7

A CHROMATIC TRANSFORMATION SYSTEM

7.1 A MODIFIED CHROMATIC SYSTEM

As recounted in the previous chapter, the line of inquiry in tonal transformation theory seems to be shifting its focus of late from theory of harmony *per se* toward models based on voice-leading and economy of transformations. Without question this approach is providing considerable insight into aspects of organization in nineteenth-century music. But concomitant with this shift is a turning away of attention from the idea that the tonal harmonic system itself might possess greater complexity than is inherent in common practice. Instead, the argument is that some chord relations beyond the diatonic must be the result of nonharmonic processes. In many cases this is most certainly true, but in others a notion of chromatic function seems appropriate. The foregoing review of nineteenth-century theory, along with my presentation of the nature of common-tone tonal harmony, argue strongly for the recognition and inclusion of chromatic common-tone relations in a comprehensive model of the appropriate musical subject. Thus I would like to propose a transformational system based on common-tone tonality. From the perspective of common-tone tonality it would be beneficial to conceive of every type of fifth relation, and every type of third relation, as unary harmonic processes. From a harmonic point of view, a plain fifth relation is more basic than two diatonic third relations, even though it may be divided into them on the musical surface or reduced to them in theory. Similarly, a fifth-change relation is harmonically more basic than the two diatonic third relations plus mode change it may be reduced to, and a chromatic third relation in context may be more basic than the diatonic third relation plus parallel mode relation it may be divided into. An approach along these lines yields a system of transformations deriving more directly from the ideas of direct fifth and third relations inherent in Riemann's later root-interval theory, as well as the views of diatonic and chromatic third relations advanced in this essay. It replaces some of the other systems' compound transformations with direct transformations and allows for transformations identified with harmonic quality, rather than specific root-interval or change operations (see Table 6.1d). The result is a set of six transformation types arranged as three pairs (refer back to Table 1.1). For each of the three common-tone root intervals (the prime, the third, and the fifth), one transformation represents mode constancy, while the other represents mode change. I will refer to this system as the chromatic transformation system, as opposed to the more diatonic and dualistic systems discussed in the previous chapter.[1] Since many of the transformation types are shared, those belonging to the system proposed in this chapter will be designated by italics: e.g. **P** in the diatonic systems, ***P*** in the chromatic system.

7.2 PRIME AND MEDIANT TRANSFORMATIONS

Lewin's system defines two transformations for the prime: IDENT, which, in preserving all the pitches of a chord, preserves mode; and PAR, in which the third of a triad changes from minor or major to the other state. The latter surpasses its literal meaning to become the all-purpose indicator of chromatic change. Cohn's **LPR** system includes identity only implicitly, and elevates **P** as one of only three basic elements. The chromatic system preserves Lewin's two transformation types, designating them as ***I*** and ***P***, respectively. The role of ***P*** is diminished, though, for the system will include other dedicated transformation types to account for direct chromatic common-tone progressions.

Unique to the chromatic system are transformations for the four chromatic mediant relations, all of which share a similar harmonic profile. They define a single transformation type: *mediant*. The mediant transformation comes in two forms: ***M***, with root motion of a major third, and ***m***, with root motion of a minor third. As with the ***D*** transformation (retained and described below), the principal direction is down;[2] upward motion by major and minor third yields $\boldsymbol{M}^{-1}$ and $\boldsymbol{m}^{-1}$.[3] Thus the upper sharp mediant will relate to the tonic by way of the ***M*** transformation: $[\mathrm{E},+] \xrightarrow{M} [\mathrm{C},+]$, while the upper flat mediant will relate by the ***m*** transformation: $[\mathrm{E}\flat,+] \xrightarrow{m} [\mathrm{C},+]$. The two relative mediant relations, which also share a similar harmonic profile, define a single transformation type: *relative*. Here the notion of relative transformation is expanded to include not only the tonic of the corresponding relative mode (vi in major, III in minor), but also the other relative mode chord (iii in major, VI in minor). Two forms are needed: ***R***, with root motion of a major third, and ***r***, with root motion of a minor third. The relative transformation, although less strongly cadential than the dominant and mediant transformations, is expressed similarly as motion toward its goal; thus, $[\mathrm{A},-] \xrightarrow{r} [\mathrm{C},+]$ and $[\mathrm{E},-] \xrightarrow{R} [\mathrm{C},+]$.[4] The

[1] This approach also provides a different kind of economy. The diatonic systems aim to describe the maximal number of musical relationships with the minimal number of transformations, although Cohn's "Square Dances With Cubes" may represent a move away from this approach. The chromatic system aims to describe the maximal number of direct musical relationships with unary transformations. One might distinguish the former as mathematically economical, the latter as musically economical.

[2] This makes for a consistent system in accordance with Lewin's. It also acknowledges the leading-tone resolution property of the descending chromatic major-third relation.

[3] Thus the lower flat mediant will relate to the tonic by way of the $\boldsymbol{M}^{-1}$ transformation: $[\mathrm{A}\flat,+] \xrightarrow{M^{-1}} [\mathrm{C},+]$, while the lower sharp mediant will relate by the $\boldsymbol{m}^{-1}$ transformation: $[\mathrm{A},+] \xrightarrow{m^{-1}} [\mathrm{C},+]$.

[4] It is perhaps forward to propose alternative and potentially conflicting labels for the diatonic system's **L** and **R** transformations, symbols which, as Cohn remarks in "Square Dances", "are by now standard." My intent is not to refute their validity or challenge their use but rather to frame a different way of thinking based on harmonic affinities, which requires a consistent approach here.

R and ***r*** transformations in this system thus correspond to the **L** and **R** transformations in the diatonic systems. While it is regrettable to lose the *leittonwechsel* concept (in name only, though, since the *leittonwechsel* relation *is* a relative mode relation), it is beneficial to gain an explicit sign of the correspondence between major- and minor-third relative mediants, analogous to that of the chromatic mediants. There could be reasons to call the *minor*-third relative mediant transformation ***R***: it represents the central relation between tonics of relative modes, and would correspond to the diatonic systems' **R** transformation. But, throughout the history of chord symbols, major and minor thirds have consistently been depicted by upper-case and lower-case letters, respectively. In that it ensures consistency of symbols in the system presented here, this notational tradition will be observed.

