To indicate the kinds of diffculties that can arise from such a crisp distinction, consider figure 8.3, which presents a synopsis of the first forty-three measures of Liszt's Consolation no. 3 in $\mathrm{D} b$ major. A complete score is given atWeb score 8.3 , and a recording is embedded in Web animation 8.9. The music consists of two sentences, both of whose presentation phrases cadence in f minor. The initial sentence classically continues through ii ${ }^{6}$ to a perfect cadence in $\mathrm{D} b$ major, executing an expanded cadential progression (Caplin 1998, 61). The second sentence continues instead to a cadence in A minor, after which a second continuation returns to $\mathrm{D} b$ major, completing a major-third division (see figure2 .11(b)).

Figure 8.3. Synopsis of Liszt Consolation no. 3. See Web score 8.3 for a complete score. ©


SECOND SENTENCE


From the standpoint of a final-state hearing, the double positioning of $f$ minor on figure 8.2 is well motivated. The initial sentence moves northward, engaging $f$ minor as a mediant on the way to a dominant. After returning to tonic at its initial position, the second sentence moves westward, engaging fminor as a Leittonwechsel that initiates a hexatonic journey. Yet this conception ignores a significant aspect of in-time experience. Arriving at the second f minor cadence, one has no reason to be aware of having embarked on a westward journey through chromatic space. Indeed, the principle of "parallel passages in parallel ways" (see chapter 3, note 11) suggests rather a retracing of the northward path toward dominant, as at m. 7. The continuation phrase forces a retrospective reevaluation of that position; we realize that we were migrating leftward, not upward. This reevaluation depends on identifying f minor on the vertical axis of figure 8.2 with its associate on the horizontal axis. But the model presents us with no means for establishing that identity: the two f minors occupy different positions, and our phenomenological journey from one to the other involves a magical wormhole for which the model has no explicit account.

An influential paragraph from a 1984 article by Lewin will help identify the problem and suggest a solution.

The nature and logic of Riemannian tonal space are not isomorphic with the nature and logic of scale-degree space. The musical objects and relations that Riemann isolates and discusses are not simply the old objects and relations dressed up in new packages with new labels; they are essentially different objects and relations, embedded in an essentially different geometry. That is so even if in some contexts the two spaces may coexist locally without apparent conflict; in this way the surface
of a Möbius strip would locally resemble the surface of a cylinder to an ant who had not fully explored the global logic of the space. (345) ${ }^{4}$

Is the geometry that Lewin envisions compatible with my cylinder and Rings's grid? Standing at a triad, one inhabits two distinct spaces, represented by the intersecting axes, "without apparent conflict." Yet Lewin's thrice-iterated conjunction of "objects and relations" suggests that the intersection of the spaces includes not only triads but also relations that pair them. He imagines the intersecting space as a surface rather than a set of discrete points. Paths intersect not only at points where they cross but also at segments where they merge. The grids of figure 8.1 show the triadic objects coexisting without apparent conflict, but the forced assignment of diatonic third relations to one axis or the other in figure 8.2 precludes the possibility that a triadic relation can coexist simultaneously in two spaces.

Brian Hyer's 1989 dissertation developed a geometry capable of simultaneously modeling both of Lewin's spaces, while situating each object and relation in a unique location. Hyer positions each triad as a point and connects it to its $\mathbf{L}, \mathbf{P}$, and $\mathbf{R}$ associate, as well as directly to its modally matched fifth. This Tonnetz models chromatic space by identifying ("gluing") enharmonically and syntonically equivalent points at opposite ends of the plane. Each such dimensional folding individually creates a cylinder like figure 8.1 (a). As Hyer phrased it in a subsequent article, "the [transformational] group as a whole disperses the functional 'significance' of [a single] triad among the harmonic consonances woven together to form its algebraic fabric; there is no one triad that forms a tonic for the group as a whole" (1995, 127).

Hyer's Tonnetz, however, has the capacity to change shape in response to how the listener hears the relations among its objects. If the triads are heard to collaborate in the definition of some tonic, then the glue loses its bond. "To assert a given triad as a tonic . . . forces us to imagine transformational relations with regard to the tonic, and to calculate them in scale degrees rather than generic semitones, in effect decircularizing [the Tonnetz], extending its [axes] in all directions" (1995, 127). Converting from a circular to a planar geometry "impos[es] a sense of perspective on the surrounding terrain, a point of view from which all the other triads appear to be near, more or less remote, or over the horizon" (127-28). Inversely, "when it becomes strained to hear relations between triads with respect to a given tonic triad, then we in fact no longer hear that triad as a tonic. At that moment . . . the circularized form of the lattice comes back into play" (Hyer 1989, 215). ${ }^{5}$

Hyer's convertible Tonnetz is ordered up to Lewin's blueprint in almost every respect. Each triad occupies a unique position, as does each direct triadic relation.

[^0]The interpretation of each relation is contingent on whether the space is closed or open. The structure of the space cannot be inferred from the standpoint of a single triad, or even of a direct relation such as the $\mathrm{D} b$ major and f minor of figure 8.3.That structure is cylindrical when the space is closed, exactly as in Lewin's metaphor. Only one detail is astray: where Lewin envisions a Möbius strip, Hyer constructs a plane. To bring the vision to full realization requires us to imagine Hyer's plane closed into a loop, with a half-twist. Candace Brower's 2008 consideration of the Tonnetz as a model of diatonic space provides a motivation for exercising our geometric imagination in exactly this way.


[^0]:    4. The italicized passages in the original 1984 publication were romanized in the 2006 reprint (194). The relevance of this passage to the present situation hinges on the interpretation of "Riemannian tonal space," whose domain of reference was mobile in Lewin's writings of the 1980s. It is nonetheless clear that the Riemann/scale degree distinction has strong affinities with binary relations that Lewin elsewhere cultivates in terms of chromatic/diatonic and atonal/tonal.
    5. The distinction between the circular and planar interpretations is equivalent to the conforming/ nonconforming distinction in Harrison 2002a.
