TEN

The Ancient *Musica speculativa* and Renaissance Musical Science

Speculative music theory at the beginning of the Renaissance, with rare exceptions, was dominated by the Pythagorean, Platonic, and Neoplatonic traditions. As stated earlier (chapter 1), the first ancient music-theoretical source that humanists rediscovered was Boethius, an author identified with these traditions. Though read and respected throughout the Middle Ages, particularly for his *Consolations of Philosophy*, Boethius needed to be repossessed as an authority on ancient music, to be reclaimed from medieval theory. The accretions of the plainchant theorists had to be brushed away, and his image altered from that of a universal musical lawgiver to that of a transmitter of ancient learning. Fifteenth-century humanists could not identify precisely Boethius' sources, but it was clear that he leaned a great deal on Nicomachus and Ptolemy and was against the Aristoxenians. Although there were Aristoxenian elements in both Nicomachus and Ptolemy, and Ptolemy did not always sympathize with the Pythagoreans, Boethius was identified as a Pythagorean.

Pythagoras is usually cited early in a treatise as the inventor of music or the discoverer of the ratios of the consonances. Typically the legend of the blacksmith's shop is recounted. Almost everyone depended upon the embroidered version of the story told by Boethius.[1] The older and fuller accounts of this legend, by Nicomachus and Gaudentius, were not known until the mid-sixteenth century.[2]

As told by Boethius the story goes as follows. By divine will Pythagoras happened to pass a blacksmith's shop, from which he heard diverse sounds as the apprentices were hammering, and these sounds blended in conso-

1. *De institutione musica* 1.10–11.

2. For an English translation of the account by Nicomachus, see Flora Rose Levin, "Nicomachus of Gerasa *Manual of Harmonics:* Translation and Commentary" (Ph.D. diss., Columbia University, 1967), pp. 28–32.

nances. Upon observing the smithies and reflecting on what he saw, he theorized that the diversity of pitches was caused by the diversity in strength of those hammering. But on testing the theory by having them exchange hammers, he found that this was not true. He then examined the weights of the hammers and found that one which weighed twice another sounded with it a diapason. Comparing other weights, he found that those in the ratio of 3:2 produced a fifth, and those in the ratio of 4:3 produced a fourth. By this means he determined the ratios of the consonances. After returning home, he made further tests. He attached weights to strings, blew on pipes of various lengths, and filled and partly filled glasses with water and struck them with a copper or iron rod. In all these experiments he found that the same ratios caused the same consonances.

Johannes Gallicus, who relied heavily on Boethius' treatise and referred to it as "that Music, which the so often mentioned Boethius turned into Latin from Greek,"[3] was dubious about Pythagoras' role in this incident. Around a figure of an anvil surrounded by four hammers bearing the numbers 6, 8, 9, and 12, Gallicus writes that it was more likely Jubal who made the discovery of the ratios of the hammers than Pythagoras, as handed down by the Greeks.[4] In the text itself Gallicus represents Jubal addressing the blacksmiths: "Exchange hammers, I pray you, and strike again, for I sense that not a small secret of nature hides either in your arms or in the hammers themselves."[5] After this experiment, Jubal concluded that the weights of the hammers and not the force of the blows determined the pitches.

Gallicus did not give an authority for his ascription of the discovery to Jubal, but Gaffurio, some years later, did. After paraphrasing the account of the story in Boethius, Gaffurio noted that Josephus attributed this investigation to Jubal before the flood, and in the appended figure Jubal is shown overseeing six smithies, five of them swinging hammers weighing 4, 6, 8, 12, and 16 pounds. In accordance with the account by Boethius, three other woodcuts show Pythagoras coaxing the same consonances from bells, water glasses, strings with weights attached, and pipes (see Figure 10.1). These figures illustrate Gaffurio's paraphrase from Boethius.[6]

In both Boethius and Gaffurio, the legend is introduced to show that,

3. *Ritus canendi*, I, 4; Coussemaker ed., IV, 304; Seay ed., 11.13: "ea namque musica, quam totiens allegatus Boetius de Graeco vertit in latinum."

4. *Ritus canendi*, I, 10; Coussemaker ed., IV, 310; Seay ed., 21.13: "Tradunt Graeci Pythagoram Hanc inveniisse fabricam./ Sed magis puto consonum/ Opinari dictum Iubal,/ Suum fratrem Tubal Cain/ Frequentasse fabricantem/ Qui ferro patet extitit/ Ac aere malleantium."

5. Ibid., I, 10; Coussemaker ed., IV, 310; Seay ed., 21.7: "Mutate, quaeso malleos ac iterum percutite, non enim parvum aut in vestris brachiis, aut in ipsis malleis latere sentio naturae secretum."

6. Gaffurio, *Theorica musice*, I, 8.

Figure 10.1.
The discovery of the ratios of the consonances by Jubal and Pythagoras, from Gaffurio, *Theorica musice,* I, 8

given the inadequacy of the hearing when confronted with a multitude of sensations, only the reason coupled with accurate observation and measurement can establish the true relationships of tones. Yet neither author gives evidence of observation or measurement, or reasoning thereon, and neither attempts to demonstrate anything geometrically, mathematically, or by logical induction or deduction. Boethius and Gaffurio simply recount a legend and remain in a narrative mode throughout these chapters. A correspondence between consonances and ratios having been established in this fashion, no further defense appears to them necessary, and this is true also of Nicomachus and Gallicus.

Of the four woodcuts in Gaffurio's figure, only the last represents phenomena that are verifiable. If pipes 4, 6, 8, 9, 12, and 16 units long are alike in other respects, the sequence of intervals that Gaffurio aimed to illustrate, a series comparable to A E a b e a′, will result when they are blown. In the other four cases—hammers, bells, glasses partly filled with water, and strings stretched by weights —the intervals will not be the same. With hammers the result is unpredictable, since the pitch emitted depends more on the metal struck than on the hammers. With bells and water glasses the relationships are complex. In the case of weights attached to strings the frequency will vary as the square of the weights. The one medium with which Gaffurio had direct experience, the single stretched string, the division of which would support the series of ratios he wished to demonstrate, is not brought into the account.

Although statements such as Gaffurio's wear some of the trappings of scientific research and demonstration, they are transparent appeals to authority and legend and cannot be considered scientific expositions at all. Hardly indicative of the current state of knowledge of sound, which in all of these authors is quite sophisticated at times, chapters such as these on the hammer story are concessions to a literary convention. Sometimes Gaffurio contrasts different opinions among the ancient authorities, but here too conventional erudition prevails over any impulse to critical choice.

Even in this indiscriminately eclectic, antique-worshipping environment, Valgulio's open-minded defense of both Pythagoreans and Aristoxenians is notable. He was not blind to their differences. The harmonists he recalls, "attribute more authority to the judgment of the ear than to that of reason, like the Aristoxenians do." The canonists "assign the first and most approved grade of judgment to the reason, as the Pythagoreans do, who with respect to genus are also harmonists." Ptolemy held to a middle way and maintained that a musician proceeds correctly "when the judgment of the ears accords with that of the reason."[7]

7. Valgulio, *Proemium,* 1530 ed., fol. 247r. See ch. 5 above, for a detailed treatment of his views.

Franchino Gaffurio

Gaffurio similarly contrasts the views of Plato and Nicomachus but is unable to choose one over the other. In a passage that is common to the *Theoricum opus* of 1480 and the *Theorica musice* of 1492, Gaffurio presents Plato's explanation of the mechanics of consonance:

Sit uero auribus ipsa consonantia secundum Platonem hoc modo: quom acutior sonus qui uelocior est grauem praecesserit in aurem celer ingreditur: offensaque extrema eiusdem corporis parte quasi pulsus iterato motu reuertitur: sed iam segnior nec ita celer ut primo impetu emissus aduenit: quo circa acutior ipse sonus nunc grauior rediens sono primum graui uenienti similis occurrit misceturque ei unam efficiens consonantiam.[8]	According to Plato, consonance strikes the ear in this way: the higher of the two sounds, which is speedier, precedes the low sound and enters the ear quickly, and when it has met the innermost part of the ear, it bounces back, as if it were impelled with repeated motion. But now it arrives more slowly, not fast as when emitted by the first impulse. For this reason this higher sound, now returning lower, presents itself as similar to the approaching low sound, and is blended with it, making one consonance

This explanation of consonance was given by Plato in *Timaeus* 80a–b, but Boethius or his source added the clarification of how the faster sound slows down to reach a correspondence with the slower sound, namely, by bouncing back and forth in the innermost part of the ear. Gaffurio in 1480 had no direct access to Plato; so he could not appreciate that he was transmitting a later interpretation along with Plato's views.

Gaffurio now finds in Boethius a competing theory, attributed to Nicomachus.[9] A sound consists of not one impulse but many in quick succession. When a string is tense, it produces frequent and dense pulsations; when it is loose, it produces slow and rare pulsations. If the percussions of the low sounds are commensurate with the percussions of the high sounds, then consonance will result, otherwise not. The words with which Boethius reports Nicomachus' thoughts are repeated almost verbatim by Gaffurio.[10]

In a section of the *Theorica musice* not held over from the 1480 version, Gaffurio went to what was probably the most enlightened source then

8. *Theoricum opus*, II, 3; *Theorica*, II, 4, fol. c5v. This is a paraphrase of Boethius *De institutione musica* 1.30.

9. This is not preserved in Nicomachus' extant works.

10. *Theoricum opus*, II, 3; *Theorica*, II, 4, fol. c5v.

available on the science of sound, the *Paraphrases* of Themistius (c. 317–38 A.D.) on the *De anima* of Aristotle in the Latin translation of Ermolao Barbaro.[11] Themistius now became Gaffurio's main source for the theory of sound and hearing. Themistius had insisted, as d'Abano was later to do, that the air struck by the sounding object was not the same as that which reached the ear. He noted, following Aristotle, that the notions of grave and acute were assigned to sounds by analogy with touch, and elucidated this by saying that the acute voice stabs the air and pungently wounds it, while the grave tone hits bluntly and spreads as it hits. Whereas the acute sound moves the sense a great deal quickly, the grave sound moves it little slowly.[12]

Gaffurio depended on Themistius also to explain the mechanism of hearing. The nature of the ear is akin to that of air in that the ear is congenitally filled with air, which is excited by the air outside and transmits the motion to little sensitized tinders inside a tissue of little breadbaskets (*paniculae*) filled with air. The outside and inside air are continuous, which explains why animals do not hear by their other bodily parts.[13]

Gaffurio made no attempt to reconcile the Aristotelian and Pythagorean-Platonic traditions in his *Theorica musice*. The split became even more intense in Gaffurio's last treatise, *De harmonia*, in which he turned to a wider variety of Greek sources, often eclectic themselves. As he darts from one to another it is nearly impossible to detect any consistent philosophy. Yet when a question touches on some of the fundamental tenets of music theory, he takes a conservative, Boethian position.

Such a question is the tuning of the diatonic scale. Despite the alternatives to the Pythagorean tuning offered by Ptolemy, some of them better suited to current musical practice, Gaffurio never departed from the system sanctioned by Boethian authority. It is characteristic of him to overlook the incompatibility of the ancient theory of intervals with the way composers employed consonances in polyphony. Whereas Boethian theory recognized only the few consonances acceptable by Pythagorean standards, later called "perfect" consonances, musical practice in the fifteenth century required that one of these, the fourth, be treated in most polyphonic situations as a dissonance and that the perfect consonances be mixed and alternated with so-called imperfect consonances, thirds and sixths. In the tuning prescribed by Boethius, the major third was a ditone, 81:64, and the minor third, 32:27, neither too displeasing as a simultaneous concord by itself, but grating

11. Themistius *Paraphrases* on Aristotle, *De anima*, Latin trans. Ermolao Barbaro (Paris, 1535), ed. Richard Heinze (Berlin, 1899).

12. Ibid., II, 30, fol. 74.

13. Gaffurio, *Theorica*, II, 2: Themistius, *Paraphrases*, Barbaro trans., II, 28, fol. 72.

when combined together in a three-part chord. One of the tunings described by Ptolemy, indeed the diatonic he most favored, permitted better-tuned thirds on most degrees of the scale, namely those in the ratios 5:4 and 6:5. This was his syntonic diatonic. Yet Gaffurio could not bring himself to accept it.

Ramos de Pareja

The mathematician Bartolomé Ramos de Pareja (c. 1440–after 1491) in 1482 had proposed a similar but not identical tuning purely as a practical strategy.[14] Ramos appears not to have read any of the Greek sources directly, but, like Gallicus and Gaffurio, had studied Boethius closely. He read him, however, more critically than his predecessors. Ramos began the prologue of his book with an encomium of Boethius, paying tribute to the profound arithmetical and philosophical foundations on which the work of Boethius rests and proclaiming that it always has been and always will be greatly prized by the learned. At the same time it always has been and always will be neglected by half-educated musicians, who find it obscure and sterile. This statement may reflect Ramos' own ambivalence toward Boethian theory. He frequently cites its authority for definitions and ancient musical lore; yet, after praising it as subtle, delightful, and useful to theorists, and with only a mild complaint that the monochord division of Boethius is "laborious and difficult for singers to learn,"[15] Ramos proceeds to overturn completely the Pythagorean system. Slily constructing a monochord division that would correct the tuning of the imperfect consonances, he proposes it simply as a method that anyone moderately educated will easily understand. Only toward the end of the book does he make it plain that his imperfect consonances have simpler ratios than those of the Pythagorean system, namely 5:4 and 6:5 for the major and minor thirds, and 5:3 and 8:5 for the major and minor sixths.[16]

According to Ramos' disciple Giovanni Spataro, Ramos arrived at his diatonic division independently of Ptolemy and Didymus,[17] although his system seems to graft the two. The string lengths shown in Figure 10.2, which Ramos does not reveal but were later calculated by John Hothby,

14. *Musica practica* (Bologna, 1482; facs. ed., Bologna, 1969), I, 1, 2; ed. Johannes Wolf in Publikationen der Internationalen Musikgesellschaft, Beihefte, II (Leipzig, 1901), p. 1.

15. Ibid., I, i, 2; Wolf ed., p. 4.

16. Ibid., III, ii, 3; Wolf ed., p. 98.

17. *Errori di Franchino Gafurio da Lodi* (Bologna, 1521), Error 17, fol. 22r: "Io non dico/ o Franchino: che el mio preceptore habia tolto el suo Monochordo da Ptolomeo: perche questo io non el scio di certo: Ma io dico/ che el suo Monochordo predicto non e dissimile da quello de Ptolomeo/ dicto di sopra."

a		b		c		d		e		f		g		a
288		256		240		216		192		180		160		144
	9:8		16:15		10:9		9:8		16:15		9:8		10:9	
			Didymus' diatonic						Ptolemy's diatonic syntonon					

Figure 10.2:
Ramos de Pareja's monochord division

result from the division of the monochord presented by Ramos.[18] All the thirds on this monochord are just, or pure, that is, 5:4 and 6:5, except B–D (32:27). However, as Hothby pointed out, there are also two poor perfect consonances, the fourth, D–G (27:20), and the fifth, G–D (40:27).[19] Ramos expanded this diatonic system into a fully chromatic scale in a later chapter, but, aside from one more pure third (B♭–D), the thirds are either larger or smaller than just intervals.[20]

Even after Gaffurio discovered that Ramos' innovation was corroborated by Ptolemy, Gaffurio continued to oppose it and attacked Ramos by name in passages he added to his *De harmonia* before publication. He refutes the proposition that a ditone may be in the 5:4 ratio by appeals to authority—Jacques Lefèvre d'Étaples (Jacobus Faber Stapulensis), Boethius, and Porphyry—and by invoking the legendary Pythagoras.