This group of mediant transformations provides some advantages. First, it better reflects the musical nature of chromatic mediant relations by eschewing diatonically based compound transformations in favor of unary chromatic ones. Second, in utilizing the ***M*** operation it draws more directly from both Riemann's later root-interval theory and his later functional theory, which is more faithful to the spirit and intent of his ultimate use of the *Tonnetz*. Third, it better portrays the interplay between the transformation types and their relation to the harmonic system. Rather than basing groupings solely on absolute root-interval type, the chromatic system groups third-relation transformations primarily by harmonic content (diatonic ***R*** vs. chromatic ***M***), and secondarily by root-interval (***R*** vs. ***r***, ***M*** vs. ***m***). Fourth, by proposing variability within the basic transformations, it accounts for the common harmonic characteristics shown by groups of progressions which involve different root-intervals, which must be accounted for by different transformations in the diatonic systems. (For example, the chromatic mediants, all of which share a similar aural profile, but which involve root-intervals of both major and minor thirds, are all ***M/m*** transformations in the chromatic system, while they are variously **PL**, **LP**, **RP**, and **PR** in the diatonic systems.) Fifth and last, it provides for simpler transformation formulas in many cases. Overall, I believe that these revisions strengthen the correspondence of the transformation system to the actual harmonic relationships it portrays.

There is an interesting mathematical ramification to the ***M*** transformations. In the diatonic systems, the **D** transformation, a plain operation which preserves mode, is commutative, meaning that it may appear at any position in a compound formula and still yield the same result.[5] But the **R**, **L**, and **P** transformations are not commutative: their positions in a compound formula affect the outcome. See for example Figure 7.1 below, in which the **LP** and **PL** transformations end up in different places. This is because the **R**, **L**, and **P** transformations are dualistic change operations, which have different outcomes depending on whether they originate with a major or minor chord. Their compound chromatic mediant transformations, likewise, are

[5] The IDENT or ***I*** transformation, since it changes nothing, is also commutative.

$$\text{C major} \xrightarrow{\mathbf{P}} \text{C minor} \xrightarrow{\mathbf{L}} \text{A}\flat\text{ major} \xrightarrow{\mathbf{R}} \text{F minor}$$
$$\text{C major} \xrightarrow{\mathbf{R}} \text{A minor} \xrightarrow{\mathbf{P}} \text{A major} \xrightarrow{\mathbf{L}} \text{C}\sharp\text{ minor}$$

$$\text{C major} \xrightarrow{\boldsymbol{M}} \text{A}\flat\text{ major} \xrightarrow{\boldsymbol{R}} \text{F minor}$$
$$\text{C major} \xrightarrow{\boldsymbol{R}} \text{A minor} \xrightarrow{\boldsymbol{M}} \text{F minor}$$

PLR≠RPL yet ***MR*=*RM***, even though **PL=*M***

Figure 7.1 Noncommutativity of the compound **PL** transformation vs. commutativity of the unary ***M*** transformation

not commutative, even though their end result is the preservation of mode. The chromatic ***M*** transformations, on the other hand, *are* commutative; they may appear anywhere in a compound formula with the same effect.[6]

Thus the commutative ***M*** transformations present a non-dualistic model for the chromatic third relations, which, like the dominants, preserve mode. Furthermore, the ***D*** and ***M*** transformations share the property of being cumulative, since they will always continue smoothly in the same direction when applied continuously. This process results in harmonic circles involving a single interval. The ***D*** transformation, applied twelve times, yields the circle of fifths; the ***M*** transformation, applied three times, the circle of major thirds; and the ***m*** transformation, applied four times, the circle of minor thirds. This common property further validates the notion of a family of unary, commutative ***M*** transformations existing along with the ***D*** transformations.

Since the transformation system specifies root-interval relationships, while labels for mediant function specify chord identity, the two approaches are complementary, not mutually exclusive. Mediant transformations may occur both within the key and between keys, as long as they make sense; functional mediants may connect to a variety of other chords expressing meaning in a key. They work together to provide a sense of coherent progression to meaningful places. Thus the progression from a C major triad to an A♭ major triad in C major may be described in the following way: the tonic moves by way of an ***M*** transformation to its lower flat mediant.[7]

[6] Hyer makes a similar argument for the nonequivalence of the unary **D** transformations to the **RL** and **LR** transformations, which also yield mode-preserving motion by fifth. "Reimag(in)ing Riemann," p. 124.

[7] Cohn, "Neo-Riemannian Operations," pp. 58–59, note 3, considers whether the compound formula **PL**, which describes this progression, should be understood to represent a direct unary process or a compound process (**P**, then **L**, with an elided C minor triad as intermediary term). He leaves this question open. Hyer, "Reimag(in)ing Riemann," p. 111, on the other hand, considers **PL** to be a unary process in which the transformations "fuse together." But to my mind, the **PL** formula is by definition compound; the two-process *Tonnetz* representations of the diatonic transformation theories bear this out. The elided intermediary term is unnecessary within chromatic tonality, in the same way that two diatonic third relations are unnecessary to model a direct fifth relation in diatonic tonality. (If the intermediary term is present as a musical event, then it can be seen in context as a subsidiary diatonic element of an overall chromatic progression; see below, section 8.2.) A unary transformation is preferable to model a unary process which occurs along a different dimension of chromatic space than the ones followed by a compound diatonic one.

In the analyses that follow, I will employ both functional and transformational language to describe harmonic meaning and activity where a sense of tonic key is maintained.

7.3 PLAIN-FIFTH AND FIFTH-CHANGE TRANSFORMATIONS

My system retains Hyer's transformation $\boldsymbol{D}/\boldsymbol{D}^{-1}$, representing mode-preserving dominant-type relations by root-interval of a fifth. $\boldsymbol{D}/\boldsymbol{D}^{-1}$ is the only fifth-relation transformation in the diatonic systems. But there is another familiar class of common-tone progressions for which no direct expression exists in any of the diatonic systems. These are the fifth relations involving mode change, the most common of which occurs between major dominant and minor tonic. Some transformation systems in Table 6.1 give this progression the formula **DP**, reflecting, for example, the conventionally understood altered-third origin of the major dominant in a minor key. Nonetheless, it is hard to deny that the V–i progression sounds straightforward and direct, with no audible twist suggesting borrowing or conspicuous alteration. (To our ears, it is the minor dominant resolving to a minor tonic that sounds peculiar.) Another common fifth-change progression, ii–V in a major key, is fully diatonic and sounds equally straightforward. Thus for reasons both of musical instinct and systematic thoroughness, there is good reason to define fundamental transformation types for plain and change progressions by fifth to mirror those defined for the thirds and the prime.

