↑ → G↑; E↓ → A↓
2	*Gegenquintschritt*	P4	II to I	G↑ → C↑; A↓ → E↓
3	*Ganztonschritt*	M2	twice I to II	F↑ → G↑; B↓ → A↓
4	*Gegenganztonschritt*	m7	twice II to I	G↑ → F↑; A↓ → B↓
5	*Terzschritt*	M3	I to III	C↑ → E↑; E↓ → C↓
6	*Sextschritt*	M6	II to III	G↑ → E↑; A↓ → C↓
7	*Leittonschritt*	M7	I to II plus I to III	F↑ → E↑; B↓ → C↓
8	*Gegenleittonschritt*	m2	II to I plus III to I	E↑ → F↑; C↓ → B↓
9	*Gegenterzschritt*	m3	III to II	E↑ → G↑; C↓ → A↓
10	*Gegenterzschritt*	m6	III to I	E↑ → C↑; C↓ → E↓
11	*Tritonusschritt*	d5/a4	twice I to II plus I to III	F↑ → B↑; B↓ → F↓

II. Wechsel

	Transformation	Definition	Examples
12	*Seitenwechsel*	Invert a klang around I	C↑ ↔ C↓
13	*Quintwechsel*	*Quintschritt*, then *Seitenwechsel*	F↑ ↔ C↓
14	*Sextwechsel*	*Sextschritt* then *Seitenwechsel*	G↑ ↔ E↓
15	*Leittonwechsel*	*Leittonscritt* then *Seitenwechsel*	C↑ ↔ B↓
16	*Ganztonwechsel*	*Ganztonschritt*, then *Seitenwechsel*	G↑ ↔ A↓
17	*Terzwechsel*	*Terzschritt*, then *Seitenwechsel*	C↑ ↔ E↓
18	*Tritonuswechsel*	*Tritonusschritt*, then *Seitenwechsel*	F↑ ↔ B↓
19	*Gegenterzwechsel*	*Gegenterzschritt*, then *Seitenwechsel*	C↓ ↔ E↑
20	*Gegenganztonwechsel*	*Gegenganztonschritt*, then *Seitenwechsel*	C↓ ↔ D↑
21	*Gegensextwechsel*	*Gegensextschritt*, then *Seitenwechsel*	E↑ ↔ G↓
22	*Gegenquintwechsel*	*Gegenquintschritt*, then *Seitenwechsel*	G↑ ↔ C↓
23	*Gegenleittonwechsel*	*Gegenleittonschritt*, then *Seitenwechsel*	C↓ ↔ B↑

Monat Mai" from Schumann's *Dichterliebe*, we can assert that the piece presents in turn the following four tonal genera: C♯ minor-major, A major, F♯ minor-major, D major-minor. Moreover, the transformations given in Table 14.1 can be shown to have individual tonal value, by referring them to trajectories on one or more of the topographies.

Secondly, the topographies form the basis from which to understand Riemann's theory of dissonant (non-triadic) events, which derive ultimately from his conceptualization of tonality – that is, his four modes of tonality – along the lines presented in Figures 14.4–14.7.

Dissonant klangs

Dissonance in Riemann's view arises from the "disruption of the unity of klang structure and klang meaning by foreign elements."[24] This disruption is carried out in two ways: the combination of one klang (or its elements) with another; and the alteration of a klang pitch affecting the constituent major third or minor third.

The first of these is of special concern here. Dissonant klangs in this class articulate the tonic or central klang of a particular topography delimiting the relevant topography's boundaries. These chords may usefully be further divided into two categories: dissonant chords that articulate the horizontal boundaries of a topography, which is to say the boundaries within the topographic rows; and chords that articulate the vertical boundaries of a topography, or the extent of the constituent columns. The first of these two classes vary significantly across the four tonal genera; the second does not.

The foremost of these combinations involve the two primary klangs on either side of the tonic klang. These two klangs, the primary dominant and subdominant functioning ones, are all that are needed to provide a sense of the central klang – which mediates the lateral two primary klangs both spatially (or topographically) and transformationally (or dynamically) – as tonic functioning. When the two lateral primary chords are presented as a single, dissonant chord they have the same effect.