> But a sesquiquartal proportion, since it is superparticular, cannot ever be divided into two equal proportions, as Boethius laid down in the third [chapter] of the first [book] of his Music. So Pythagoras despised all intervals that deviated from the purity of the multiple and superparticular [ratios], omitting in his investigation of consonant and equisonant tones intervals made agreeable sounding by the addition or subtraction of a minimal increment, because a very small error is not evident to the sense of hearing. But Ptolemy does not seem to have agreed with him altogether, for he constituted the incomposite ditonic interval in the enharmonic by subtracting a minimal interval, assigning to [the remaining interval] the proposed superparticular ratio that singers call major third, granted that it is a ditone diminished. We, however, were led to demonstrate [the intervals] with reason, even if the sense does not perceive the

18. *Musica practica*, I, i, 2. Ramos translates the points on his string h to p into mese to nete hyperbolaeon and also to letters in the Guidonian gamut, a to a' in his figure of the following chapter.

19. John Hothby, *Excitatio quaedam musicae artis per refutationem*, in Johannes Octobi, *Tres tractatuli contra Bartholomeum Ramum*, ed. Albert Seay (American Institute of Musicology, 1964), p. 25.

20. *Musica practica*, I, ii, 5.

minimal differences, for harmonics, as Porphyry says, hinges on the examination of differences."[21]

Although Gaffurio cites the favorable attitude of Ptolemy toward the sesquiquartal third, he is obviously not swayed from his loyalty to the Boethian-Pythagorean heritage, and his final appeal is to a defender of the rationalist position, Porphyry.

Giovanni Spataro

The defense of Ramos' position was assumed by his pupil, Spataro, choirmaster at San Petronio in Bologna. He was at a considerable disadvantage, for he could not read Latin and had to use an Augustinian friar to translate for him. This also meant that most of the humanist literature was unavailable to him. Spataro nevertheless boldly pointed out errors in Gaffurio's reading of Boethius and other authors. On the point made in the above quotation, Spataro pleads that Ramos should not be blamed for describing the tuning that singers actually use, namely a ditone of 5:4 proportion and not the theoretical one of 81:64. The difference between them, 81:80, is not, as Gaffurio claims, inaudible. Ramos considered it significant and distinctly audible.[22]

Spataro insinuates that Gaffurio admitted the defeat of his own and Pythagoras' theories when he acknowledged that musicians tempered certain intervals by ear, purposely altering consonances from their rational proportions. This *participatio*, as it was called, Spataro argues, means that all intervals besides the octave deviate from the Pythagorean proportions; in other words, the Pythagorean doctrine is unsuited to musical practice, "for if the Pythagorean arrangement followed by you needs the aid of heightening and lowering, such an arrangement in the sole Pythagorean genus cannot suit musical practice. Through this adjustment of the Pythagorean diatonic genus, one passes from this genus to that called by Ptolemy intense diatonic. I say that you *tacite* conclude that the Pythagorean doctrine, as far as practice is concerned, is altogether useless, deceptive, and futile."[23]

21. Gaffurio, *De harmonia*, II, 34, fol. 52v. All of this quotation dates from 1500 except the last sentence, which was added before publication in 1518. The subsequent three chapters similarly reject the 6:5, 5:3, and 8:5 ratios for the remaining imperfect consonances.

22. Spataro, *Errori*, Error 22, fol. 21v.

23. Ibid., Error 26, fols. 22v–23r: "perche se la pythagorica institutione (da te seguitata) ha bisogno de aiuto per intensione: et remissione/ tale institutione non potrà conuenire per se al Musico exercitio: in lo solo diatonico genere pythagorico: & perche (per tale adiuuamento) del genere diatonico pythagorico, se passa in quello genere chiamato da Ptolomeo intentum diatonicum genus. Dico che da te (tacite) e concluso/ che la pythagorica doctrina (in quanto a la exercitatione) essere omnino inutile: frustatoria: & uana."

Spataro's case was built entirely on his observation of practice. He was sure that the syntonic diatonic tuning of Ptolemy, "which divides the tetrachord by the ratios 16:15 at the bottom, then 9:8 and 10:9—a monochord produced by Ptolemy—is that practiced in active music today."[24] Spataro's knowledge of Ptolemy evidently came from Gaffurio and Boethius, for like them he made the mistake of attributing a Hypermixolydian octave species to Ptolemy.[25]

Lodovico Fogliano

It was not until Lodovico Fogliano's treatise *Musica theorica* (1529) that the imperfect consonances in just tuning received a logically developed defense. Fogliano was exceptionally well qualified to deal with questions of Greek music theory. He had experience as a singer and composer, and he knew Greek well enough to contemplate the translation of the works of Aristotle into Italian. Pietro Aretino wrote to him: "If you start to render in our vernacular the Greek of Aristotle, you will be the cause of making bigger than men those people who, not understanding the language of others, cannot derive benefit from a gift of nature. Surely you alone are qualified to clarify the obscure with your plain speech, sweetly opening the senses, confused in the clouds of the material. Therefore get on with your honored translation, providing for the enrichment of ambitious intellects."[26]

All that is left of Fogliano's work on Greek authors is a collection of extracts, definitions, and compendia, arranged by subject, in a manuscript headed "Flosculi ex philosophia Aristo. et Auerroijs A ludouico foliano mutinensi excerpti et in hunc vtilissimum ordinem redacti."[27]

Zarlino had a high opinion of Fogliano's work and in response to an inquiry from Gian Vincenzo Pinelli, Giuseppe Moleto prompted Zarlino to report what he knew of him. "I spoke to S. Zerlino on the subject of Foliano. He says that he was neither priest, friar, nor monk, and he never practiced music in public, but that he lived in Venice for a very long time. He was Modenese. He says that for someone who went slowly into musical

24. Ibid., Error 16, fol. 21v: "quale diuide el tetrachordo/ per semitonio sesquintadecimo in graue & per tono sesquioctauo/ & tono sesquinono: & perche tale monochordo (da Ptolomeo producto) e quello/ che in la actiua Musica oggi se exercita."

25. Ibid., Errori 25–26, fols. 36r–37r.

26. Pietro Aretino to Lodovico Fogliano, 30 November 1537, quoted by Girolamo Tiraboschi, *Biblioteca modenese* (Modena, 1781–86), II, 307.

27. Paris, Bibliothèque Nationale, MS lat. 6757, fols. 1–74v. At folio 74v we read: "Expliciunt flosculi doctrina aristo. et auerroijs. Incipiunt quaedam fragmenta diuersarum materiarum." The manuscript ends on fol. 88. Included in the "Flosculi" is material on harmonics, music in education, and the moral effects of music, drawn from Aristotle's *De anima*, *Politics*, and Averroës' commentaries on the *Metaphysics*, *Ethics*, *Posterior Analytics*, and *De anima*.

things, he wrote better than anyone else on the subject."[28] Zarlino's admiration for Fogliano is understandable, since, unlike many of his predecessors, he was not a compiler but sought to investigate questions of music theory by observation and deduction. Fogliano espoused the method of Aristotle's *Posterior Analytics* and based his chapters on sound, consonance, and hearing on his *De anima* and *Physics*.

Fogliano establishes at the outset that the subject of the discipline of music is sonorous number, namely the number that measures the parts of a string. For example, if a string is divided into five parts, and a bridge is placed so that two parts are on one side of it and three on the other, and the two sides are struck at the same time, we know that the sounds issuing from them will compare as 3:2. Thus sonorous number is considered the subject of music. But music, insofar as it consists of sound and this is caused by motion, is not a mathematical but a natural phenomenon. This places music as a science in an intermediate position between the mathematical and natural.[29] Fogliano recognizes the existence of both consonance and dissonance on the grounds that if consonance is perceived, its contrary must also be perceptible.[30] Before giving his own analysis of the circumstances of consonance and dissonance, Fogliano reviews the position of the Pythagoreans, who accepted as forming consonance the multiple ratios 2:1, 3:1, and 4:1 and the superparticular 3:2 and 4:3.

Nec plures his posuerunt consonantias: ut apparet ex suis quae ad nos peruenerunt opinionibus:
Sed haec positio licet maxima innitatur auctoritate nihilominus mihi uidetur falsa: quum sensui contradicat: quis enim nisi sensu aurium diminutus neget plures alias a praedictis quinque: inueniri consonantias? infra enim diapason nonne praeter istas inuenitur: Semidytonus:

They reckoned among the consonances no more than these, as it appears from those opinions of his [Pythagoras] that have reached us.
Although this position leans upon the greatest authority, nevertheless it seems false to me, since it contradicts sensation. For who—unless he were deprived of the sense of hearing—would deny that consonances other than the five established ones could be found? Are there not found below the octave besides these the semiditone, the

28. Giuseppe Moleto to G. V. Pinelli, 20 January 1580, Milan, Biblioteca Ambrosiana, MS S.105 sup., fol. 49r: "ho parlato col S. Zerlino in materia del Foliano, egli dice che non era ne prete, ne frate, ne monaco. et che non esercito la musica in luogo publico, ma che sene é vissuto à Venetia lunghissimo tempo. Esso modonese, et dice di più che per huomo che andasse à lentone nelle cose della musica, ha scritto meglio d'ognun' altro intorno à tal cose."

29. *Musica theorica*, I, 1, fol. 1r–v. The notion that harmonics combines physical science and mathematics is expressed by Aristotle *Physics* 2.2.194a.

30. Ibid., II, 2, fol. 15r.

Dytonus: Hexachordum minus: & Hexachordum maius: similiter supra Diapason: nonne inuenitur Diapason cum semidytono: & Diapason cum dytono: & diapasondiatessaron: Quam posuit Ptholomaeus? necnon diapason cum minori hexachordo: & diapason cum maiori hexachordo: hae autem quas addimus: sunt consonantiae: quae a practicis appellantur Tertia minor: Tertia maior: Sexta minor: Sexta maior: Decima minor: Decima maior: Vndecima: Tertiadecima minor: Tertiadecima maior: quae omnia interualla esse ueras & ualde delectabiles consonantias non potest negari: nisi negato sensu: quod est inconueniens: omnes enim concentuum auctores in suis compositionibus: similiter: Omnes organistae: Omnes cytharoedi: Et omnes naturaliter fine aliqua arte canentes huiusmodi utuntur consonantiis: ut scit quilibet in hac facultate mediocriter eruditus.[31]

ditone, the minor hexad, and the major hexad; similarly above the octave, are there not found the diapason-plus-semiditone; the diapason-plus-ditone, the diapason-plus-diatessaron, which Ptolemy included? Are there not also the diapason-plus-minor hexad, and the diapason-plus-major hexad? These, which we yet add, are consonances, and they are called by practicing musicians minor third, major third, minor sixth, major sixth, minor tenth, major tenth, eleventh, minor thirteenth, major thirteenth, all of which intervals, it cannot be denied, are true and very delightful consonances, unless the sense is denied, which is inappropriate. For all authors of part music in their compositions, and, similarly, all organists, all singers to the lute, and in the end all others who make music use consonances of this kind, as anyone moderately learned in this discipline knows.

The experience of the ear and of working musicians and composers have determined that these are all consonances. Indeed, Fogliano defines consonance in purely sensory terms: "a mixture of two sounds which are separated with respect to high and low pitch that is pleasing to the ears." Dissonance, on the contrary "is a mixture of two sounds separated with respect to high and low pitch that is displeasing to the ears." Granted that all the intervals mentioned above are consonances, Fogliano proceeds to show that a string may be divided through superparticular proportions other than those accepted by the Pythagoreans and thereby produce consonances. The ditone (5:4) and semiditone (6:5) are two. Moreover there are ratios of the multiple superparticular class that generate consonances: 5:2, or dupla sesquialtera, the diapason-plus-ditone; 10:3, the tripla sesquitertia, the diapason-plus-major hexad; 16:5, the tripla sesquiquinta, the diapason-plus-minor hexad. Further, the superpartient genus of ratios generates the following consonances: 5:3, the bipartiens tertia, major hexad; 8:5, superbi-

31. Ibid., II, 1, fol. 11v.

partiens quinta, minor hexad. Finally, the multiple superpartient genus of ratio elicits consonances: 8:3, dupla superbipartiens tertia, the diapason-plus-diatessaron; 12:5, the dupla superbipartiens quinta, diapason-plus-semiditone.

Fogliano defends the determination of consonance and dissonance by sense experience through Aristotelian physics, psychology, and logic. First he analyzes the interaction of the sounding body and the air:

Sonum igitur: uniuersaliter generari per expulsionem aeris uiolentiam: ab omnibus concessum est & ratione comprobatum: Aer enim sic expulsus antequam natus sit cedere per naturam: necessario frangitur: unde sic fractus emittit sonum: talem autem uiolentam Aeris expulsionem pluribus modis contingit fieri. Aliquando enim fit ex percussione duorum corporum adinuicem: Quae solida sunt & dura: Aliquando etiam ex concursu unius corporis solidi & firmi ad corpus fluidum: ut quando uirga impetuose mota per aerem generat sonum: aer enim sic scissus uelocissime congregatur: & confluit ex omni parte: uacuum abhorrente natura: unde fit uelocissima quaedam aeris condensatio: quae resistit uirgae percutienti: & talis condensatio fungitur uice corporis solidi.[32]	Sound, therefore, is universally generated by the violent expulsion of air; this is agreed to by all and is corroborated by reason. The air thus expelled, made to give way before it was intended to by nature, is necessarily broken up. Thus fractured, it emits sound. This violent expulsion of air may happen in several ways. Sometimes it is through the striking of two bodies together that are solid and hard. At other times it is through the collision of a solid and firm body with a fluid one as when a switch impetuously swung through the air generates sound. Air thus torn is very quickly compressed and flows together from every direction, since nature abhors a vacuum. Thus a very rapid condensation of air comes about that resists the striking switch, and this condensation is discharged by exchange with the solid body.

Now Fogliano applies Aristotelian logic to distinguish the formal relationships among the elements in the interaction. Three things concur in the generation of sound: that which violently expels the air, the air violently expelled, and the motion of the expulsion. None of these three is formally the cause of sound. The agent expelling the air and the air itself are bodies, species of the genus substance. But sound is an occurrence (*accidens*), not a substance. Sound is also not the motion or the expulsion of the air, because it is a *sensibilis proprium*, an object of a particular sense, not a *sensibilis communis*, an object common to all senses. In *De anima* 2.6 Aristotle makes

32. Ibid., II, 2, fol. 15r.

this distinction: color is a special object of sight, sound of hearing, flavor of taste; movement, rest, number, figure, and magnitude, on the other hand, are objects common to all the senses. Since motion is a "common sensible," whereas sound is a "sensible particular," Fogliano argues, motion of air cannot be sound. Fogliano thus moves away from Aristotle's position, which was that sound was motion, toward the view that it is an effect of motion.[33] Fogliano concludes that sound is a passive or affective quality:

Dico quod sonus est passibilis qualitas proueniens ex motu aeris uiolento ac praecipiti habens esse in aequali mensura cum illo: dicitur autem passibilis qualitas: quoniam: quicquid potens est: immutare sensum est passibilis qualitas: sonus potest immutare sensum: ergo sonus est passibilis qualitas.[34]	I say that sound is a sensible quality arising from a violent and precipitous motion of the air that is commensurate with it. It is said to be a passive quality because whatever is able to alter the sense is a passive quality. Sound is capable of altering the sense; therefore sound is a passive quality.

Sound, then, is a sensible quality arising from the violent motion of the air, is commensurate with it in that it lasts as long as the motion, and has the potential of altering the sense. By altering the sense, Fogliano means that sound acts upon the natural potency of the hearing by producing in it sound's own species. Both sound and hearing being natural potentials, the hearing has definitive cognition of consonance and dissonance.

Freed of the necessity of determining the limits of consonance by numerical definition, Fogliano proposes a new enumeration and classification of consonances. He limits the consonances to seven within the octave, for after the octave they seem to return as if by a cyclical motion, just as numbers do after ten. This happens only with the octave, which, although it has two sounds, strikes the sense as if it were a single sound. All diversity of consonances is limited to the compass of the diapason, so far as the judgment

33. For example, in *De anima* 2.8.420a Aristotle states that: "sound is the movement of what can be moved, in the way that things rebound from a smooth surface when struck against it" (trans. W. S. Hett [Cambridge, Mass., 1967], p. 115). Fogliano may have derived this argument from Albertus Magnus, who in the section *De homine* of Part II of the *Summa*, holds that sound cannot be motion, because motion is an object of the common sense, whereas sound is an object of the hearing only (1498 ed., fol. 120v).