Because the harmonic distance covered in chromatic mediant relations is so great, it is relatively easy to distinguish the harmonic and voice-leading aspects of these progressions. Juxtaposition of chromatically third-related key areas relays a sense of considerable, quick harmonic motion, while common tones and semitone motion provide smooth, close connections. For the fifth-change relations, an analogous distinction may be more difficult to make at first, since the harmonic distance between members is not as great. Moreover, these progressions juxtapose different tonics less often than the chromatic mediant relations do. Rather, fifth-change relations usually consist of a tonic and its modifier, or of chords participating in a single cadential progression. Thus fifth-change is not usually identified as a particularly chromatic process. To the extent that fifth-change does invoke some sense of the mixture of parallel modes, there is cause to formalize this by portraying them as compound transformations containing $\boldsymbol{P}$, as do Lewin and Hyer. But I believe that there are compelling reasons to do otherwise in most cases. First, fifth-change relations do not always unequivocally establish a tonic. For example, in the case of a minor tonic and major subdominant, both members of the relation may exhibit some tonic quality, since the progression can be heard, depending on context, either as i–IV or V–i. Second, the common fifth-change progression ii–V in no way involves the parallel mode. Third, nineteenth-century practice often blurs the distinction between

Table 7.1. *The four fifth-change operations*

Progressions between triads	Examples	Formula
major down to minor	V–i in minor; I–iv in major	F
minor up to major	iv–I in major; i–V in minor	F^{-1}
minor down to major	ii–V in major; i–IV in minor	F
major up to minor	IV–i in minor; V–ii in minor	F^{-1}

parallel modes.[8] Tonic major and minor may come to represent essentially the same harmonic area. In this context, the perception of key remains constant: little or none of the sense of a passage between parallel mode-related keys is invoked by the transitory appearance of a tonic of opposite mode. In such circumstances a compound transformation involving ***P*** actually seems inappropriate. What is needed instead is a unary transformation, ***F***, representing the direct relation between elements of the fifth-change.[9] ***F*** will need to account for all four instances of this progression class, shown above in the first column of Table 7.1.[10]

The first fifth-change pair, with major triad above and minor triad below, embodies the relationship between major dominant and its minor tonic, and major tonic and minor subdominant. The second fifth-change pair, with minor triad above and major triad below, embodies both the relationship between minor tonic and its major subdominant, and that between ii and V in a major key. While the appearance of the major triads IV and V in minor is normally explained as alterations to scalar motion for melodic reasons, or as borrowings from the parallel major key, these explanations do not affect their innate legitimacy as members of direct common-tone relations. It is worth noting that the two reciprocal fifth-change pairs are not identical. In the major-above pair, the two moving voices move by semitone, while in the minor-above pair they move by whole tone. Moreover, the major-above pair inverts about the pitch which forms a major third with the third of the chord, while the minor-above pair inverts about the pitch which forms a minor third with the third. In order to define the ***F*** transformations as reciprocal change relations, it would therefore be

[8] Harrison, *Harmonic Function*, pp. 19–21, discusses theoretical reception of this phenomenon.

[9] As noted above in section 6.6, Cohn has recently identified this transformation in "Square Dances" and named it **N** for *Nebenverwandt*. ***F***, introduced by me in "A Systematic Theory of Chromatic Mediant Relations," is retained here as a unary alternative, since the term **N** derives from the reciprocal (dualistic) process discussed in this section.

[10] A possible analogy with the ***M*** transformation, which also has four varieties, presents itself. The two forms ***M*** and ***m*** were necessary to accommodate two different interval sizes. ***F*** involves two reciprocal pairs defined by exactly the same process and involving the same interval size: one with a major triad above and a minor triad below, the other with a minor triad above and a major triad below. A solitary form of ***F*** defining downward interval motion of a fifth along with mode change, and its inverse, F^{-1}, thus accounts fully for all four fifth-change progressions.

necessary to define two different types of $\boldsymbol{F}$ in order to specify the interval around which the inversion takes place in any triad.[11]

But musical practice and intuition suggest that fifth-change, unlike prime-change and third-change, does not spring entirely from reciprocal operations. Whereas prime-change and third-change involve change relations between parallel and relative modes, fifth-change usually involves relations between primary chords within a single key. In this it is more like the plain fifth relations represented by the $\boldsymbol{D}$ transformation. Both G major → C major and G major → C minor, for example, represent dominant-to-tonic progressions. Thus root-interval appears to outweigh mode change in influencing the harmonic perception of fifth-change. Defining the $\boldsymbol{F}$ transformation by root-interval motion rather than reciprocal operations gives it simpler and stronger properties. In this model $\boldsymbol{F}$ represents downward fifth-change from the initiating triad, while its inverse, $\boldsymbol{F}^{-1}$, represents upward fifth-change from the initiating triad. This formulation works equally well no matter what the mode of the initiating triad is, requiring only one transformation type. Furthermore, $\boldsymbol{F}$ defined this way is commutative, as are $\boldsymbol{D}$ and $\boldsymbol{M}$. The commutativity aspect is essential for a notion of fifth-change which incorporates dominant behavior, which is direction-specific. It also makes good sense for yet another reason: $\boldsymbol{F}$, like $\boldsymbol{D}$ and $\boldsymbol{M}$, involves only a single common tone, while $\boldsymbol{P}$ and $\boldsymbol{R}$ entail two.

7.4 GENERAL TRANSFORMATION CHARACTERISTICS

Thus the complete system of chromatic common-tone transformations includes three root-interval types, each with two principal varieties – one preserving mode, the other changing mode. For the prime, $\boldsymbol{I}$ preserves mode, while $\boldsymbol{P}$ changes it. For the third, $\boldsymbol{M/m}$ preserves mode, while $\boldsymbol{R/r}$ changes it. For the fifth, $\boldsymbol{D}$ preserves mode, while $\boldsymbol{F}$ changes it. All of these transformations are shown in relation to the other systems in Table 6.1d.