In the major and minor topographies presented in Figures 14.4 and 14.5, the relevant dissonant combinations are generated by the transformation *Gegenganztonschritt*, which in each topography maps the leftmost primary klang to the rightmost, and the rightmost secondary klang to the leftmost. In the case of C major, then, *Gegenganztonschritt* adds F↑ to G↑ in the primary rank and B↓ to A↓ in the secondary rank. The same combinations arise within the E minor topography of Figure 14.5. By suppressing various pitches in the combined klangs, Riemann generates a series of non-triadic structures. Accordingly, the combination of [G B D] and [F A C], which articulate the boundaries of the primary rank in C major and the secondary rank in E minor, can yield G^{7}, G^{9}, and B° (in standard, Weberian notation). Correspondingly, the combination of [E G B] and [D F A], which articulates the boundaries of the primary rank in E minor and the secondary rank in C major, can yield $B^{\phi 7}$, G^{9}, and B°.

The boundaries of the primary ranks of the two mixed genera given in Figures 14.6 and 14.7 are defined not by *Gegenganztonschritt* but by *Gegenquintwechsel*: since all *Wechsel* are their own inverses *Gegenquintwechsel* maps both the leftmost klang to the rightmost, and the rightmost to the leftmost. In the case of C major-minor, the transformation combines G↑ and C↓. Accordingly, by suppressing various pitches, the combination of [G B D] and [F Ab C] can yield G^{7}, $G^{7\flat 9}$, $B^{\circ 7}$, $D^{\phi 7}$, $B^{\circ 7\flat 9}$, and $B^{\phi 7}$ (again, in standard, Weberian notation). Correspondingly, the combination of [E G♯ B] and [D F A], the lateral limits of E minor-major, can yield $B^{\phi 7}$, $G\sharp^{\circ 7\flat 9}$, $G\sharp^{\circ 7}$, E^{7}, $E^{7\flat 9}$, and $G\sharp^{\phi 7}$.

24 Riemann, *Harmonielehre*, p. 138.

The vertical limits of the tonal genera are defined, according to Riemann, by *Terzwechsel* and *Leittonwechsel*. Combining *Terzwechsel*-related chords in C major(-minor) yields various "minor seventh" chords: A | C E | G; D | F A | C; and E | G B | D. Combining *Leittonwechsel*-related klangs in those genera produces F major 7 (F | A C | E) and C major 7 (C | E G | B).

The chords discussed here do not exhaust all possible dissonant structures in Riemann's catalogue. But they do constitute the major classes of such chords and illustrate Riemann's and Oettingen's conception of dissonant structures and their role. This approach to seventh chords seems to have become widespread, surviving – to the embarrassment of some – even in Schenker's *Harmonielehre*. In spite of its current discredited status, such an approach to dissonance seems especially suggestive in the context of atonal works of Schoenberg, Webern, Berg, and others, and may provide particularly fruitful access to certain harmonic aspects of that music.

Closing remarks

Almost all of Riemann's theoretical conceits have current advocates. The use of function theory (in some form or another) is widespread. Siegmund Levarie has written on the benefits of harmonic dualism, although leaning much more heavily on Goethe's *Naturwissenschaft* than on Riemann or Oettingen. Daniel Harrison, as already mentioned, has proposed his own revised theory of harmonic dualism (see p. 460 above, n. 7). And David Lewin has revived and further developed – with special reference to group theory – Riemann's transformational categories (see **Chapter 10, pp. 295–96**). However, these three aspects of Riemann's tonal theory are rarely as integrated as they are in Riemann's own thought. Harmonic dualism is altogether evaded in current applications of functionalism and in Lewin's reconstruction of transformational structures; Levarie's harmonic dualism exists outside of the context of functionalism or klang transformations.

A growing number of researchers in North America find themselves engaged in some way or another with some aspects of Lewin's original articulation of transformational Riemannian theory, in particular, Richard Cohn, Brian Hyer, and John Clough. Papers presented at a recent symposium concerning neo-Riemannian theory have appeared in a special issue of the *Journal of Music Theory* (vol. 42, 1999). The work carried out in this symposium is especially broad in scope, and includes Carol Krumhansl's investigations of certain neo-Riemannian conceits along purely music-psychological lines. The recent work of John Clough, Jack Douthett, Norman Carey, and David Clampitt integrates Lewin's and Cohn's work with an already existing tradition of examining the purely structural characteristics of the diatonic collection, the pentatonic collection, set class 3–11, and other tonally meaningful set-classes, as well as extending the discussion from Cohn's three parsimonius transformations to include

the entire *Schritt/ Wechsel* group. Others – I have in mind here Edward Gollin, David Kopp, and Michael Mooney – continue to revise and extend neo-Riemannian theory, often with more emphasis on the work of Riemann and Oettingen themselves, and with a particular interest in concrete music-analytical situations.

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