34. *Musica theorica*, II, 2, fol. 15v. Here too Fogliano appears indebted to Albertus Magnus, whose words are similar: "Dicimus ergo quod sonus est qualitas sensibilis perueniens ex fractione motus aeris et ens cum illo. dico autem qualitas sensibilis propter sensum auditus et dico ex fractione motus: quia non quilibet motus aeris facit sonum: sed motus frequens aerem ante quam diuisibilis sit per naturam," *De homine*, 1498 ed., fol. 120v.

of the ear is concerned.[35] The seven consonances, then, are semiditone, ditone, diatessaron, diapente, minor hexad, major hexad, and diapason.

Fogliano limits the perfect consonances to the diapason and diapente. The rest are imperfect, including the diatessaron, which was traditionally a perfect consonance. He proves this by definition:

Probatur sic: eorum quae ab aliqua potentia sub ratione alicuis communis appreheduntur illa sunt perfecta: quae in suo genere uirtutem habent quietandi & complendi appetitum talis potentiae: reliqua uero quibus hoc repugnat: sunt imperfecta.[36]	It is proved this way: of those things which are comprehended by some potential by reason of having something in common, those are perfect which in their genus have the power of quieting and fulfilling the appetite for such a potential. The rest, to which this is opposed, are imperfect.

The diapason, diapente, and bisdiapason are capable of fulfilling the appetite of the auditory sense; hence they are perfect.

Apart from the seven consonances named and their compounds with the octave, all other intervals recognized by musicians are dissonances. These are essential to the progression of the consonances, as in going from the diatessaron to the diapente. Fogliano proposes six dissonances: major tone, minor tone, major semitone, minor semitone, minimal semitone, and comma.[37] In his determination of the ratios of these dissonances Fogliano adopts a system of just intonation. The ratios are 9:8, major tone; 10:9, minor tone; 27:25, major semitone; 16:15, minor semitone; 25:24, minimal semitone; 81:80, comma.

Fogliano applied his empirical methodology to the tuning of the practical musical scale. He proposes dividing the monochord in "a new way, almost according to the sense, and materially (nouo modo quasi secundum sensum: & materialiter)"[38] in contrast to the usual mathematical method. Like Ramos' division, Fogliano's permitted not only pure fifths and fourths, as in the Pythagorean tuning, but also pure major and minor thirds. His diatonic division corresponds to the scale shown in Figure 10.3.

The central tetrachord is identical to Ptolemy's syntonic diatonic, descending 10:9, 9:8, 16:15. However, unlike Ramos', which is laid out on the A octave, Fogliano's is on the C octave, so that there are two identical

35. *Musica theorica*, II, 4, fol. 16v: "septem sint consonantiae: quarum maxima est diapason: ad quam tota: quo ad iudicium sensus: terminatur consonantiarum diuersitas."

36. Ibid., II, 5, fol. 17r.

37. Ibid., II, 7, fol. 18r.

38. Ibid., III, 1, fol. 33r.

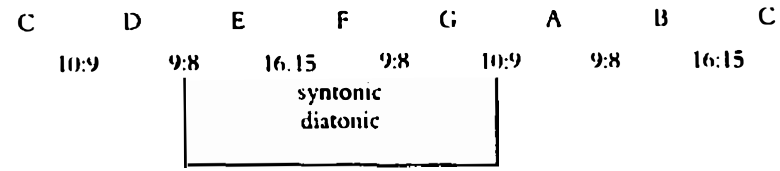

Figure 10.3:
Fogliano's monochord division

tetrachords rising 10:9, 9:8, 16:15, the reverse of Ptolemy's descending pattern. Fogliano was probably aware of these similarities and differences, but he did not name either Ramos or Ptolemy. His choice of the C octave he justifies as *more practicorum*—in the manner of practitioners. But it has important theoretical advantages, because it affords a number of harmonic means to aid in the division of the monochord. The octave c-c′ is divided harmonically with the fifth below and the fourth above, which yields the best-sounding combination of these two intervals (see Figure 10.4). The diapente c-g in turn may be harmonically divided to produce a ditone below and a semiditone above, again offering the best sounding combination of the two thirds. Similarly the diapente f-c′ is divided harmonically by a. The Roman numerals in Figure 10.4 indicate the number of the step in the division. Fogliano further divides the string to obtain a chromatic scale. But in order for each note of the chromatic scale to have a corresponding major and minor third above and below it is necessary to have alternate notes a comma apart, two D's, and two B♭'s. Then the alternate D will be a pure minor third against F, which otherwise would be too small, whereas the normal D will make a perfect fourth with G. Similarly an alternate higher B♭ permits a just minor third with G, whereas the normal B♭ makes a perfect fourth with F. Fogliano admits that having two D's and two B♭'s is an inconvenience in musical practice. Therefore he proposes dividing the spaces between the duplicate notes into two equal parts and at the midpoints placing a compromise D and B♭, which, though not affording precisely just intervals, produce intervals that deviate a mere half comma from purity.[39]

The space that needs to be divided is the comma, 81:80. According to Pythagorean mathematics, this is not possible, as there is no mean proportional between the terms of a superparticular ratio. Fogliano proposes a geometric solution for the required division, relying upon Euclid's construction of Book VI, Proposition 9.[40] Fogliano illustrates the construction in a figure (Figure 10.5). In the figure, AB:BD = 81:80. According to

39. Ibid., III, 2, fol. 35v.

40. Cited in ibid., III, 2, fol. 36r.

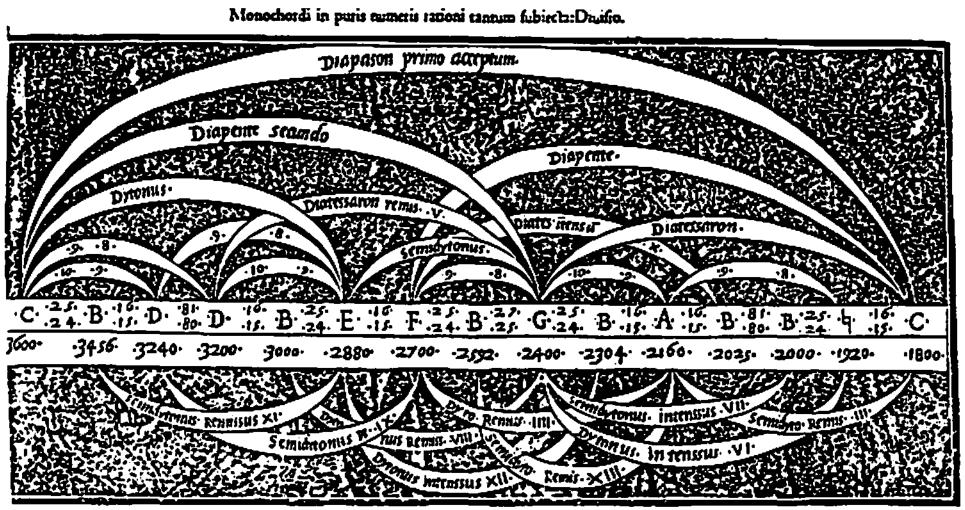

Figure 10.4.
Division of the monochord subject to the ratio of pure numbers, from Fogliano, *Musica theorica*, III, 1, fol. 34v

Euclid, if a semicircle is described around the line AD and a perpendicular to the circumference is drawn from B, BC is the required geometric mean. Then AB:BC = BC:BD. The string length BC, which cannot be represented by a whole number, will sound the desired intermediate note.

Fogliano was not the first to challenge the impossibility of finding a mean proportional between the two terms of a superparticular ratio. Those who preceded him in this had profited, as he had, by the revival of interest in the *Elements* of Euclid on the part of humanist mathematicians. The medieval translation by Campano had been published in 1482.[41] In 1496 Jacques Lefèvre d'Étaples showed how Euclid VI, 9, and VI, 13, could be applied to find the mean proportional between two string lengths.[42] His object was to find the geometric mean that would divide the intervals formed by the fractions 9:8 (whole tone), 4:3 (fourth), 3:2 (fifth), and 2:1 (octave), where ab:bc = 8:9; ab:bd = 4:3, ae:be = 3:2; and ab:bf = 2:1. A circle is constructed around line abc; similarly around abd, abe, and abf (see Figure 10.6). Then a perpendicular to abc is drawn at b to intersect the circles. The distance from b to the intersection with the circle is the geometric mean. So bg is the mean of 9:8, bh of 4:3, bi of 3:2, and bf of 2:1. These

41. *Praeclarissimus liber elementorum in artem geometrie*, trans. Campano of Novara (Augsburg, 1482).

42. *Musica libris demonstrata quatuor* (Paris, 1496), III, 35, fol. g6v. (Paris, 1552 ed., fol. 29v).

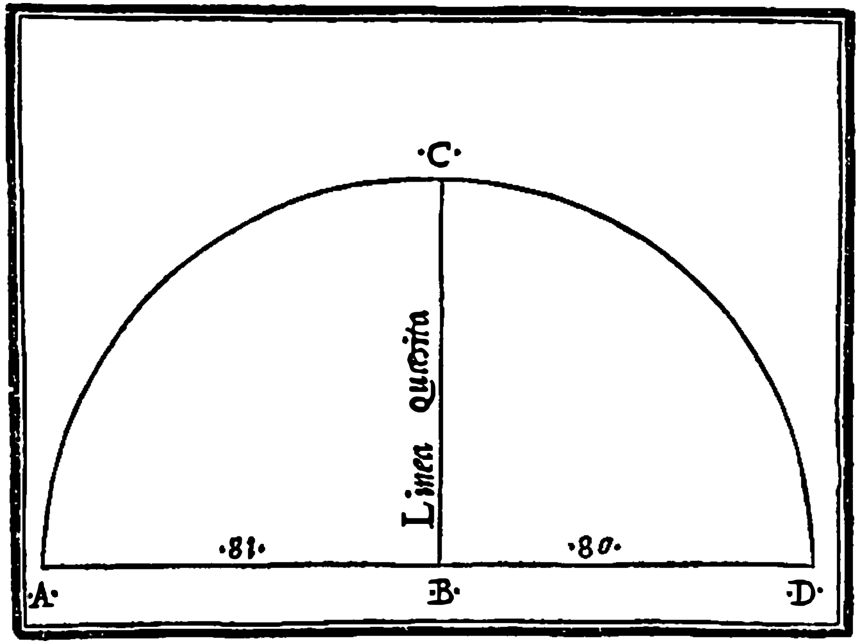

Figure 10.5.
Geometric division of the comma, from Fogliano, *Musica theorica*, III, 2, fol. 36r

lengths are marked on the string bc. The only geometric means of practical interest are those of the whole tone (marking off a mean semitone) and the octave (a tritone).

Lefèvre's demonstration is purely theoretical. Heinrich Schreiber (Grammateus), on the other hand, in 1518 applied the construction to locate a mean-tone between two diatonic steps, for example, the tone between G and A that could serve as both G♯ and A♭.[43]

Erasmus of Höritz, in his unpublished treatise *Musica* of around 1506, showed how the 9:8 tone may be divided by computation and proved the method by Euclidian propositions.[44]

So the revival and spread of Euclid's *Elements* contributed to solving some practical problems that surfaced once theorists began to shed prejudices about numbers. Of those who applied the geometric method, Fogliano was

43. *Ayn new kunstlich Buech* (Nuremberg, 1518).

44. Rome, Biblioteca Apostolica Vaticana, MS Reg. lat. 1245, Book VI, Proposition 17, fols. 66r–67r. See Palisca, "The *Musica* of Erasmus of Höritz" in *Aspects of Medieval and Renaissance Music*, ed. Jan LaRue (New York, 1966), p. 640.

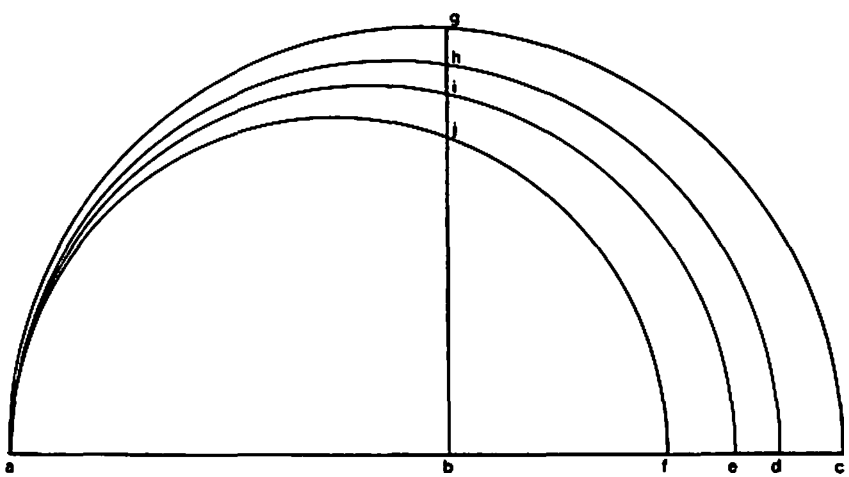

Figure 10.6.
Geometric division of the whole tone, fourth, fifth, and octave, from Lefèvre d'Étaples, *Musica libris quatuor demonstrata*, III, 35

surely the most aware of the practical implications and the most deliberate in his methodology and objectives.

Gioseffo Zarlino

Zarlino's relationship to classical sources, to Boethius, and to more contemporary writers is a very complex one. He read very widely and constantly quoted authority. He cited sources when they advanced his argument and if they were ancient. (The citations are more precise in the 1573 edition of the *Le Istitutioni harmoniche*, where he gives title, book, and chapter, than in that of 1558.) Modern authors—as far back as Gaffurio or as recent as Fogliano and Glarean—he utilized also, sometimes even paraphrased, but without acknowledgment. Zarlino did not depend on any one school of thought, nor did he accept any body of theory as a foundation. He constructed a system of his own. How much of it was owed to his teacher and mentor Adrian Willaert cannot be ascertained, as Willaert left no theoretical writing. It is probable that he owed more to him in the area of musical practice than in that of speculative theory.

Zarlino fervently believed in the possibility of a rational explanation for musical practice and aesthetic preferences. To do something without a reason was the ultimate error. The first two parts (called books in the second and later editions) of the *Istitutioni* are conceived as a preparation for the

third and fourth, which are practical treatises on counterpoint and the modes. Thus the two speculative books were not intended to have an independent existence, like those of Gaffurio or Boethius, but to serve as a foundation for practice.

Zarlino did not simply accept classical authority, which in any case was full of contradictions. To fulfill the goal he had set for himself he saw that he had to raise every question anew, to doubt every previous solution, to reason out and prove the most obvious principles. If this mode of operation was unimpeachable, his facts, proofs, and solutions often were not. Zarlino was not a Pythagorean, although he was fond of number theories. He cannot be called a Neoplatonist, although Plato's ideas, which he knew through Ficino's translations and commentaries, appealed to him more than did those of the Aristotelians. He had a strong belief in the uniformity, wisdom, and rationality of nature— *la Natura*—whose secrets he thought he could discover through reason, theology, or by consulting authority, but without further observation or experiment. He was quite consistent in applying Aristotle's categories and dialectics. Of the ancient musical authors, he most admired Ptolemy, whose balancing of reason and sense experience harmonized with his own inclination. Zarlino did not read him thoroughly, however, and he disagreed with some of what he did read. He shows no evidence of having studied Aristoxenus directly in preparation for the *Istitutioni*. Only in the *Sopplimenti musicali* (1588) is his influence felt. In the *Istitutioni* Zarlino used Plutarch, Pliny, and Athenaeus for historical information, and he cited the treatises of Aristides Quintilianus, Cleonides (whom he calls Euclid), and Gaudentius, but there is no evidence in this work of his acquaintance with the Bellermann-Najock anonymi, Nicomachus, or Alypius.[45] In addition he relied on a vast number of general Greek and Latin sources that contain musical, mathematical, humanistic, and philosophical erudition. He had some acquaintance with Greek, as he shows in his book, but it must not have been much, as he requested Antonio Gogava to translate Aristoxenus' *Harmonics*.