These transformations differ in more than essential harmonic content: each displays a different set of properties. Summarized in Table 7.2, these give a more complete picture of the ways in which groups of transformations may act musically. They also serve to define each individual transformation more closely in terms of a unique combination of properties. Important properties include:

- *Accretion* – This distinction has ramifications beyond the obvious one of harmonic quality. The $\boldsymbol{D}$, $\boldsymbol{F}$, and $\boldsymbol{M/m}$ transformations are cumulative: successive applications of the same transformation propel harmony further in the same direction along the circles of fifths or thirds. Thus repeated applications of $\boldsymbol{D}$ to (D,+) yield (G,+), (C,+), and so on. Mathematically, these transformations are commutative (as is $\boldsymbol{I}$). The $\boldsymbol{P}$ and $\boldsymbol{R/r}$ transformations, on the other hand, are tightly circular: successive applications of the same transformation result in the

[11] For example, $\boldsymbol{F}$, inverting about the major third, would move from C major to F minor, while $\boldsymbol{f}$, inverting about the minor third, would move from C major to G minor.

Table 7.2. *Some aspects of the chromatic transformation system*

	Dominant (D)	Fifth-change (F)	Mediant (M/m)	Relative (R/r)	Identity (I)	Parallel (P)
Mode relation	plain	change	plain	change	plain	change
Root relation	perfect fifth	perfect fifth	major/minor third	major/minor third	prime	prime
Common tones	one	one	one	two	three	two
Varieties	two	two	four	two	one	one
Harm. range and outcome	1 ♭ ↔ 1♯; diatonic	4–2 ♭ ↔ 2–4 ♯; diatonic/chromatic	4–3 ♭ ↔ 3–4 ♯; chromatic	1–0 ♭ ↔ 0–1 ♯; diatonic	0	3 ♭ ↔ 3 ♯; chromatic
Accretion	cumulative	cumulative	cumulative	reciprocal (same) cumulative (mixed)	none	reciprocal
Commutativity	yes	yes	yes	no	yes	no
Inverse	D^{-1}	F^{-1}	M^{-1}/m^{-1}	R/r	none	P
Identity expr.	D^{12}	F^{12}	M^3/m^4	$R^2/r^2/R^{12}r^{12}$	none	P^2

alternation of just two values, yielding little cumulative harmonic motion. Thus repeated applications of ***R*** to (C,+) yield (A,−), then (C,+) again, and so on. The ***P*** and ***R/r*** transformations can further be understood as dualistic processes: the progress from minor to major is the mirror image of the progress from major to minor (see below, section 7.8).

- *Harmonic range* – some transformations, namely ***D*** and ***R/r***, yield chords closely related harmonically to the original; the diatonic sets of the keys associated with these chords vary by no more than one accidental from the original. The ***F*** transformations, mainly local phenomena, would vary diatonic sets by two to four accidentals. Other transformations more often involved in direct key relations, namely ***P*** and especially ***M/m***, yield chords whose associated keys differ from the original by three to four accidentals.
- *Varieties* – the number of individual progression types belonging to a group of transformations ranges from four (***M/m***) to two (***D***, ***F***, and ***R/r***) to one (***P*** and ***I***). This is a direct result of the number of root relations defining each transformation type. The chromatic mediants involve two root-interval sizes, resulting in double the number of transformations as in the dominant and relative groups, each of which involve only one root-interval. The parallel transformation, limited by its constant root, and the identity transformation, which preserves everything, have only one possible outcome.

Figures 7.2a and b propose a revamping and expansion of Hyer's relation-grid from Plate 6.4. Separate planes for major and minor chords replace the *Tonnetz*'s traditional conflation of major and minor chords with shared tonics on a single point; this allows for the distinction between plain and change transformations. Also present are the direct chromatic mediant relations of the ***M/m*** transformations. The affinities between the plain ***D*** and ***M*** transformation types, and the change transformation types ***R/r***, ***F***, and ***P***, are also made more apparent. Since this system contains six transformation types while separating out plain and change transformations, it is denser. Hyer's diagram entails 36 connections; this one entails 120.[12] While this scheme appears more complex, it reflects the greater richness of a common-tone harmonic space containing both diatonic and chromatic connections.[13] To begin, Figure 7.2a separates out major and minor triads into two Hyer-planes, equally defined by the plain ***D***, ***M***, and ***m*** transformations. Figure 7.2b adds the change transformations, which occur between planes: $\boldsymbol{F}/\boldsymbol{F}^{-1}$, ***r/R***, and ***P***. Whereas the plain transformations continue on in four-dimensional Hyer-space, the change transformations ***r/R*** and ***P*** simply bounce back and forth between the same two points in different planes.

[12] Hyer's diagram shows the following individual relations: 12 **D**, 12 **R**, and 12 **L** (and perhaps 12 **P** relations at the pitch-loci). Since this diagram has two planes and adds two relation types, it includes 24 ***D***, 24 ***F***, 24 ***M***, 24 ***m***, 12 ***r***, and 12 ***R***. This provides for a nexus of 120 individual common-tone relations. This diagram does not show connections outside its boundaries but extends an extra fifth horizontally.

[13] This representation of the multiple connection-paths and harmonic dimensions of the tonal system is a manifestation of what Robert Morris calls a "compositional space" in *Composition with Pitch Classes* (New Haven: Yale University Press, 1987), chap. 2.

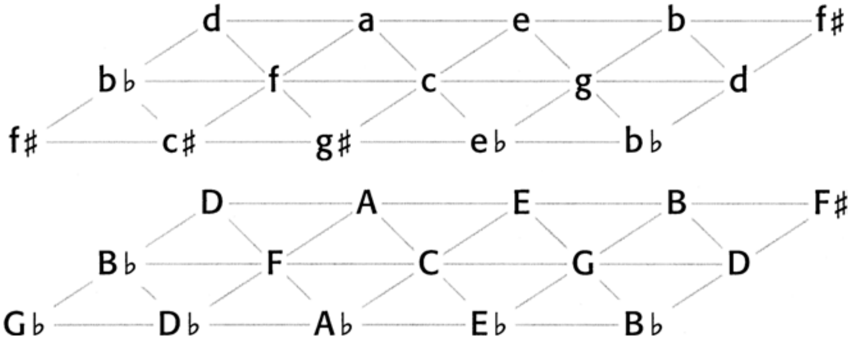


Figure 7.2a Separate *Tonnetz* planes for major and minor

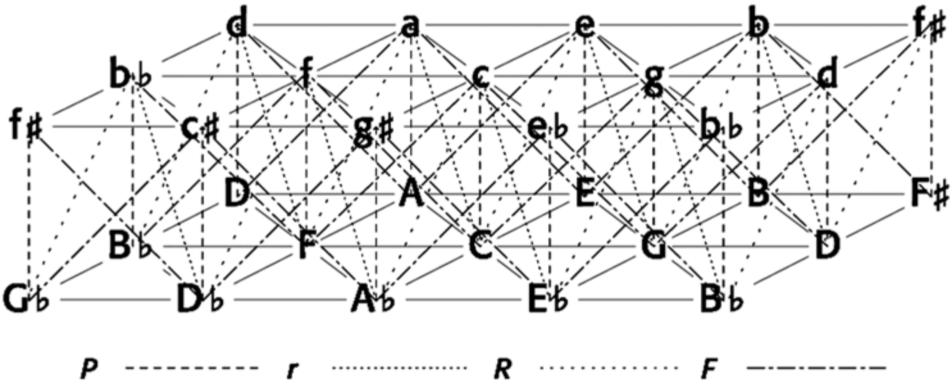


Figure 7.2b Expanded *Tonnetz* showing common-tone change relations

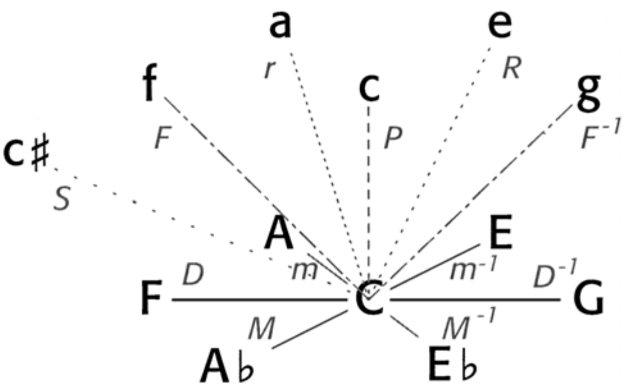


Figure 7.3 Full set of common-tone relations to a C major triad

$\boldsymbol{F}/\boldsymbol{F}^{-1}$, on the other hand, travel in a sawtooth pattern between planes until traversing their perimeters in four-dimensional space.[14] Zooming in, Figure 7.3 shows the complete set of unary transformations associated with a single locus, including the $\boldsymbol{S}$ transformation described immediately below.[15]

7.5 THE SLIDE TRANSFORMATION

One more common-tone relation resides at the bottom of Table 6.1. In this relation, the third is held common between both chords, while either chord inverts around this third to become the other. The result is a pair of opposite-mode triads a semitone apart, the major triad below and the minor triad above. This is the semitone change relation noticed by Marx (Plate 3.6), mind-twistingly dubbed *Doppelterzwechsel* by Riemann (Table 4.2), and recognized by Lewin as the SLIDE transformation. It is the consummate common-tone relation, whose harmonic strength, such as it is, is defined almost wholly by the common tone itself. The relation it represents is more distant than in any other common-tone relation: root motion by chromatic semitone joining keys four accidentals apart, with mode change. On the other hand, the common-tone relation is very strong: it forms the third of both chords. Only the parallel-mode relation, with its constant root and fifth, has a stronger common-tone component. Thus in SLIDE, the constant third greatly outweighs the harmonic connection. Voice-leading factors are also important, since the other two voices move by semitone, the privileged interval of parsimonious voice-leading theory.

Given the insignificance of its root relation, it may seem unnecessary to include SLIDE as a formal element of a systematic transformational theory. It occupies an odd place in Table 6.1. It appears in nineteenth-century music but not with the profile of the chromatic mediants. But there is one common harmonic relation which SLIDE fits like a glove: the progression from the Neapolitan chord to the dominant. The traditional explanation has it that the Neapolitan acts like a chromatic member of the class of dominant preparation chords resembling either ii through alteration or iv through 6-5 substitution. Scale-degree designation by Roman numeral for the Neapolitan chord is often withheld, since its "root" is located on lowered $\hat{2}$, which belongs to neither mode. Its only diatonic element is $\hat{4}$, which, although clearly the most important pitch connecting the Neapolitan to the dominant, cannot serve as the scale-degree root with which the chord is identified. But this common tone $\hat{4}$ between these chords suggests a transformational interpretation, shown in Figure 7.4. To begin with, the model for all dominant preparation progressions is the descending fifth progression ii–V, an instance of the fifth-change transformation $\boldsymbol{F}$. Next, the difference between ii in major and the Neapolitan chord is exactly SLIDE, or $\boldsymbol{S}$.[16] Thus, where the formula for ii–V is $\boldsymbol{F}$, the formula for the

[14] The difference between the dynamic paths traced by some transformations and the static paths traced by others relates to the concepts of essential and reciprocal dualism, to be introduced in section 7.8.

[15] Another schematic representation of tonal relations based on both harmonic and common-tone connections is advanced by Fred Lehrdal in "Tonal Pitch Space," *Music Perception*, 5, 3 (Spring 1988), pp. 315–349.

[16] $\boldsymbol{S}$ for SLIDE is not to be confused with **S** for SUBD in Lewin's system, which is replaced by $\mathbf{D}^{-1}$ in Hyer's.

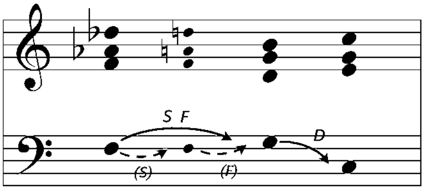


Figure 7.4 Involvement of ***S*** and ***F*** in the Neapolitan progression in major

progression N–V is ***SF***. This expression exactly captures the harmonic sense of the progression.[17]