Zarlino was selective in what he took from both the ancient and modern authors. For example, he did not accept the principle that musical intervals are built up from an indivisible unit, like numbers from unity. He attributes to Aristoxenus the theory expressed by Aristotle that the diesis is such a basic unit.[46] He prefers the theory transmitted by Ficino from Plato's *Epinomis*, that all consonances and intervals begin in the diapason, since 2:1 is

45. See the "Index of Classical Passages Cited" in Zarlino, *On the Modes*, trans. Vered Cohen, ed. Claude V. Palisca (New Haven, 1983), for a sampling of his reading. All of these authors became known to him, however, before he wrote the *Soppliplimenti*.
46. *Metaphysics* 10.1.1053a.

the beginning of proportion.[47] Ptolemy's exclusion of all but superparticular ratios from his approved tetrachords in the various genera was unacceptable to Zarlino, even though Boethius seemed to go along with it, because it was an unnecessary limitation that did not advance his purposes.

It is instructive to compare Zarlino with Fogliano, from whom he borrowed a number of concepts and principles. Like Fogliano, Zarlino concluded that the division of the octave sung by contemporary musicians was the one that provided both perfect and imperfect consonances in their simplest ratios. It was based on the species of tetrachord called by Ptolemy the syntonic diatonic, which Fogliano did not identify by either author or name. Fogliano chose it on the grounds of aural experience, and he deemed this sufficient reason, since the ear was the final judge. Zarlino was not confident of the rightness of the ear's choice; rational arguments and authority for the inclusion of intervals within the consonant class had to be found. Thus Fogliano established his classification on the basis of usage and aural preference, whereas Zarlino devised numerical criteria that did not contradict the sense.

Zarlino accepts Fogliano's resolution of the status of musical science as midway between mathematics and natural science. He adds that this is confirmed by Avicenna, who held that music received its principles from natural science and from the science of numbers.[48] Having accepted Fogliano's proposition that the subject of music is the sonorous number, Zarlino (right) quotes, without attribution, his definition of this phenomenon (left):

Numerus sonorus . . . nihil aliud est: nisi numerus partium sonori corporis: utputa: chordae: Quae numeri ac discreti accipiens rationem: nos certiores reddit de quantitate soni ab ea producti.	il Numero sonoro non è altro, che il numero delle parti d'un Corpo sonoro, come sarebbe di vna chorda, la quale pigliando ragione di quantità discreta, ne fa certi della quantità del suono da lei produtto.[49]

Sonorous number is nothing other than the number of the parts of a sounding body, such as a string, which, subjected to an accounting of the discrete quantity, renders us certain [Fogliano: more certain] of the quantity of the sound produced by it.

Zarlino finds this definition incomplete. His objections are semantic and hairsplitting, however, and introduce irrelevant metaphysical considera-

47. *Istitutioni*, II, 48, p. 142. Plato *Epinomis* 991a, Novotny ed., p. 40; Harward trans., p. 107.

48. *Le Istitutioni harmoniche*, I, 20, p. 31.

49. Fogliano, *Musica theorica*, I, 1, fol. 1r; Zarlino, *Istitutioni*, I, 19, p. 29.

tions. He objects to the use of the term *soni*, because vocal tones *(voci)*, not sounds, are what the musician considers and those on which he bases what instruments do. He therefore modifies the definition to read:

Numero sonoro è Numero relato alle voci, & a i suoni; il quale si ritroua artificiosamente in vn corpo sonoro, si come in alcuna chorda, la qual riceuendo la ragione di alcun numero nelle sue parti, ne fa certi della quantità del suono produtto da essa, & della quantità delle voci, riferendo, ouero applicando essi suoni ad esse voci.[50]	Sonorous number is number related to vocal and instrumental sounds. It is found artificially in a sounding body, when a string is subjected to an accounting of the number of its arts, for this renders us certain of the quantity of the sound produced by it and, by referring or applying these sounds to vocal tones, the the quantity of the vocal tones.

Fogliano's original definition was better, because it included voices or any other sound source in a more concise formulation. Zarlino's rephrasing is simply an accommodation to his questionable bias for voices as natural, human, and therefore superior and more fundamental than instrumental sounds.

Zarlino evidently was also not satisfied with Fogliano's treatment of the nature of sound and consonance, for he goes back to Aristotle for the generation of sound and develops his own analysis of the causes of consonance. He attributes to Aristotle the principle that the generation of sound requires three things: that which strikes, the object struck, and a medium.[51] He then gives some of the same examples of sound production as Fogliano. So far as consonance is concerned, sounds are the material, numerical proportions the form. However number is not the cause, either proximate or intrinsic, of musical proportions or of consonances. Four things must concur: the goal of the action (playing in harmony), which is to profit and delight; the agent or efficient cause, that is, the musician; the material or material cause, which are the strings; and the form or formal cause, namely proportion. The first two are extrinsic, the last two intrinsic.[52]

Despite the elaborate proof that number cannot be the cause of consonance but only a means for measuring the terms of a proportion, Zarlino conceives a sacred precinct, the *senario*—the set of numbers from one to six—to contain the realm of consonance. His chapter on the virtues of this number is pure numerology. Of the twelve signs of the zodiac, six are always in our hemisphere, the others hidden below the earth. There are six errant bodies in

50. *Istitutioni*, I, 19, p. 29.

51. Ibid., II, 10.

52. Ibid., I, 41.

the sky: Saturn, Jupiter, Mars, Venus, Mercury, and the moon. There are six substantial qualities of the elements: acuity, rarity, movement, and their opposites, obtuseness, density, and stillness. Six circumstances are necessary to existence: size, color, shape, interval, state, and motion. Six are the species of movement: generation, corruption, increase, decrease, alteration, and change of location. According to Plato there are six differences of direction: up, down, ahead, behind, right, and left. Closer to home, the intervals (*voci musicali*) are of six types: unisone, aequisone, consone, emmele, dissone, and ekmele. And the modern modes come in sixes: six authentic, and six plagal! He gives a number of further examples of this ilk[53] before turning to the mathematical and musical properties of the six-part number.[54] Six is the first perfect number, meaning that it is the sum of all the numbers of which it is a multiple, that is, one, two, and three. Any two numbers from one to six yield the ratio of either a simple or composite consonance. (See Figure 10.7.) The two largest perfect consonances are formed from the first three numbers and are divided by harmonic means to produce the next perfect consonances. The diapason, 2:1, in the form 4:2, divided harmonically by 3, yields the diapente, 3:2, and the diatessaron, 4:3. The diapente, 3:2, in the form 6:4, divided by 5, produces the ditone, 5:4, and semiditone, 6:5. The major hexad, 5:3, harmonically divided by 4, yields the diatessaron and ditone. Any of the numbers multiplied by any other will always produce, when juxtaposed with another so generated, a harmonic relation. Further, if the six numbers as they occur in sequence are each squared, the adjacent squares will form the dissonances that separate the consonances, the tones and semitones.

The major hexad, 5:3, is regarded as distinct from the other consonances of the senario for two reasons: it is formed from a superpartient ratio, unlike the others, which are all superparticular in their minimal terms. It is also a composite consonance, made up of a diatessaron and ditone, because in its minimal terms, 5:3, it can be mediated by another number, namely 4. Similarly the minor hexad, 8:5, is mediated by 6, producing a diatessaron and semiditone. Here Zarlino is confronted with a contradictory element, a consonance the terms of whose ratio are not both in the senario. His rationalization for its inclusion is ingenious:

Et benche essa tra le parti del Senario non si troui in atto, si troua nondimeno in potenza: conciosiache dalle parti conte- nute tra esso piglia la sua forma,	Although it is not found in actua- lity among the parts of the senario, it is found in potential, for it takes its form from the parts of which it is a composite,

53. Ibid., I, 14.
54. Ibid., I, 15.

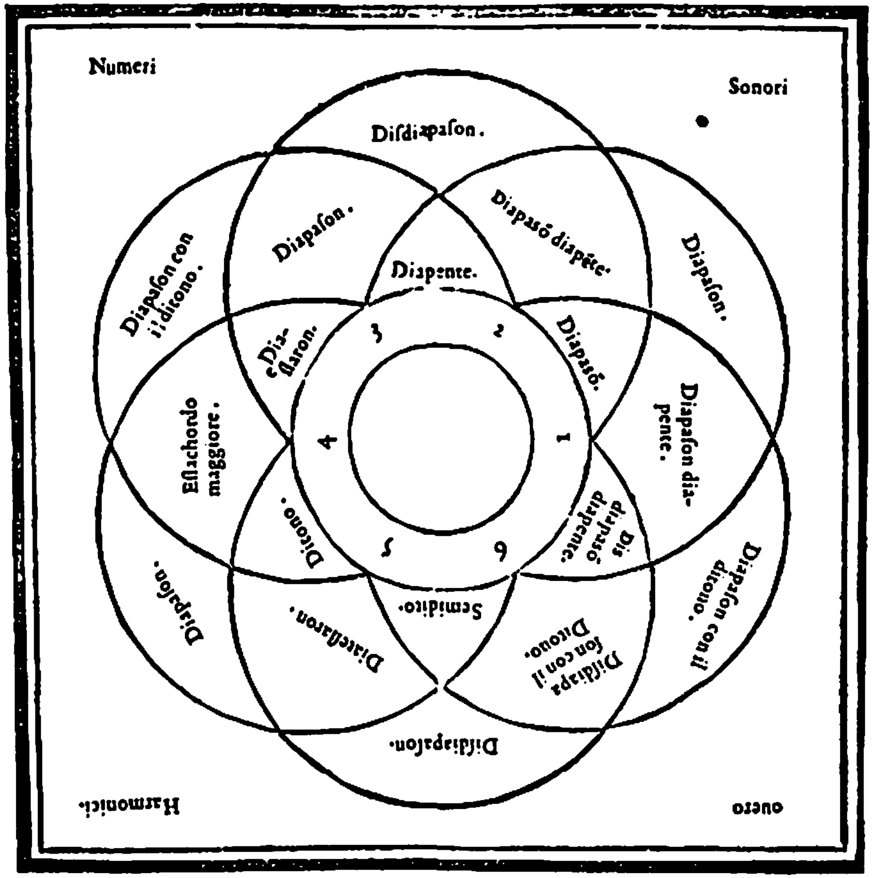

Figure 10.7.
Sonorous or harmonic numbers, from Zarlino, *Istitutioni*, I, 15, p. 25

cioè dalla Diatessaron & dal Semiditono: perche di queste due consonanze si compone: la onde tra'l primo numero Cubo, il quale è 8. viene ad hauer in atto la sua forma.[55]	that is, from the diatessaron and semiditone, because it is composed of these two consonances. For within the first cubic number, 8, its form attains actuality.

Zarlino did not in this book extend the realm of consonance to the *ottonario*, and this with good reason, since it would have admitted the ratios in which one of the terms is seven, all falling outside the circle of consonances.

55. Ibid., I, 16, p. 27.

Zarlino was too pragmatic a musician to insist on the just consonances for instrumental music. He recognized that it was not possible to tune a chromatic keyboard so that every fifth, fourth, and third was in a ratio of the senario. He was willing to admit compromises in tuning these intervals in instruments, provided vocal music remained pure. His faith in nature demanded that the ideal ratios be operative in the natural medium of voices.

> If it were true that in voices as well as instruments we hear only the consonances and intervals out of their natural ratios, it would result that those which are born of the true harmonic numbers would never reach actuality but would remain always potential. This potential would be futile and frustrated, for every potential that is not put into action is without utility in nature. And yet we see that God and nature never do anything in vain.[56]

For instruments, he thus feels free to devise an "equally tempered diatonic monochord" which is a compromise between the Pythagorean diatonic ditoniaion and Ptolemy's diatonic syntonon. He divides the comma into seven equal parts and subtracts two of these parts from each fifth,[57] resulting in major thirds that are one-seventh comma smaller than 5:4. For dividing a ratio into equal parts, Zarlino gives the same construction as Fogliano but goes beyond the construction to refer to Euclid's proof.[58] He then presents an instrument for finding two mean proportional lines between two given lines that he learned of from Giorgio Valla's *De geometria*.[59]

By relying excessively upon reason and authority, Zarlino laid himself open to attack from those who were bent on testing some of his premises. To counter the attacks that inevitably came, Zarlino explored further the Greek authors on music. We shall, therefore, come back to him after we have considered some theories that rival his.

Francisco de Salinas

The remarkable *De musica libri septem* of Francisco de Salinas (1513–90) belongs more properly to a history of Spanish than Italian humanism. Yet it deserves some discussion here, because Salinas lived in Rome and Naples between 1538 and 1558, years during which he studied the ancient Greek sources and probably drafted parts of his treatise. Blind from an early age, he was trained as a singer and organist. Yearning for a broader education, he exchanged organ lessons for lessons in Latin and later went to the Uni-

56. Ibid., II, 45.
57. Ibid., II, 43.
58. Ibid., II, 25; *Elements*, VI, 8.
59. Ibid., II, 25; Giorgio Valla, in *De geometria* IV (*De expetendis*, XIII), 2, fols. u6r–x1r, presents several solutions to this problem, one of which is the mesolabio.

versity of Salamanca, where he studied Greek, philosophy, and the arts. Service with Pedro Goméz Sarmiento, archbishop of Compostella, gave him the opportunity to go to Rome when Sarmiento was made a cardinal by Pope Paul III in 1538. There he became immersed in the study of music theory, for he realized that to be proficient with one's hands, as Vitruvius said of architects, was not sufficient if one sought to acquire real authority. He gives a partial list of the ancient sources he consulted in an autobiographical account in the early pages of his book:

> Those who aided me very greatly in this task, besides Boethius, whom every musician has on his lips, were manuscript books of ancient Greek authors not yet translated into Latin of which I still found a great plenty, above all the three books on harmonics of Claudius Ptolemy in the Vatican Library, to whom I do not know whether astronomy or music owes more, and the very instructive commentaries on them by Porphyry—of which the cardinal of Carpi made me a copy—containing most precious things collected from his reading of the ancients; two books of Nicomachus, whom Boethius follows; also one of Bacchius; three books of Aristides [Quintilianus]; also three of Bryennius, which the cardinal of Burgos of Venice himself attended to transcribing. . . . In this inquiry and investigation I spent more than twenty-three years.[60]

The Porphyry manuscript mentioned must be one of two from Valla's library that had belonged to Cardinal Rodolfo Pio di Carpi, now in Modena, Biblioteca Estense.[61] There were several manuscripts of Ptolemy in the Vatican, and Salinas would have found most of the other treatises he mentioned there also. The two books of Nicomachus may refer to the two books of the *Introduction to Arithmetic* which Boethius practically translated in his *De institutione arithmetica libri duo*, rather than the *Manual of Harmonics*, which is in a single book. Salinas relies on Nicomachus' arithmetic quite heavily in the mathematical sections of the first book. On the other hand, Salinas' list is otherwise an exclusively musical one, and by "two books" Salinas may therefore have meant the two works of Nicomachus—one on music and one on arithmetic—as in the inventory made under Sixtus IV in 1475, which describes item 365 (the present Vat. gr. 198) as "Nicomachi arithmetica et musica."[62] The reference in the quotation to Boethius following Nicomachus could apply to either alternative, since later Salinas refers to Boethius as having "followed Nicomachus in the two books con-

60. Francisco Salinas, *De musica libri septem* (Salamanca, 1577; facs. ed. Macario Santiago Kastner, Kassel, 1958), fol. 5r.
61. Numbers 149 and 152 in Puntoni's catalog of that library's Greek manuscripts. "Indice dei codici greci della Biblioteca Estense di Modena," *Studi italiani di filologia classica* 4 (1896):379–536.
62. Robert Devreesse, *Le fonds grec de la Bibliothèque vaticane des origines à Paul V* (Vatican City, 1965) p. 60. It also includes Ptolemy, Porphyry, and Bryennius.

cerning arithmetic, and in the first four concerning music."[63] Salinas' list was not intended to be exhaustive. Plutarch, Euclid, Cleonides, Gaudentius, and Alypius are some of the obvious omissions. Indeed, Salinas cites Plutarch's *De musica*, "Euclid's Isagoge" (the title shows he means Cleonides), and Gaudentius "Introductorium" elsewhere in the book.[64]

From the very title page, where Salinas advertises that he demonstrates the true doctrine of harmonics and rhythmics "according to the judgment of the sense and the reason," he professes his faith in the method of Ptolemy. Critical of both the Aristoxenians and the Pythagoreans, Salinas took a middle road: "In harmonics the judges are the sense and the reason, but not both the same way, because, as Ptolemy asserted, the sense judges concerning the matter and affection, the reason, concerning the form and cause. From these words we can draw the conclusion that, just as matter is completed by form, so sensory judgment is completed by the rational."[65]

Salinas did not disdain modern authors. Although he borrowed a great deal from Fogliano and Zarlino, he hardly mentioned them until he dedicated to each a critical review in a separate chapter of the fourth book.[66] Some of the debts to Fogliano are the theory of sonorous number, the break with the Pythagorean definition of consonance, the espousal of the syntonic diatonic tuning as the basis of modern vocal intonation, and the geometric division of the comma. To Zarlino he owed the theory of the senario, the treatment of the sixths as composite intervals, and the use of the mesolabio, among other doctrines. Salinas was a more perspicacious humanist than Zarlino in that he knew the contents of the ancient treatises more thoroughly and understood them better. But he was less of an antiquarian; he really had little interest in classical civilization as such and was bent on applying to modern music whatever he found useful in the older theories. Zarlino, on the contrary, was deeply interested in classical literature and the lore about Greek music but found little in it that was applicable to an already perfect art.