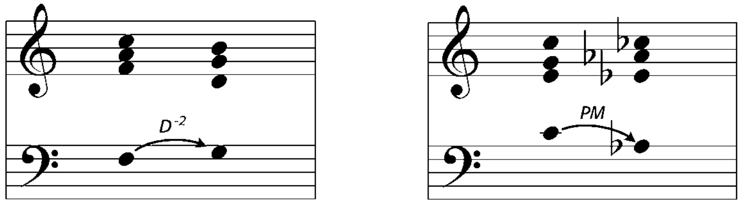
Without the ***S*** transformation, other formulas for N–V are possible, but fall short as precise, concise description. N–V is a mode-preserving tritone progression, and could thus be defined as $\boldsymbol{m}^2$, the product of two plain minor-third transformations. This expression, however, has little to do with how the Neapolitan acts in music, and is more suitable for analysis of circles of minor thirds (see below, section 8.6). Alternatively, the Neapolitan's affinity with iv in minor could suggest the influence of ***R***, after which the path to V is completed by $\boldsymbol{D}^{-1}\boldsymbol{F}^{-1}$. The complete formula $\boldsymbol{R}\boldsymbol{D}^{-1}\boldsymbol{F}^{-1}$ is plausible yet awkward, while ***SF***, with its descending-fifth relation, is simpler, more accurate, and more powerful. Another option is a translation of Riemann's functional interpretation of the Neapolitan as a subdominant *Leittonwechselklang* borrowed from the minor mode.[18] This suggests the transformational relation ***PDR*** between Neapolitan and tonic, but does not illuminate the actual N–V progression. Thus, thinking in terms of SLIDE is an elegant and appropriate way of accounting for the direct sense of the chromatic Neapolitan progression to the dominant. It also shows the usefulness of ***S*** beyond the specific progression which defines it. ***S*** will also prove useful in other ways in some of the analyses that follow.

7.6 STEP PROGRESSIONS

The relatively fewer compound transformations which exist in the chromatic transformation system take on more specific meanings. They invariably represent non-common-tone relations, such as the N–V relation just described. As another example, Figure 7.5a shows a chromatic progression containing no common tones – a disjunct mediant relation between C major and A♭ minor. Among the possible ways of expressing this relation, the compound transformation formula ***PM***, which describes a parallel-mode change accompanied by a chromatic mediant relation, makes good musical sense. This order of transformations reflects the norm in musical contexts where the intermediary step is expressed, but since ***M*** is commutative, either order

[17] In minor, the Neapolitan more closely resembles iv than ii. But the ***SF*** formula maintains a concrete reference, since the mediating chord, minor ii, arises from the melodic form.

[18] Cited in Robert Gjerdingen, "Guide to Terminology," in Dahlhaus, *Studies in the Origin of Harmonic Tonality*, p. xv.



a) diatonic step progression b) chromatic disjunct mediant

Figure 7.5 Some compound transformations in the chromatic system

is possible. This clear-cut notion of parallel-mode displacement of a strong mediant relation captures this progression more precisely than the conventional yet vague term "double mixture."

Some compound transformations may also be diatonic. Here is where transformation theories revive one of the more problematic aspects of common-tone theory: the unavoidable conclusion that step progressions are not unary and direct. Progressions between triads with roots a second apart (Fig. 7.5a), with the exception of SLIDE, contain no common tones, and strict common-tone theory must find that no direct connection exists in such cases unless extra tones such as sevenths are introduced.

Historically, harmonic theories have recognized both the special nature of the step progression and its difference from the other progression types. Rameau, for example, allowed thirds, fourth, fifths, and sixths freely in the fundamental bass; the second, a dissonance, could only occur as a byproduct of dissonance resolution or as the result of a disruptive progression. He interpreted some common step progressions as involving different fundamental bass intervals: the deceptive cadence, for instance, was at times explained as the arrival to a 6-5 substitution over a tonic bass, and thus as a fifth progression.[19] Moreover, Rameau's concept of *double emploi* redefined the common step progression IV–V as something else entirely, explaining that (in modern terms) IV, always carrying the essential dissonance of an added sixth whether or not it is not actually present, acts as ii when moving to V.[20] Thus, in order to explain its clarity and strength, he reinterpreted this step progression as a fifth progression.

In the early nineteenth century, Weber, for whom Roman numerals constituted the limit of explanation, could point to the validity of any chord succession and, in the case of IV–V, a relationship between principal chords with common membership in the key. Interval of root relation was not an issue for him. Riemann also accommodated step progressions as easily as anything else in his root-interval system, which was designed to account for all possible progressions. His dualistic

[19] Rameau, *Traité de l'harmonie*, bk. 2, ch. 1. Among the many discussions of this cadence are Allan Keiler's "Music as Metalanguage: Rameau's Fundamental Bass," in *Music Theory: Special Topics* (New York: Academic Press, 1981), and Thomas Christensen's *Rameau and Musical Thought in the Enlightenment*, p. 123.

[20] Rameau, *Génération harmonique*, p. 115.

theory meant, however, that in his system step progressions between chords of opposite mode occur between the lowest note of the major chord and the highest note of the minor chord (refer to Table 4.1). Since Riemann's functional theory defined categories which were not tied to the presence of specific pitches, step progressions were readily explained as the succession of two functions. The strength of the progression IV–V was easily accounted for as involving two pure function archetypes (S–D). In other step progressions, at least one function would appear as other than the basic form (e.g. V−vi = D−Tp). Schoenberg's harmonic theory also allowed for all possible progressions. He did distinguish step progressions from all others, though, isolating them in the category of rougher disjunct relations.[21] In Schenkerian theory, fifth and third progressions generally arise as the result of arpeggiation, a chord-based or harmonic process.[22] Step progressions principally arise as neighbor notes or as the filling in of an interval with passing tones, a more linear or contrapuntal process joining chord members. Thus step relations, while perfectly legitimate, will appear in Schenkerian analysis as lower-level occurrences with regard to the chord-based processes on which they depend. In sum, while many of these theories allow for the direct coherence of step progressions, all accord them a status somewhat inferior to fifth and third relations.

The common-tone theories, on the other hand, did not allow for direct step progressions. The purer theories of Hauptmann and Dehn accounted for step progressions by positing an unrealized mediating chord having tones in common with both chords in the actual progression, providing needed cohesion. In the whole-step progression between major triads, such as IV–V, this mediating chord would lie a fifth above the lower chord and a fifth below the upper one; in other words, these two chords refer to their tonic. Marx, however, felt it important to directly validate at least the IV–V progression. As recounted above in section 3.5, he observed that, for V^7 at least, the seventh provides a common tone with IV, lending this progression direct coherence. However, Marx limited his explanation to this single case. Other step progressions, including plain IV–V, were not direct for him, and must derive their meaning from context. Reicha took the opposite tack, finding a common tone in the subdominant's added sixth, à la Rameau.