Their attitudes toward the chromatic and enharmonic genera illustrate the nature of the contrast. Rather than defining these two tetrachords in classical terms as dense in the lower pitches and sparse in the higher, as Zarlino and the older authors did, Salinas followed Nicola Vicentino in

63. Salinas, *De musica*, II, 18, p. 73: "Boethius autem totus Pythagoricus est, & in libris duobus de Arithmetica, & quatuor primis de Musica Nicomachum secutus."

64. Plutarch, in III, 4, p. 109; IV, 25, p. 217; Cleonides and Gaudentius, in II, 9, p. 55.

65. Salinas, *De musica*, I, 3. He devotes IV, 16–21, to a refutation of Pythagorean theories; IV, 22–24, to a critique of Aristoxenian harmonics. These chapters are translated in Arthur Michael Daniels, "The *De musica libri vii* of Francisco de Salinas" (Ph.D. diss., University of Southern California, 1962), pp. 364–94.

66. Salinas, *De musica*, IV, 32–33, trans. in Daniels, "The *De musica*," pp. 422–36.

making them dense throughout, that is, dividing the entire chromatic tetrachord into semitones and the entire enharmonic tetrachord into dieses.[67] The inspiration for the revival of the chromatic and enharmonic genera was surely the example of the Greeks, but neither of these authors modeled his theory of the genera on the ancient one, which was well known from Boethius, Gaffurio, and other authors.[68] Vicentino, particularly, is vague about how the two dense genera were practiced in ancient times. He says they were put to other uses than the diatonic, which was meant for common ears in public festivals, the chromatic and enharmonic being addressed to "purified ears" (*purgate orecchie*) in the private entertainments of gentlemen and princes, when great men and heroes were praised.[69] Vicentino gives no source for this, and although in his book he occasionally names ancient authors—Aristoxenus, Nicomachus, Ptolemy—there is no sign that he had read any of them. In humanist circles in Ferrara he certainly must have heard the virtues of the genera extolled, possibly by Francesco Patrizi, but Vicentino himself was by nature uninclined toward historical scholarship. Salinas, on the other hand, shows that he read Plutarch, and in Greek, for he gives in the original language the locus classicus from the speech of Soterichus on the virtues of the enharmonic and follows it with a translation:

At verò Musici nostri temporis pulcherrimum omnium, maximéque decorum genus, quod veteres propter maiestatem, grauitatém- que ipsius colebant, penitus repudiarunt, adeò vt ne qualis- cunque perceptio curáque sit plerísque Enharmononiorum inter- uallorum. Et tanquam ignauia, atque secordia inuasit eos, vt Diesim Enharmonion, ne speciem quidem omnino cadentium sub sensum praebere putent, eámque de canticis, atque modulaminibus exterminent.[70]	The musicians of our time, however, have repudiated altogether the most beautiful and charming genus, which the ancients, because of its majesty and severity, cultivated, so much so that the majority have no knowledge or concern at all about the enharmonic's inter- vals. So much laziness and sloth overcomes them, that they believe that the enharmonic diesis, of all things falling under the sense, is not perceptible, and they banish it from songs and melodic compositions.

This quotation, however, is not adduced in defense of the enharmonic but to substantiate its neglect and thereby prove a point against Didymus. In

67. For a detailed study of the different approaches to the genera, see Karol Berger, *Theories of Chromatic and Enharmonic Music in Late 16th-Century Italy* (Ann Arbor, 1980).

68. Salinas reports the shades of Aristoxenus, Didymus, Ptolemy, and others but only to show that they were erroneous solutions: *De musica*, IV, 22–29, pp. 212–22.

69. Nicola Vicentino, *L'Antica musica ridotta alla moderna prattica* (Rome, 1555; facs. ed. Edward E. Lowinsky, Kassel, 1959), I, 4, fol. 10v.

70. Plutarch *De musica* 1145A. Salinas, *De musica*, IV, 25, p. 217.

the chapter in which he introduces the dense genera[71] Salinas calls the enharmonic the best and most adaptable genus, but quite typically he depends on logical arguments to prove this rather than classical authority.

Salinas was the first modern scholar to distinguish between the tonoi and octave species and between these and the plainchant modes in a published book. (Mei preceded him in an unpublished book.) As will be shown in the next chapter, Salinas' treatment was too brief to give the reader a good idea of how these systems worked, but he apparently understood their functions. On the other hand, he obscured some aspects of the theory while clarifying others. He attributed eight rather than seven tonoi to Ptolemy. He unjustly charged that Boethius confused the tonoi and modes, when all he did was to translate *tonos* and *tropos* usually as "modus." Salinas was right, though, in criticizing Glarean and Gaffurio for having applied to the modes attributes that belonged to the tonoi.[72] Salinas falls into a similar error, though, when he associates the ancient harmoniai of Plato with the modes and then, by dividing six of them through the species of fifths and fourths, derives twelve.[73]

Salinas admired the work of Fogliano, of whom he says, "he has come far closer to a true understanding of the science of harmonics than all of the ancient and more recent [writers]."[74] He makes this statement at the end of a chapter in which he enumerates what he considers serious errors on the part of Fogliano. These are not scientific, logical, or scholarly errors but differences of opinion, and we need not go into them here. There is a similar chapter on Zarlino, whom he praises as having surpassed all those who wrote on music before him. The disagreements with Zarlino are also mainly matters of opinion, and some of the criticisms are founded on misreadings.[75]

The really significant challenge to the foundations of Fogliano's and Zarlino's speculative theory came from other quarters, from the scientist Giovanni Battista Benedetti and from Zarlino's pupil Vincenzo Galilei, and preparatory to their work were the findings of Girolamo Fracastoro.

Girolamo Fracastoro

With the work of Girolamo Fracastoro and Giovanni Battista Benedetti musical science enters a new period of discovery. Up to that time no significant advances had been made over the state of knowledge represented

71. Salinas, *De musica*, III, 2.
72. Ibid., IV, 12–13, pp. 198–201.
73. Ibid., IV, 7–8, pp. 187–91.
74. Ibid., IV, 32, p. 231; Daniels trans., p. 430.
75. Ibid., IV, 33, pp. 231–34.

by Aristotle's *De anima*, the Aristotelian *De audibilibus* and *Problems*, and the commentary of Themistius. Fracastoro and Benedetti, like Fogliano, worked within the Aristotelian tradition, but they were able to correct him and make notable advances.

Fracastoro (1483–1553) studied at the university in Padua, where he pursued literature, mathematics, astronomy, philosophy under Pietro Pomponazzi and Nicolò Leonico Tomeo, and medicine under Girolamo Della Torre and his son Marcantonio. He wrote poetry and practiced medicine and even combined the two in his famous Latin poem *Syphilis sive morbus gallicus* (1530). His *Naugerius, sive de poetica dialogus* (c. 1540) proposes beauty of expression as the distinctive end of poetry and criticizes the theory of imitation. In his scientific work he regarded nature as autonomous, independent of supernatural intervention, a reality that could be studied to reveal its regulating principles. He was contemptuous of astrological and numerological explanations, such as in theories of the critical days of a disease. He sought explanations in immediate causes of concrete events.

His clarification of the action of air waves in the transmission of sound came out of his analysis of contraries in *De sympathia et antipathia rerum liber unus* (Venice, 1546). It is agreed, he says, that material elements tend to return to their natural place. Thus something that is rarefied (*rarefacta*) tends to be condensed (*condensata*), and something condensed tends to be rarefied. Sound, which depends on this principle, requires a dense medium:

Soni quidem, nisi addensetur aer,	Unless air is compressed,
non sentiuntur, quoniam	sounds are not heard, because all
qualitates, quae sensus	qualities that move the sense
mouent, omnes quidem	require the
subiectum, in quo per se sunt,	substance in which they exist—
densum amant, medium vero,	the medium—to be dense.
per quod feruntur earum species,	This medium, through which the
non omnes densum volunt, sed	qualities' species are made, need not
quaedam rarum exposcunt, quaedam	always be dense but sometimes rare,
densius: . . . dico autem densum	sometimes dense. . . . I say dense
non per admistionem terrae, sed	not through the admixture of earth,
vi addensatum, quod in	but in the sense of the compression
aere accidit facto ictu.	that occurs in the air when it is hit.
Inde enim facta prius distrac-	After a drawing apart and rarefaction
tione, tum subita fit	has first been made, a condensation
addensatio partis post	immediately follows, part for
partem, more vndarum, vnde	part, in the manner of a wave, whence
circulationes conflantur, quod	the circles are stirred up that
non aliud est, quàm successive	are nothing but a successive
quaedam aeris addensatio in orbem	compression of the air in a circle,
facta, per quam delata species à	through which the species is carried

primo profecta
sensum dimouere potest.[76]

from where it first set out and is
able to move the sense.

The comparison of the sequence of compressions and rarefactions to a wave is significant although not carried far enough. The ancient authors compared the propagation of sound to the circular waves made in a pond when a pebble was thrown into it. Fracastoro goes a step further in saying that the cycle of compression and decompression itself resembles a wave, which, like the circular waves of water, moves in all directions. At the same time he clings to the idea of *species*, that "something audible" carried by the air waves which "moves the sense of hearing," by activating its potential for sound.

In a later chapter Fracastoro applies this model to explain sympathetic vibration:

Unisonum autem aliud unisonum
commotat, quoniam, quae similiter
tensae sunt cordae, consimiles
aeris vndationes & facere, &
recipere natae sunt, quae vero
dissimiliter sunt tensae, non
eisdem circulationibus aptae sunt
moueri, sed una circulatio aliam
impedit: ictus enim
cordae, motus est compositus è
duobus motibus, vno quidem,
quo corda pellitur ante,
hoc est versus aeris circu-
lationes, alio vero, qui retro
fit, corda reducente sese ad
situm proprium: si igitur mota
vna corda debet & alia moueri,
oportet, vt in secunda talis
proportio sit, vt vndationes &
circulationes aeris,
quae impellunt,
& faciunt motum ante, non
impediant motum, qui retro fit
à corda: quam proportionem
solum eae cordae habent, quae
etiam consimilem tensionem habent:
quae vero dissimilem sortitae
sunt tensionem, non sese commo-

A unison stirs another unison,
since strings that are stretched
to the same tension are set up to
make and receive similar waves of
the air. Those that are
unequally stretched are not
apt to be moved by the same circu-
lations; rather one circulation
impedes the other. The stroke
of a string is composed of
two movements, one in which
the string is impelled *ante*,
that is, toward the circulation
of the air; in the other, which is made
retro, the string returns to its
original position. When, therefore,
one string is moved, the other
must move too. But the second
must contain such a proportion [in
relation to the first] that the waves
and circulations of the air impelling
and making the motion *ante* do not
impede the motion that the string
makes in return, and this proportion
only those strings have that
have equal tension.
Those, however, that are stretched
to an unlike tension do not cause

76. *De sympathia et antipathia rerum liber*, ch. 4.

tant, quoniam dum secundus fit
motus, id est
reditus cordae retro, circula-
tio secunda illi obuiat, & sese
impediunt: vnde nec motus fit
vllus praeter primam impulsa-
tionem, quae intensibilis est.[77]

each other to move, since as the
second motion is made, that is, the
return of the string *retro*, the
second circulation resists the return
and they impede each other. Therefore
it is not moved beyond the first
impulse, which is inaudible.

Here Fracastoro shows how two strings of equal length stretched to the same tension will be susceptible to each other's vibrations. The impulse or compression given to the air by the first string as it moves from its stationary position will be communicated to the second string. When the first string returns to its position, rarefying the air, the second will also. Were it not in the same tension, the second string would impede the motion of the air produced by the first string, evidently because it takes a longer or shorter time for its rarefaction-condensation cycle. So the second string will cease to move.

Fracastoro does not report any experiment that led to these conclusions, but he describes two analogous observations. When the bell rings in church certain of the statues high above the sanctuary begin to tremble, but others do not. Another experience cited is that of trying to reverse the swing of a pendulum before it has completed its period, which requires a great deal of effort, whereas at the right moment it is easy.

No further progress on sympathetic vibration is known to have been made until Marin Mersenne, citing Fracastoro's explanation, applied it to strings that were not in unison but in simple ratios to each other.[78]

Giovanni Battista Benedetti

In a letter to Piero Vettori of August 1560 from Rome, Girolamo Mei tells of hearing a certain Doctor Benedetti, about thirty to thirty-four years of age, read the natural science, *De coelo*, and *De generatione animalium* of Aristotle, and of regretting that he missed him lecture on the *Physics*. He praises Benedetti highly for his fluency, memory, languages, acumen, and independence of mind. The description fits our Benedetti, and if it was indeed he whom Mei heard, this is the only specific information we have of his teaching in Rome or of his having been there.[79] Benedetti (1530–90) admitted that he had no formal education but studied Euclid's first four

77. Ibid., ch. 11.
78. *Harmonicorum libri* (Paris, 1635), Bk. IV, Proposition 27, pp. 65–68.
79. G. Mei to P. Vettori, 31 August 1560, London, British Library, MS Add. 10,268, fols. 214r–15r.

books with Niccolò Tartaglia, probably in 1546–48. By the age of eighteen he was said to have become a mathematician, philosopher, and musician. From 1558 he was court mathematician to Duke Ottavio Farnese in Parma, and in 1567 he moved to Turin, where he taught mathematics and science at the court of Duke Emanuele Filiberto. In 1553 Benedetti published a theory that bodies of the same material but of different weights would fall through a given medium at the same speed, and not a speed proportional to their weights, as maintained by Aristotle. The demonstration of this in 1554 was plagiarized by Jean Taisnier, another mathematician-musician. Benedetti made numerous other contributions to mathematics and physics.[80] For Benedetti, as for so many of his contemporaries, Aristotle's works were a point of departure, and often the renewed investigation of problems found there led to fresh insights.

In two letters of around 1563 addressed to the composer Cipriano de Rore (1516–65), published in *Diversarum speculationum mathematicarum & physicorum liber* of 1585,[81] he confronted the age-old dilemma of the nature of consonance and its cause. This matter is broached only at the end of the second letter, as if it were an afterthought. For the letters concern theories of intonation. From the standpoint of acoustical theory, however, the remarks constitute an important revelation.

Nec alienum mihi videtur à proposito instituto, speculari modum generationis ipsarum simplicium consonantiarum; qui quidem modus fit ex quadam aequa- tione percussionum, seu aequali concursu undarum aeris, vel conterminatione earum.[82]	It does not seem foreign to my chosen purpose to speculate on the way the simple consonances are generated. This way is through a certain equalizing of the percussions or through the equal concurrence of air waves, or their cotermination.

There is no doubt, he says, that the unison is the consonance most friendly to the ear, after which comes the diapason, next the diapente, then the others. "Let us see, therefore," he invites the reader, "the order of the concurrence of the termination of percussions or air waves from which sound is generated."[83] Benedetti asks the reader to imagine a stretched string that is divided in half by a movable bridge. If the two halves are each plucked, a unison will be heard.

80. See Stillman Drake, "Benedetti," in *Dictionary of Scientific Biography*, I, 604–09.
81. Turin, 1585, pp. 277–83. The two letters are reprinted in Josef Reiss, "Jo. Bapt. Benedictus, *De intervallis musicis*," *Zeitschrift für Musikwissenschaft* 7 (1924–25):13–20.
82. Benedetti, *Diversarum*, p. 283.
83. Ibid., p. 283: "Videamus igitur ordinem concursus percussionum terminorum, seu vndarum aeris, vnde sonus generatur."

tot percussiones in aere faciet vna partium illius chordae, quot et altera; ita vt vndae aeris simul eant, et aequaliter concurrant, absque intersectione, vel fractione illarum inuicem.[84]	One part of the string will make as many percussions in the air as the other. Thus the waves of the air go at the same time and concur equally without their cutting in or fractioning each other.

Everyone knows, he says, that the longer a string, the more slowly it moves. If a string is divided by a bridge so that two-thirds are on one side and one-third on the other side, and if the two parts are each plucked, the consonance of the octave will be heard. The larger portion of the string will complete one period of vibration (*intervallum tremoris*) during the time it takes the shorter to complete two. If two-fifths of the string are on one side of the bridge and three-fifths on the other, the consonance of the fifth will be generated, the longer portion of the string completing two periods of vibration during the time the lesser portion completes three. Benedetti then arrives at the law which states that the product of the number representing the string length and the number of periods of the longer portion of the string will equal the product of the number representing the string length of the shorter portion and the number of periods of this portion. For example, in the case of the fifth, string length 3 will have 2 periods and the product will be 6; string length 2 will have 3 periods, and the product will also be 6. He proceeds to calculate the products for each of the consonances recognized by Fogliano, which are: diapason 2, diapente 6, diatessaron 12, major sixth 15, ditone 20, semiditone 30, and minor sixth 40. He notes that these numbers agree among themselves with a wonderful reasonableness (*mirabili analogia*).[85]

There are a number of tacit assumptions in this statement: that pitch is caused by periodic vibrations, that air waves transmit sound, and that the frequency of vibration varies inversely with the string length. For Fracastoro waves of air were still a metaphor, and he believed that they began slowly, picked up speed, then slowed down at the end. By contrast, Benedetti assumed that air waves caused sound and that the percussions determining a pitch occurred at a constant frequency. Aristotelian writers implied that frequency varied inversely with string length, giving the ratio of frequencies of the higher to the lower note of the octave as two to one. Benedetti builds on two Aristotelian *Problems*, 19.35 and 19.39, that imply this. In the first of these is stated: "For since *nete* is double *hypate*, as *nete* is two, so *hypate*

84. Ibid., p. 283.
85. For the Latin text and translation into English of this passage, see Palisca, "Scientific Empiricism in Musical Thought," in *Seventeenth Century Science and the Arts*, ed. H. H. Rhys (Princeton, 1961), pp. 106–08.

is one; and as *hypate* is two, *nete* is four; and so on." Since the nete string is half hypate, either the author was confused, or he was truly referring to frequency of vibration. That the latter is likely is made probable by 19.39, which introduces the idea of the concurrence of the terminations of vibrations: "Furthermore, *hypate* happens to have the same conclusions to the periods in its sounds as *nete*, for the second stroke which *nete* makes upon the air is *hypate*."[86]

What the Aristotelian problems only imply, Benedetti affirms unequivocally: frequency varies inversely with string length. Whether he observed the periods of vibration or simply assumed that vibrating strings must display such periodicity, it is impossible to say. It is unlikely that he succeeded in counting the vibrations. Exactly what is meant by the equalizing (*equatione*, which can also mean equal distribution) of percussions in unison strings is not explained. But it must be that the number of percussions of the air caused by the string per unit of time is the same in both string segments. Cotermination of percussions is clearer. The end of the compression of the air near one string segment will coincide with the end of the compression of the air near the other segment every time in a unison, every second time in an octave, every sixth percussion in a fifth, and so on. Given the relative number of percussions, it is possible, Benedetti discovers, to establish the interval of cotermination by multiplying the terms of the consonance's ratio and to compare this in various consonances. This product becomes, as it were, an index of consonance. Having created such an index, Benedetti does not pursue it to any conclusion, although the series of products strongly suggests a hierarchy of consonances. This is reinforced by the remark that introduces the demonstration:

Nam, nulli dubium est, quin vnisonus sit prima principalis audituque amicissima, nec non magis propria consonantia; et si intelligatur, vt punctus in linea, vel vnitas	For there is no doubt that the unison is the first, principal [consonance] and friendliest to the hearing, and also quite properly a consonance, if it is thought of as a point is to a line or unity to number.

86. The translations are by E. S. Forster in *The Works of Aristotle*, ed. W. D. Ross (Oxford, 1927), VII. Nicomachus also associated higher pitches with higher numbers, but he attributed these numbers to tension on a string. Flora R. Levin would have us believe that because Nicomachus' numbers, such as 12 and 6 for the octave, would not yield consonances if applied to weights suspended from strings— for it would take the ratio of 4 to 1 to produce the octave—he must have been thinking of rates of vibration. But this is not likely, for Nicomachus clearly describes the way the weights are hung and couples the numbers with the word *holkōn*, which Levin translates "pounds." Nicomachus *Manual* 6.7; Levin ed., p. 30; von Jan ed., p. 247, line 13; see also Levin, pp. 158–61.

in numero, quam immediate sequitur diapason, ei simillima, post hanc verò diapente, caeteráeque. Videamus igitur ordinem concursus percussionum terminorum, seu undarum aeris, vnde sonus generatur."[87]	This [unison] the diapason, most similar to it, directly follows, then the diapente, and so forth. Let us see, therefore, the order of concurrence of the terminations of percussions of waves of the air through which sounds are generated.

It is clear that Benedetti sees no break between the so-called perfect and imperfect consonances. Nor are the sixths in any way inferior to the thirds; indeed the major sixth precedes either of the thirds. The progression from greater to lesser consonance appears to be a continuum rather than a stratification. Had Benedetti carried his investigation into the so-called dissonances, he would have found that the diminished fifth (7:5), with the product of 35, fell between the minor third and the minor sixth.

Benedetti's findings support the claims of just intonation, and therefore Zarlino's theories about tuning, for the consonances in the simple ratios have the most frequent concurrences of vibrations, and Benedetti's scale of consonance could even be interpreted to support the rule of the senario, because, by stopping at the product 30, the problems of the diminished fifth and the nonsenarian minor sixth are sidestepped. Benedetti made no such claim, however, and in the two letters demonstrates, in fact, the opposite, that just intonation is not practicable, whether in instruments or voices. Benedetti was well aware that two eminent music theorists, Fogliano and Zarlino, supported just intonation; he names them both. He also probably knew that his correspondent, de Rore, like Zarlino, was a pupil of Willaert. Yet he, an amateur in music, boldly proceeded to demolish the case for the syntonic diatonic.

Benedetti begins the first letter by telling de Rore that Hector Eusonius is wrong when he says that one can understand the ratios of musical consonances without experiencing them with the senses. Nor can one know the theory of music without being versed in its practice, Benedetti adds. A theorist can no more understand what a diapente is without mastering practice than a pure practitioner can know what a fifth is without adding theory to practice. (Benedetti uses the two sets of terms: diapason, diapente, etc., and octave, fifth, etc., quite deliberately here.) Benedetti himself was obviously trained in both theory and practice, for he mentions in the same letter some motets that he wrote to Latin texts. He enumerates the intervals

87. Benedetti, *Diversarum*, p. 283. D. P. Walker, in *Studies in Musical Science in the Late Renaissance* (London, 1978), p. 31, has contested my claim in "Scientific Empiricism," p. 109, that Benedetti was establishing any kind of hierarchy of consonances.

and their ratios as found in Fogliano's monochord, which, he says, the Modenese author selected from Ptolemy's syntonic diatonic. He then presents seven musical examples (which he promised de Rore he would send) to illustrate the use of these intervals. Among the examples are some excerpts from de Rore's chanson *Hellas comment voules-vous*. The examples demonstrate that there are three sizes of semitones and two of whole tones:

inter diesim, et.b. in	between the b natural and b flat in
superiori, agnosces	the superius, you recognize the
interuallum minimi semitonij	interval of the minimal semitone.
et si ibi sit diesis,	If you take the b natural
tanquam terminus ad quem,	as a *terminus ad quem*,
et.b. tanquam terminus à quo:	and b flat as a *terminus a quo*,
quod autem inter diesim et.b.	then between b natural and b flat
sit semitonium minimum, facilè	there is a minimal semitone. This you
agnosces si subtraxeris	readily admit if you subtract
decimam minorem à maiori, quam	a minor from the major tenth that
facit superius cum inferiori,	the superius makes with the lower
idest cum bassu.[88]	part, that is, the base.

Benedetti does not show the numerical computation. If the major tenth minus the minor tenth equals the minimal semitone, then we have for the ratio of the latter 10:4 / 12:5 = 25:24. A similar analysis shows that in the third example there appears between d and ɑ a major semitone: the seventh (product of the fifth and minor third) minus the major sixth thus equals the major semitone: 3:2 × 6:5 = 9:5; 9:5 / 5:3 = 27:25. The fourth example shows the minor semitone: 4:3 / 5:4 = 16:15. By similar means Benedetti illustrates the two sizes of the whole tone. The fifth example shows, in the tenor, a sequence of a minor (10:9) followed by a major (9:8) whole tone; the sixth, in the tenor, two minor whole tones; and the seventh, in the superius part, two major whole tones. Thus Fogliano's monochord assumed three sizes of semitones and two of whole tones. (See Figure 10.8.)

The point of the demonstration is not brought home until the second letter. Here Benedetti declares that if these different sizes of semitones and whole tones are used, as they must be if the consonances are tuned justly, a vocal composition will not end on the same pitch as it began but either higher or lower. Utilizing the same method of calculating the smaller intervals through the addition or subtraction of successive or simultaneous consonances as in the preceding demonstrations, Benedetti now presents two sets of examples. The first (Figure 10.9) consists of a simple diatonic progression in which at each return of the note g′ in the superius, the actual pitch rises a comma. By the end of the example the pitch has risen by four

88. *Diversarum*, p. 278.

Figure 10.8.
Demonstration of the variety of semitones and whole tones, from Benedetti, *Diversarum*, p. 278 (to which have been added the ratios according to Benedetti's prose)

commas. This is because in each of the four repetitions of the pattern the upward step g′-a′ was a large whole tone, whereas the downward step a′-g′ was a small whole tone. Benedetti's final example (Figure 10.10) shows a parallel process involving a sharped note, in which the consonances between the superius and the tenor dictate the size of the semitones, which are always large (27:25) descending and small (16:15) ascending, thereby realizing a descent by a comma (81:80) with each statement of the pattern.

This phenomenon, Benedetti remarks, does not occur in organs and

Figure 10.9.
Demonstration of the rise in pitch in a diatonic passage, from Benedetti, *Diversarum*, p. 279

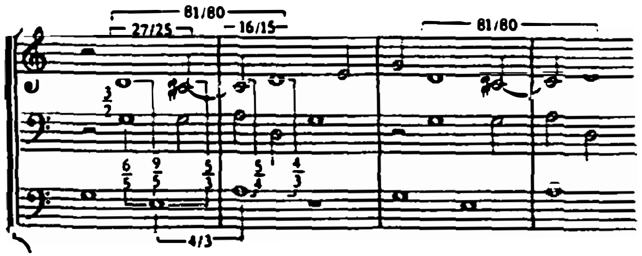

Figure 10.10.
Demonstration of the descent in pitch when a sharp is introduced, from Benedetti, *Diversarum*, p. 280

harpsichords, because all the consonances besides the diapason or octave are imperfect, that is, they are less than or greater than their just sizes (*diminutae, aut superantes a iusto*). The alteration is done, he explains, because if you take three successive sesquialter proportions, you get a major thirteenth (3:2 × 3:2 × 3:2 = 27:8), as from G to e'. This interval sounds "hateful" to the ear (*odiosus esset sensui auditus*). When an octave is subtracted from it, a major sixth that is "unfriendly" (*inimica*) results; this is in the ratio 13:8, which differs from the just major sixth (5:3) by a comma (81:80). For this reason "the learned and most excellent Zarlino" distributed parts of this comma over all the perfect consonances. But because the sense of hearing cannot distinguish the proper increment by which to raise or lower each string, Benedetti devised a purely aural tuning procedure for realizing this distribution of the comma error.

Benedetti began his tuning by making G consonant *grosso modo* with E♭ above it. He then tuned a series of "imperfect" fifths until he got a C, which he tested with an E♭, a major sixth below. If the sixth was "tolerable," he left the fifths alone; otherwise he retuned them until the major sixth was somewhat large (*aliquantulum excessiva*) but tolerable (*consonet tolerabiliter*). Modern tuners use the major third for testing the perfect consonances; Benedetti may have preferred the major sixth, because its number in his scale, 15, the lowest number among the imperfect consonances, ranks it higher than the major third, 20. Benedetti continued tuning the fifths slightly small until he reached a G♯, remaining, however, within a three-octave span by shifting to a lower octave whenever room was needed to complete the upward spiral.

Benedetti did not say that his tuning was an equal temperament, but since his demonstrations show that all semitones and whole tones should be equalized, this would have been a logical goal. Indeed, his tuning method is not unlike that proposed by Giovanni Lanfranco in 1533, which J. Murray Barbour has interpreted as equal temperament.[89] Lanfranco, however, began with F, alternating tempered fifths and fourths, and used a tolerably sharp third rather than a major sixth as a guide for tempering the fifths and fourths. Both Benedetti and Lanfranco went into the sharps only as far as G♯ and into the flats no farther than E♭. In their keyboards G♯ doubled as A♭, and E♭ similarly functioned as D♯. I assume that Benedetti's starting point, E♭, sounded a good tempered fifth against the G♯ that terminated the cycle.

It is striking that in the very letter in which Benedetti demonstrated the coincidence of vibrations in pitches related as simple ratios, he proposed a system in which the consonances deviate from these simple ratios. Whereas Gaffurio and Zarlino expected art somehow to conform to and follow nature, Benedetti realized that this was impossible, that musical practice was not science.

Girolamo Mei and Vincenzo Galilei

No one resolved the conflicting demands of science and art more clear-headedly than Girolamo Mei. It was to Mei that Vincenzo Galilei turned in 1572 when he found glaring contradictions between the ancient and modern authors. Mei had established the reputation of being the best-informed scholar on ancient Greek music, mainly through correspondence with his teacher Piero Vettori. Although a native of Florence, where he was born in 1519, Mei spent most of his mature life elsewhere, from 1546 to 1554 in France, then in Padua, and from 1559 until his death in 1594 in Rome. He had begun his studies of Greek music theory in 1551 while in Lyon, working as tutor and companion to Guglielmo Guadagni, but he had had relatively little time to pursue this subject until ten years later, when he had committed himself to make a thorough study of the sources of ancient Greek music theory, as he reported to Vettori. He describes the surviving sources in a letter of 21 February 1562, to which he appended a list that, unfortunately, is lost, but it must not have been unlike the bibliography he later sent to Galilei.