Transformation theories formalize the common-tone theorists' mediating-chord approach to step progressions. IV–V, for instance, receives the formula $\boldsymbol{D}^{-2}$, representing one inverse dominant transformation from the initiating chord to the mediating chord, followed by another from the mediating chord to the goal chord.[23] This formula represents the shortest path from one chord to another along the Öttingen grid. The semitone progression from a minor triad to a major triad, such as iii–IV,

[21] This category was described above in section 5.2.1.

[22] As argued above in section 5.1, Schenker's mature theory is not principally a harmonic theory, but insofar as it derives from ideas put forth in *Harmonielehre*, and shows relationships between harmonic areas and *Stufen*, some general remarks seem fair.

[23] Lewin's analytic network diagrams make explicit use of mediating chords. An analysis containing mediating chords is found below in Figure 7.8.

requires a similar mediating chord located either a diatonic third or fifth below the minor triad, since a $\boldsymbol{D}^{-2}$ formula would involve a questionable diminished fifth. In the chromatic system, the transformation formulas for these progressions would be ***RD*** or ***DR***, respectively.[24] Other step progressions may appear even more distant in common-tone terms.

Thus while the more celebrated harmonic systems described above – Rameau, Riemann, Schoenberg, Schenker – all incorporate some notion of direct step progressions, the common-tone theories, old and new, do not. Rather than indicating a weakness on the part of the common-tone theories, this difference shows their value for revealing aspects of harmonic relations which other theories pass over. The conclusions one can draw seem reasonable: step progressions, containing no common element, must either imply a third harmony to which they both relate, or draw on contexts outside themselves for coherence. Step progressions between diatonic chords having strong significance in their key can bring powerful contexts to bear. Common-tone progressions, on the other hand, make immediate, local sense, which may then be interpreted in a larger context. For chromatic mediant relations, which cannot draw on a context of shared key identity, local sense is paramount. For fifth relations, context and connection are equally potent, and the relationship is the strongest of all.

The disjunction between IV and V may in fact be an important aspect of the capability of the progression to introduce the feeling of strong arrival characteristic of fifth-directed cadential motion. Prominent common-tone connections may sometimes provide too smooth a path to the penultimate point. Thus in the ii–V cadential progression, the melodic progression $\hat{2}$–$\hat{2}$, emphasizing the common tone between the chords, is much less favored than the melodic progression $\hat{2}$–$\hat{7}$. The latter emphasizes melodic disjunction, while implying the passing tone $\hat{1}$, which suggests IV and its concomitant step progression.

Furthermore, while IV–V is a common progression at the level of chord relation, direct modulations by whole step between tonic triads of similar mode are not as frequent as other types. This may be the true test of the innate coherence of any progression: if two chords do in fact possess a direct connection, then they should be able to bring about a direct modulation, serving as successive tonics. The fifth relations work well this way; so do the parallel and relative mode relations and the chromatic third relations. But the step relations work less well. When direct modulations by step do occur, it is often in the context of a sequence, which, although an important structural device, is not determined by chord identities expressed in cadential relationship, but rather by a more syntactic principle of displacement.[25] Hauptmann's observation that keys of the third and fourth degree appear more closely related to the tonic than those of the second degree applies to this case: chromatic

[24] All things being equal, the third makes more intuitive sense.

[25] Of course, upward direct modulations of a semitone or whole tone also occur in many styles of music as a simple device for increasing intensity.

mediant relations make for better *Tonalitätssprünge* than do mode-preserving step relations.

The common-tone theories' mediating-chord explanations do not imply that step progressions do not cohere, nor that they lack harmonic sense. They do imply that step progressions are operative within the key, making local syntactic sense, but are dependent for it on outside information. Other factors, such as neighbor motion in the bass and other parts, or parallel or sequential structure, figure more strongly than they might in progressions where the direct harmonic connection is more powerful. Tonal transformation theory models the content of specific relations; it does not measure other aspects of musical language which may also be operative. Hence it is perfectly reasonable to assert that some chromatic progressions are harmonically direct, while some diatonic ones are not.

7.7 THE AUGMENTED SIXTH

The transformational structure of the augmented-sixth progression from the exposition of Schubert's B♭ major sonata, D960, discussed in section 2.4, suggests an intriguing interpretation of a stock progression within the key. It was established that the earlier move from tonic to lower flat mediant is direct: ***M***, in chromatic transformation terms. But the ensuing progression appeared more complex, with dissonance transforming the stable LFM into an unstable German sixth, leading to a cadential $^6_4$ and beyond. A similar progression occurs near the opening of Chopin's mazurka op. 56 no. 1, analyzed below in section 9.2. Here a tonicized G major, potentially the real tonic, is proved false by the addition of an augmented sixth, leading like the Schubert to a cadential $^6_4$ and a cadence in the (true) tonic, B major. This is immediately followed by another occurrence of the same progression type at the transition from the opening section to the first inner section, modulating up another major third to E♭ major (Fig. 9.1, end; Fig. 9.2, beginning). In traditional terms, the triads and the German sixths embody different analytic entities. The former are usually given Roman numerals (e.g. ♭VI) while the latter are not. Both are considered to be unstable in relation to the key. But in chromatic tonality, the non-diatonic location of the fundamental pitch of these chords does not necessarily signify instability. In transformation theory strictly speaking, the motion of every note in a chord progression must be specified mathematically. The greater variety of four-note chords in the tonal system, and the greater complexity of their interrelation to each other and to triads, presents a theoretical challenge which is being addressed at present. For this study I am taking a more informal harmonic-theory approach, treating added dissonance as an extra-transformational adjunct to triadic relations. Accordingly, both V–I and V^7–I are instances of ***D***, and by analogy both LFM–I and Ger^{+6}–I are instances of $\boldsymbol{M^{-1}}$. Thus in the Schubert, the addition of the augmented sixth to the lower flat mediant does not alter the basic relationship to the following chord, which constitutes an $\boldsymbol{M^{-1}}$ relation on the surface as well as to the goal chord of the cadence. Likewise in the Chopin, where these progressions are the first in a number of chromatic mediant relationships which define much of the harmonic structure of the piece. Hence the formula for this common progression is basically $\boldsymbol{M^{-1}}$, enhanced by the characteristic dissonance. This sets the German augmented-sixth resolution to major $^6_4$, with its distinctive punch, apart from the others, none of which involves the power of unary ***M/m***. The variant progression in which the German sixth resolves directly to root-position tonic (section 8.3.2, Figure 8.9 below) makes for an even more clear-cut case. The formula for the resolution of a German sixth to a minor $^6_4$ would be ***R***, since there are two common tones in the progression. The less common resolution of a German sixth directly to the dominant would require a compound formula, since there are no common tones, thus $\boldsymbol{M^{-1}D^{-1}}$ to major, $\boldsymbol{RF^{-1}}$ to minor. The greater propensity of the French augmented sixth to resolve directly to the dominant is in part due to the presence of a common tone; thus its formula would be unary. But the interval structure of the French sixth makes it difficult to think of $\flat\hat{6}$ as the focal pitch of the transformation for this resolution. The old-fashioned approach to the French sixth, taking the common tone ($\hat{2}$, an augmented fourth from the bass) as the fundamental pitch of the chord, makes surprising transformational sense. Its resolution to the dominant would then correspond to unary ***D***, grouping it appropriately with other predominant harmonies. (When resolving to the tonic, the French sixth would have similar transformational behavior to the German sixth.) The unary German sixth transformations, which more readily resolve to the pitches of the tonic triad, would group with other direct relations with the tonic: modal, subdominant, chromatic. Their compound-formula relations with the dominant group them appropriately with other predominant harmonies such as the subdominant and relative mediant, both of which also have no tones in common with the dominant yet often stand in direct relation to it.