> The Greek writers that survive of which I have knowledge and who write professionally about this matter, as you will see by a list enclosed in the letter, are eighteen. The oldest of them is Aristoxenus, but we do not have him

89. Lanfranco, *Scintille di musica* (Brescia, 1533), p. 132. See J. Murray Barbour, *Tuning and Temperament* (East Lansing, 1953), pp. 45ff. Mark Lindley, in "Temperaments," *New Grove Dictionary*, XVIII, 662, states that "Lanfranco's keyboard tuning instructions of 1533 are unequivocally for some form of mean-tone." See also Lindley, "Early 16th-Century Keyboard Temperaments," *Musica Disciplina* 28 (1974):129–51, esp. 144–51. Benedetti probably did not know Lanfranco's treatise, since it was an elementary practical tutor rather than a scientific work such as those of Fogliano and Zarlino.

complete. After him, as I understand it, there is Plutarch or Ptolemy, though Plutarch in this matter is of slight importance. But Ptolemy, from what I see, from the standpoint of diligence and intellect I judge to be ahead of all those I have read, twelve in all until today. Of the Latins we have Boethius, marvelous for the most part, and almost, as we say, an ape of Ptolemy. But to want to tell and demonstrate and prove by every path and in many ways every proposition according to his habit is necessarily very long, and for someone whose objective is not entirely this, perhaps tedious. But worst of all is that he is lacking just at the conclusion. Of Ptolemy there is also lacking the end of the third, or last, book I don't know how many chapters, but these do not pertain, so far as one can tell from his words, to things altogether essential to the science [of music]. They were supplemented by the nonsense of a Nicephorus who commented on him. Now, of all these, I have resolved to take as my foundation Ptolemy, for I judge him to be the most complete and most conclusive of all. So I have transcribed a [copy] by my own hand. And to make it a good one I am taking every care possible and I hope to finish it, and being the first to fish in these depths, this will not be a small thing. In Rome there are four exemplars, all of which I can see repeatedly. I utilize the other writers as interpreters. My object is to endeavor to understand the thing first, and, once understood, to resolve, with your advice and that of others, to leave some record for those who would like to see the truth better. At the same time I want to exercise myself and not get entirely rusty."[90]

The list of authors appended to Mei's first letter to Galilei of 8 May 1572 contains nineteen ancient authors, one more than the number in the list sent to Vettori. It reads as follows:

Notice of the writers on music that are still found today whom I have seen
Aristoxenus, two books and a half or a little more, and perhaps half of the second book of the Rhythmics
Aristides Quintilianus three books
Alypius with the signs that they used to notate the steps of all the modes and the tones in each genus, with I don't know how much missing at the end
Anonymous book without name printed under the name of Harmonic Introduction of Euclid, also found under the name of Cleoneda or Cleomede, one book
Baccheius Senior introduction, one book
Gaudentius introduction, one book
Emanuel Bryennius, three books
Nicomachus "Strazeno" introduction
Plutarch is printed

90. See the letter printed in Palisca, *Girolamo Mei, Letters on Ancient and Modern Music to Vincenzo Galilei and Giovanni Bardi*, pp. 180–82.

Ptolemy, three books
Porphyry on about a book and a half of the music [treatise] of Ptolemy
Psellus introduction. I am told that it is found also printed
Theon, brief compilation, one book
Racendito Josefo, compilation or compendium in one book
Several fragments by diverse authors without name
Ancient Latins
St. Augustine
Boethius
Censorinus
Martianus Capella in the notes of his Philology"[91]

The project that Mei described in the letter to Vettori continued to occupy him until 1573, when he completed his principal work on music, *De modis musicis antiquorum*, dedicated to Vettori. Numerous scholars in Florence knew of Mei's studies on music, and it was one of these who recommended him to Galilei.

It was in replying to one of Galilei's questions that Mei formulated his theory of the separation of musical science and musical practice. Evidently Galilei was puzzled as to why the ancients were so concerned about the consonances yet did not use them in singing and playing together; Mei responded:

> The true end of the sciences is altogether different from that of the arts, since the end and proper aim of science is to consider every contingency of its subject and the causes and qualities of these purely for the sake of knowing truth from falsehood, without caring further how the arts will use this knowledge as an instrument or material or for otherwise gaining their ends. . . . The science of music goes about diligently investigating and considering all the qualities and properties of the constitutions, systems, and order of musical tones, whether these are simple qualities or comparative, like the consonances, and this for no other purpose than to come to know the truth itself, the perfect goal of all speculation, and as a by-product the false. It then lets art exploit as it sees fit without any limitation those tones about which science has learned the truth."[92]

Galilei, in his *Dialogo della musica antica et della moderna* of 1581, rephrased this thought. The principal interlocutor, Giovanni Bardi, replies to Piero Strozzi's query as to why the ancients wrote so much about consonances, when they sang only in unison:

> My reply to you is this, that the sciences have a different procedure and different goal for their operations than do the arts. The sciences search for the truth of

91. For bibliographical notes concerning these authors and their works, see Palisca, *Girolamo Mei*, pp. 118–21, nn. 58–77.
92. Mei to V. Galilei, 8 May 1572, in Palisca, *Girolamo Mei*, p. 103.

all the contingencies and properties of their subject, and together with them their causes, having as a goal the truth of knowledge and nothing more, whereas the arts have as their aim to operate, something different from understanding."[93]

As late as 17 January 1578 Mei was trying to show Galilei why the imperfect consonances were not recognized in ancient Greek theory and why, consequently, the Pythagorean tuning was perfectly satisfactory for their purposes. But such historical considerations aside, Mei thought of a better way to settle the tuning question:

> In the end it is not necessary to adduce these objections (if I am not mistaken) to ascertain whether the genus that is sung today is the syntonic or ditonic, because the very division of the strings will offer indubitable testimony of it. Stretch out over a lute (the larger it is, the more obvious will be what we wish to prove to the ear) two strings, either treble [*canti:* g'] or mean [*mezzane:* a], or whatever you want to call them, of length and thickness as equal as possible, which sound a unison together, and mark underneath them accurately the frets according to the distribution of the intervals of each of the two genera—the syntonic and ditonic—and then, taking the notes of the tetrachord one by one by means of the frets of each string, observe which of the two strings gives the notes that correspond to what is sung today. Thus without any further doubt the answer will result clear to anyone, even if what I have sometimes fancied on my own more as a matter of opinion than judgment is not proved true.[94]

Galilei must have proceeded to make this experiment, because that very year he sent to Zarlino under a pseudonym a discourse, not extant, that outlined his objections to Zarlino's theories about intonation. Zarlino in the *proemio* of his *Sopplimenti* speaks of receiving with a letter of 7 June 1578 a "Trattato di Musica," and in the letter the author apologizes for not having written to him or spoken to him after de Rore left the service of San Marco in Venice.[95] The author of the treatise, who is obviously Galilei, is quoted as saying in the letter that he studied counterpoint and other aspects of theory with Zarlino but profited little from the study. Zarlino claims to have answered the letter[96] and then received another from his "Discepolo," as he calls him, dated 19 July 1578. This time Galilei evidently spoke of Valgulio's coming to the defense of Aristoxenus (to which Zarlino replied by quoting a page-long section from Valgulio's discourse).[97] In another place Zarlino refers again to what must be the same letter, saying that his disciple sent him "a nice discourse by a gentleman of his who is very

93. *Dialogo*, p. 105.
94. Mei to Galilei, 17 January 1578, in Palisca, *Girolamo Mei*, p. 140.
95. Zarlino, *Sopplimenti musicali*, proemio, pp. 5–6.
96. Ibid., IV, 17, p. 172.
97. Ibid., IV, 17, pp. 173–74.

learned." The gentleman is quoted as saying that "he never found any mention among the ancient writers" of the senario, although "of the Greek authors he carefully read fifteen or sixteen, besides many fragments, and of the Latin authors as many as he could get."[98] The gentleman is obviously Mei. The "nice discourse" must have been taken from a letter of Mei, perhaps that of 17 January 1578, where Mei explains that the ancients did not recognize any consonances except those later called perfect, all of which were determined by multiple or superparticular ratios, but that Ptolemy maintained that the diapason-plus-diatessaron should be added. Ptolemy did not use such simplistic arguments as the senario or similar trivialities, however, Mei added.[99]

These arguments later became the core of the first part of the published *Dialogo*. At the very beginning of this work Galilei reaffirmed his empirical stance in a speech put in the mouth of the interlocutor Piero Strozzi:

> Before your Lordship begins to untie the knot of the proposed questions, I wish in those things which sensation can reach that authority always be set aside (as Aristotle says in the Eighth Book of the *Physics*), and with it the tainted reason that contradicts any perception whatever of truth. For it seems to me that those who for the sake of proving some conclusion of theirs want us to believe them purely on the basis of authority without adducing any further arguments are doing something ridiculous, not to say (with the Philosopher) acting like silly fools. This privilege is not conferred on anyone but the most wise Pythagoras, to whom you referred a moment ago, by his followers.[100]

Toward the end of his career, in an unpublished response to Zarlino's *Sopplimenti* of 1588, Galilei reaffirmed his belief in close observation with the senses as against the acceptance of authority:

gl'huomini che come professori	men who profess
d'un arte o d'una scienza, non	an art or a science do not
sogliono nello scriuerne	in writing about
andarsene presi	it go off half
alla grida come fa il Zarlino.	cocked as does Zarlino.
ma quando trouano uno scrittore	But when they find a writer who
che allega l'autorità d'un altro	cites the authority of another more
piu di lui antico, cerca di uedere	ancient than he, they seek to get
in fonte quella tal cosa; et il	to the bottom of the thing, and
medesimo si fa quando si scriuono	even more when writing
cose udite da gl'amici piu oltre.	about matters heard from friends.
quando anco sono uedute in fonte	When someone has seen to the bottom
le cose di qual sia scrittore,	of the things of any author who

98. Ibid., III, 3, p. 93.
99. Palisca, *Girolamo Mei*, p. 138.
100. *Dialogo*, p. 2.

che tratti pero di quelle cose
che sono poste al senso; si
esaminano s'elle sono uere, o no;
et dopo hauere recitato le ope-
nioni loro et conosciuto realmente
ch'elle non passano per quel uerso
gli si agiugne il parere suo con
quella modestia che conuiene.[101]

treats of matters that
are subject to sensation, he
examines whether they are true or
not, and after reporting their
opinions and recognizing that
they do not truly pass muster
he adds his own opinion with
all due modesty.

Galilei gives several examples of Zarlino's lapses from this method, of which two are of particular interest. One is the case of the ratios of the octave in pipes. Zarlino says that they follow the same rule as strings, namely that a pipe of half the length of another will sound an octave higher than the first.[102] They must be of the same width and thickness also, Galilei objects. Zarlino should have experimented (*esperimentato*) first, which would have been very easy to do. Even though Aristotle makes the same mistake,[103] Zarlino, a musician, is not so easily excused. Similarly Plutarch says that weights attached to strings produce an octave when they are in duple proportion.[104] This is false, Galilei maintains, for they have to be in quadruple proportion.[105]

At what point in his career Galilei developed the laws governing the numerical proportions obtained by measuring the dimensions of different types of sounding bodies—strings, pipes, disks, bells —is not documented. He first revealed some of his findings in the *Discorso intorno all'opere di messer Gioseffo Zarlino da Chioggia* of 1589. At the time he wrote the *Dialogo* he was still unaware of the fact that different ratios could determine the same consonances, depending on whether one measured a length, a surface, a volume, or the tension of a string. In the *Dialogo* his differences with Zarlino on matters of speculative music had revolved mainly around the definition of the tuning currently sung and played. Obviously unaware of Benedetti's critique, Galilei approached the problem in a more conventional manner. Benedetti had analyzed what would happen if punctiliously accurate singers continually adjusted their pitch to each other to maintain at all times the consonances of the simple ratios, both in simultaneous chords and in leaps. It would have been virtually impossible to find four singers capable of doing this; so his findings are true only in an ideal sense. Nevertheless, his analysis shows conclusively that if singers kept to the ideal consonances, their ref-

101. Florence, Biblioteca Nazionale Centrale, MS Galilei 5, fol. 42r.
102. *Sopplimenti musicali*, II, 13, p. 68.
103. *Problems* 19.50.922b–923a.
104. Plutarch, *De anima procreatione in Timeo*, in *Moralia* 1021; Zarlino, *Sopplimenti*, II, 13, p. 68.
105. MS Galilei 5, fol. 113r–v.

erence pitch would be constantly changing, something that Zarlino, had he considered it, would have found totally unacceptable.

Galilei's approach was more static. He pointed that if all the intervals in the gamut of notes normally used were calculated on the basis of the syntonic diatonic as a stationary tuning laid out on a monochord, an excessive number of the perfect and imperfect consonances would be intolerably out of tune. The minor third d–f (32:27), he shows, is not the same as e–g (6:5), nor is the major third a–c♯' (81:64) the same as c'–e' (5:4). The fourth a–d' (27:20) and the fifth d–a (40:27) are out of tune. These are some of the most common of the troublesome consonances. Galilei names many more.[106] The practicing musicians of his day, Galilei's observations showed him, did not adhere to any of the diatonic species described by the ancient authors. They mixed, without knowing it, the intense diatonic of Aristoxenus—an equal temperament—and the tunings Ptolemy called diatonic ditoniaion and diatonic syntonon. The viola d'arco, the lute, and the fretted lyra play the intense diatonic of Aristoxenus, which has equal semitones. The organ, harpsichord, and harp use two unequal semitones. Transverse flutes, cornetti, and similar instruments, in the hands of expert players, adjust to one or another species, depending on the situation, and voices do this also. In composing and singing, the intervals are formed in a tuning somewhere between the diatonic ditoniaion and the diatonic syntonon. Only the octave is found in its true ratio.[107]

Galilei later describes the intense diatonic of Aristoxenus in greater detail. It is one of six distributions proposed by the ancient author, two of which were diatonic, three chromatic, and one enharmonic. Aristoxenus, Galilei explains, divided the diatessaron into sixty "particles" (*particelle*), assigning twelve to the lowest interval of the tetrachord, and twenty-four to each of the higher intervals. Actually Aristoxenus did not divide the diatessaron into sixty parts. He spoke of twelfths of a tone, which was equivalent to dividing the diatessaron into thirty parts. It was Ptolemy who divided the diatessaron into sixty parts in his discussion of Aristoxenus.[108] Galilei had an Italian translation of Gogava's Latin of Aristoxenus, but he must not have looked up this passage.[109] The error does not affect the result, however.

106. *Dialogo* pp. 9–19 is mainly given over to calculating the size of these consonances and showing their impracticability. The other intervals, both smaller and larger, are also considered in the preceding or subsequent pages.
107. Ibid., pp. 30–31.
108. *Harmonics* 1.12.
109. The translation is in Florence, Biblioteca Nazionale Centrale, MS Galilei 8. On the basis of watermarks it seems to date from the first half of the 1570's. Approximately the same watermark is on fol. 38r, for example, as on letters of Giorgio Bartoli from 1572 to 1574 in Florence, Biblioteca Riccardiana, MS 2438–bis, vol. III, and his translation of Boethius, Flor-

All the semitones in this sytem are equal and have half as many particles as the whole tones. The result is an equal division, but a purely theoretical one, since Aristoxenus did not propose any arithmetical, geometrical, or other practical means to achieve it. Galilei did not at this time recommend this tuning, except for fretted string instruments, although he became proponent of its more general application years later.

Zarlino's reply to his "discepolo" was erudite, full of elaborate logical and rhetorical constructions, though exceedingly repetitious; it hardly conceded a single point. Zarlino was deeply affected by what Galilei wrote, nevertheless; indeed, he embraced some of his pupil's ideas while appearing to reject them. The cornerstone of Zarlino's defense was the distinction between natural and artificial music making. Voices are natural instruments; all others are artificial. Voices use natural intervals and consonances; instruments must be content with those produced by art. Whatever is produced by nature is superior. Zarlino reaffirms his faith in the syntonic diatonic, but now the justification is not simply numerology, as in the *Istitutioni*, but also philosophic truth.

> The forms of the consonances and other intervals that we use in our times in vocal and natural compositions are not products of art nor inventions of man but primarily of nature itself, collocated and registered among many things and especially among the parts of the perfect number, which is the senario, as I declared in the *Istitutioni*, in which they find their true forms. They are then ordered and rediscovered by art in the species that I call and shall always call natural, named syntonic diatonic by Ptolemy.[110]

Zarlino had to admit the "imperfections" that arose in the syntonic diatonic, but voices, being natural and completely flexible, could steer the harmony to a good consonance when an impure one would result from following the preordained tuning system.[111] The object of musical science is to defend and demonstrate the natural canon or monochord. The proof of this thesis leads Zarlino into a lengthy survey of the quarrels between the Pythagoreans and Aristoxenians. Some of this is of great interest from the point of view of the penetration of humanism into music theory. Zarlino shows that toward the end of his life he read quite extensively in Aristoxenus, Ptolemy, and Porphyry, as well as in other ancient authors. The contents of the *Sopplimenti* substantiates the claim made in the first chapter:

ence, Biblioteca Nazionale Centrale, MS Magl. XIX.75, finished in 1579. Bartoli was the copyist of the only existing manuscript of the Mei letters, Biblioteca Apostolica Vaticana, MS Reg. lat. 2021.

110. *Sopplimenti musicali*, I, 1, p. 8.

111. Ibid., IV, 6–7, pp. 141–46.

> I have not failed to see and read all those writers, Greek as well as Latin, that I have been able to get my hands on who treat of musical matters, as among the Greeks are Aristoxenus, Euclid [i.e., Cleonides], Nicomachus, Ptolemy, Aristides Quintilianus, Emmanuel Bryennius, Gaudentius the philosopher, Bacchius, Psellus, and Alypius, together with some other writings that are incomplete and by other anonymous authors, although the majority of the exemplars are (I lament over this), partly because of antiquity, partly because of the ignorance of the scribes, imperfect and incorrect. But of the Latins I have not missed seeing and reading many many, some printed, and some handwriten, among them Boethius, the monk Guido of Arezzo, Faber Stopulense [sic], Franchino Gaffurio of Lodi, Lodovico Fogliano of Modena, Glarean, and many others of the best who have written in this discipline, from whom I have learned many things.[112]

Stimulated apparently by reading Valgulio, Zarlino surveyed ancient opinion on the question of whether pitch differences reside in quantities or qualities. The authors he reviews are Archytas, Ptolemy, Aristotle, Theophrastus, Panaetius, Plutarch, and Porphyry, to each of whom he dedicates a separate chapter.[113] In the course of this he interpolates many opinions of his own, so that it is not always easy to pick out those of the ancient authors. He finally decides for the view of Porphyry, that pitch difference is both a quality and a quantity.

Zarlino's *Sopplimenti* is too rich a book to do justice to here. It is an eloquent testimony to the diffusion of ancient learning. Much of the erudition exchanged in letters and esoteric discourses earlier in the century has now become common property. Zarlino does not pass up any opportunity to cite an ancient Greek, Hebrew, or Latin author, quoting him, when he does, in the original language. Some of Zarlino's uses of antiquity are apropos, but much of the time he shows an indifference to the context, and, indeed, a certain contempt for what must have seemed to him the primitive ends and means of ancient music.

Galilei did not delay long in replying in print. The letter of dedication, to Zarlino, of his *Discorso intorno all'opere di . . . Zarlino* (Florence, 1589) is dated the last day of August 1588. Galilei challenged the idea that some intervals are natural, others artificial. To him all musical intervals were equally natural, whether their ratios were within or outside the senario. "The third contained in the 81:64 ratio is as natural as that in the 5:4 ratio. For the seventh to be dissonant in the 9:5 ratio is as natural as for the octave to be consonant in the 2:1 ratio." Sounds produced by instruments are as

112. Ibid., I, 1, pp. 7–8. Among the anonymous authors Zarlino probably numbered the Bellermann-Najock anonymi. Compare this list with his reading before the *Istitutioni* of 1558, p. 245, above.

113. Ibid., II, 7–15, pp. 57–74.

natural as those made by voices, for in both cases the material from which the sounds are made is natural.[114] Art is not necessarily inferior to nature. In those things that art can do and nature cannot, art is superior. In those things that nature can do and art cannot, art is inferior. Art and nature are both efficient causes. In making artificial things nature cannot rival art; nor can art rival nature in making natural things. Art, however, can improve on nature. Painting can represent not only natural and artificial things but also anything that it is possible to imagine. It can surpass nature in providing the eye with everything it can desire in the way of excellence of line and color.[115]

In replying to Galilei's inventory of the variety of poor-sounding intervals occurring in the syntonic, Zarlino took refuge, as we saw, in the flexibility of voices and their ability to seek out the best consonances. Galilei was thus forced to point out that if voices departed from the established intervals of the syntonic to seek better sounding consonances, they were no longer following the syntonic. Galilei proceeds to show in a manner perhaps inspired by Benedetti what happens when voices adjust to each other and converge on a just consonance. "We have two parts that sing this interval C–c. Then we make the lower part ascend by a fifth to G, and the upper part by a tone to d, this tone being a whole 9:8. I demonstrate this as follows: between C and G is a fifth, and from the same G to c is a fourth, which will become a fifth every time it is augmented by 9:8, by which the upper part will have risen."[116] This may be represented as in Figure 10.11, example a.

Galilei then goes on to describe a progression by a 9:8 tone in the upper part that would lead to a tenth (an octave plus the 81:64 proportion), an unpleasant "dissonance" (example b). The singers would, therefore, aim for a tone smaller than 9:8. Galilei describes two further progressions (examples c and d) that cause diverse whole tones, but he does not provide the calculations (given in brackets in the example). Moreover there are three sizes of semitones: 16:15 in going from a major third to a fourth, 135:128 from a major third to a tritone, and 25:24 from a major to a minor third. Although this discussion would appear to confirm Zarlino's view, Galilei insists that it does not, since the repetition or alternation of the two sizes of semitones depends on the piece and not on the distribution of the syntonic diatonic. He recognizes, further, the phenomenon pointed out by Benedetti: "According to whether more major or minor [tones] have occurred in the piece, ascending or descending, the singers will find at the end of it to have

114. Galilei, *Discorso*, pp. 92–94.
115. Ibid., p. 78.
116. Ibid., p. 119.

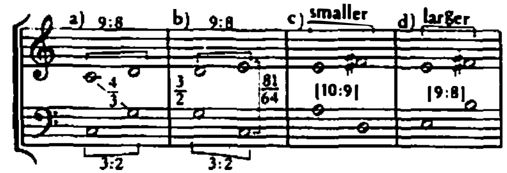

Figure 10.11.
Demonstration of the variety of whole tones, from Galilei, *Discorso*, pp. 119, 122

raised or lowered the steps from the intonation of the beginning."[117] Were Zarlino willing to accept these facts and abandon his chimerical "natural" and "artificial," Galilei confesses, he would concede that the genre "that we sing today agrees more with the very syntonic of Ptolemy than with any other distribution."[118]

Subjecting Zarlino's theories to a thoughtful review was in itself an important undertaking. (The arrogance of some passages was, to be sure, regrettable.) But even more significant were some revelations that Galilei buried in the dense prose—never relieved by a paragraph break—of this discourse. After spending several pages praising the ancient music theorists - Pythagoras, Didymus, Ptolemy, and Aristoxenus—Galilei comes to the account of Pythagoras and the hammers related by Boethius on the basis of the testimony of Macrobius:

> In this connection I wish to point out two false opinions of which men have been persuaded by various writings and which I myself shared until I ascertained the truth by means of experiment, the teacher of all things. They believe that the weights Pythagoras attached to the strings, better to hear the consonances, were the same as those of the hammers from which he first heard them. Now that this could not in any way be so, experiment, as I said, demonstrates. For if someone wished to hear from two strings of equal length, thickness, and quality, the sound of the diapason, it would be necessary for him to suspend weights, not in the duple but in the quadruple proportion. The diapente will be heard every time that from the same strings are hung weights in the 9:4 proportion, the diatessaron when in the 16:9 proportion, and the 9:8 tone when in the 81:64 proportion. . . . It is not true, therefore (and this is the other fallacy) that the consonances cannot be obtained through other genres of ratios than the multiple and superparticular.[119]

117. Ibid., p. 121.
118. Ibid, pp. 124–25. D. P. Walker has quite rightly noted this virtual agreement of the two polemicists and has some interesting reflections on this controversy in *Studies in Musical Science*, pp. 14–26.
119. *Discorso*, pp. 103–04.

Galilei goes on to say that in pipes (*canne*) the diapason will be obtained "whenever the length and the void (*vacuo*), or shall we say diameter, of the lower pipe is double that of the higher,"[120] the diapente when the two are in the sesquialteral proportion, and the diatessaron when they are in the sesquitertian ratio. Thus, he concludes, the volumes correspond to a cube, weights suspended from strings to a surface, and strings simply stretched to a line. These last remarks are somewhat cryptic, but Galilei later clarified them in an essay entitled "A Particular Discourse Concerning the Diversity of the Ratios of the Octave," of around 1589–90, in which he reported on experiments with strings of different materials, with weights attached to strings, and with coins and pipes. The octave, Galilei concludes, may be obtained through three different ratios: 2:1 in terms of string lengths, which corresponds to linear measurement; 4:1 in terms of weights attached to strings, which is analogous to area or surface measurements; and 8:1 in terms of volumes of concave bodies like organ pipes, which corresponds to cubic measurements.[121]

Galilei was the first to reveal the falsity of the famous story about Pythagoras that had been repeated in almost every book about music. Of the observations Pythagoras was said to have made, only that of the division of the string can have been true. Only in that circumstance would the traditional ratios hold. In pipes, if length alone were measured, these numbers would be approximately correct.[122] Galilei's laws for the correspondence between ratios of consonances and various physical measurements themselves needed to be refined. The behavior of volumes of air is particularly complex, though Galilei's formula is a good approximation.

Galilei's discovery that a variety of ratios could cause consonances, even superpartient proportions such as 9:4 and 16:9, was a fatal blow to numerology in general and the senario in particular. It is true that Vincenzo's son, Galileo, was to restore the traditional numbers by showing that frequencies are the real cause of pitch differences and that they vary inversely with string lengths. We have seen that Benedetti adumbrated this theory but brought forth no experimental proof. Until this new theory of frequencies was firmly

120. Ibid., p. 105.

121. "Discorso particolare intorno alla diversità delle forme del diapason," Florence, Biblioteca Nazionale Centrale, MS Galilei 3, fols. 44r–54v. Further on this essay see Palisca, "Scientific Empiricism in Musical Thought," pp. 129–30. I am planning to publish editions and translations of Galilei's scientific essays in a forthcoming volume of the Yale Music Theory Translation Series called *Documents of the Florentine Camerata*.

122. An anonymous author, probably of the fifteenth century, in Biblioteca Apostolica Vaticana, MS Barb. lat. 283, fols. 37 ff., shows that with pipes one must consider not only length but diameter also. He also makes some observations about cymbals and acoustical properties of various materials, but he maintains that weights attached to strings will give the consonances when in the usual proportions.

established there was no reason to favor the duple ratio for the octave over the quadruple, or 3:2 for the fifth over 9:4.

Perhaps because Galilei was relieved of the tyranny of numbers, he was able to give the first favorable account of the intense diatonic (*diatonico incitato*) of Aristoxenus that is to be found in the theoretical literature. He introduces the discussion in the *Discorso* by saying that he is fulfilling the desire of a number of his Aristoxenian friends. The case is first presented from a modern point of view. If the tone is divided into two unequal semitones, many inconveniences arise: D♯ is not the same as E♭; the semidiapente is larger than the tritone; the major seventh exceeds the diminished octave; D♯ to F is larger than a whole tone; the minor sixth is larger than the augmented fifth; and so on. Turning then to the situation in which Aristoxenus found himself, Galilei imagined that he must have studied every contingency of the two famous distributions then known, that of Pythagoras and that of Didymus, whom Galilei assumed to be older than Aristoxenus. In the system of Pythagoras the tone was divided into two unequal semitones of which the larger was above the smaller; in that of Didymus the reverse was true. In the Pythagorean system the tritone was equal to the semidiapente; in that of Didymus the tritone was larger than the semidiapente. Aristoxenus resolved that there should be only one semitone, the true half of the tone, and thus six tones or twelve semitones in the octave. The remaining intervals were built up from these, so that the minor second contained one semitone, the major second two, the minor third three, and so on. The uniform semitone permitted every interval to be measured exactly, just as one measures weight with the pound of twelve ounces. Galilei then sums up the advantages of this system with these words:

> No demonstrable distribution besides this one can be found among stable steps that is simpler and more perfect and more powerful, whether played or sung, or in which what part of the whole each interval comprises can be comprehended exactly by the sense with as great facility and clarity as could be desired. For the subject of music, which is vocal and instrumental sound, is a continuous and not discrete quantity.[123]

Unlike discrete quantities, which are numbers, continuous quantities can be infinitely divided without running into the difficulties that arise with ratios. One of the benefits of this division is that the tritone and semidiapente, being equal, rise to a new special category of perfect dissonance. Like the perfect consonances, of which there is only one form (rather than major and minor), the tritone-diminished fifth has a single size.[124]

Galilei saw nothing outrageous in Aristoxenus' reasoning, as others had,

123. *Discorso*, p. 113.

124. Ibid., pp. 115–17.

Figure 10.12.
Demonstration of the need for equal temperament, from Galilei, *Discorso intorno all'unisono*, Florence, Biblioteca Nazionale Centrale, MS Galilei 3, fol. 61r (barlines added)

possibly because as a lutanist he had experienced equal division in tuning his lute, in which accurate quantitative measurement did not enter. The octave in the lute and viol consisted of five whole tones and two semitones. In another of his last essays, the unpublished "Discorso particolare intorno all'unisono" of around 1590, he proposed that equal temperament was a necessary compromise for all instrumental music, not only that of lutes and viols. To prove his point he devised a short musical example that could be played flawlessly only by instruments tuned to the "intense diatonic" of Aristoxenus (Figure 10.12).[125]

Having given this defense of the Aristoxenian system, Galilei could not, however, claim that it is the tuning currently sung, because the ear preferred the fifth, for example, in its sesquialteral form. Experienced singers would always seek the most perfect intervals possible, but it was not feasible to describe or demonstrate with numbers the system that they used in polyphonic music. In an aside he reflects that it is just as difficult to regulate and make proportional through stable canons the movements of the celestial bodies, and with cosmic irony Galilei adds, "and this may be a good part

125. "Discorso particolare intorno all'unisono," Florence, Biblioteca Nazionale Centrale, MS Galilei 3, fols. 55r–61v.

of the congruence that Pythagoras judged there was between the celestial and human harmony," as if to say that what the heavens and human harmony, including polyphonic singing, have in common is a lack of stable proportion. In short, Pythagorean universal harmony is not truly, only wishfully, harmonious.

It is no coincidence that the three men who laid the foundations for modern acoustics, Fracastoro, Benedetti, and Galilei, were all ardent students of ancient learning. Before attempting new solutions, it was reasonable to search first in the ancient writers who were dedicated to investigating the truth of physical phenomena. These ancient writers were mainly in the Aristotelian tradition, and it was there that all three modern investigators found preparatory explorations of the questions they posed. They were able to modify and sometimes overturn the Aristotelian solutions through reflecting upon sense experience, real experiments, and thought experiments. But what they could discover by these efforts was only a beginning. The definitive mathematical and experimental work on these problems was to occupy a host of others in the seventeenth century: Galileo, Beeckman, Francis Bacon, Mersenne, Euler, Christian Huyghens, Kepler, Newton, Stevin, Wallis, and Sauveur, among others.[126]

126. See Sigalia Dostrovsky's essay on the history of acoustics in *Geschichte der Musiktheorie*, VI, in press.