Thus, as with the formula for the Neapolitan chord which includes ***S***, we have the case of a standard chromatic progression within diatonic tonality which is best modeled by a transformation drawn from chromatic tonality. This further demonstrates the utility of the chromatic transformations beyond their dedicated meanings. It also negates the idea of a fixed point of demarcation between diatonic and mid-nineteenth-century chromatic tonality.

7.8 DUALISM

The ways in which transformations work in all four systems discussed here invoke the concept of harmonic dualism. Dualism is present when similar harmonic operations work in a symmetrically opposite way from major and minor chords. In other words, what goes up from a major chord will go down from a minor chord. Two significantly different types of dualism are evident in these sections. The first type appears in the **R**(/**r**) and **P** transformations of all the systems, which work in mirror-image ways for major and minor triads, alternating between the members of one major-minor pair; that is, they are their own inverses, shown in Figure 7.6.

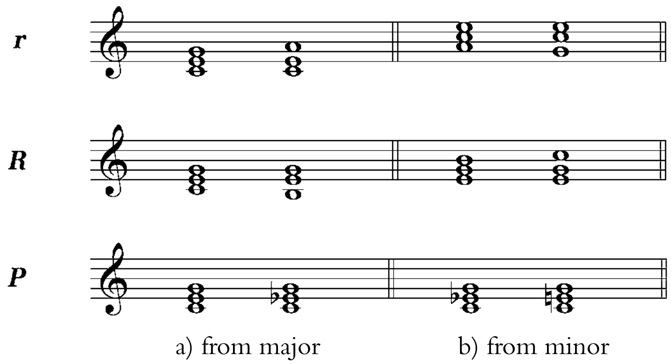


Figure 7.6 Mirror-image reciprocal change transformations in the chromatic system

Accordingly, I call this type *reciprocal dualism*. Reciprocal dualism's change relations result from fundamental properties of the major-minor harmonic system as we conceive it; their nature is evident and undeniable.

The second type appears in the compound diatonic transformations representing chromatic third relations. These do not result in the two-element change loops of the **R** and **P** operations, but rather in directed chains of mode-preserving progressions projecting in opposite directions from major and minor chords. Their appearance is that of old-fashioned dualism in the style of Öttingen, Riemann, *et al.*, in which motion in opposite directions is the natural concomitant of the symmetrically opposite nature and orientation of major and minor. Thus, I call this type *essential dualism* (Table 6.2 shows instances of this). Assertions of essential dualism occur throughout the history of tonal theory, from Rameau's undertones in the eighteenth century to the theories of Hauptmann and Öttingen in the nineteenth, leading to Riemann's youthful assertion that the dominant of a minor triad is located a fifth below it. Of late, dualism has received considerable renewed interest in the theoretical community, both with regard to transformation theory and for its own sake.[26] In his 1982 article on transformations, Lewin himself revived serious consideration of systemic aspects of essential dualism. Subsequently, Lewin has invoked essential dualism in an analysis of motives from Wagner's *Das Rheingold*, which provides a clear example of the different conclusions offered by dualistic and non-dualistic transformation systems.[27] Lewin's point of departure is the familiar

[26] Cohn has also noted these different types of dualism. Harrison bases an entire system of harmonic function in chromatic music on dualistic principles arising from oppositions inherent in tendencies of individual tones in the major and minor key systems, rather than in major and minor triads. He also provides a sympathetic history of the concept in German harmonic theory, including strictly dualist post-Riemann functional theories. Mooney, in "The 'Table of Relations'," also traces the development of nineteenth-century dualistic theory and provides a lucid explanation of its role in Riemann's early work.

[27] Lewin, *GMIT*, and "Some Notes on Analyzing Wagner." Plates 7.1 and 7.2 reproduce Figure 1, p. 50, and Figure 3, p. 52, from the latter.

a. Tarnhelm motive, Das Rheingold, sc. 3, mm. 37*ff*

b. Modulating section of Valhalla theme, Das Rheingold, sc. 2, mm. 5*ff.*

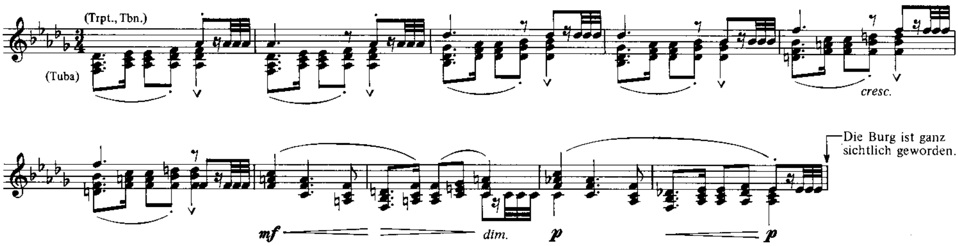


Plate 7.1 Two excerpts from Wagner's *Das Rheingold*

Tarnhelm and *Valhalla* motives (Plate 7.1), which sound similar despite their different harmonic content.

Lewin provides identical graphical node-and-network analyses for each, arguing for mathematical equivalence of harmonic motion in the two motives (Plate 7.2).[28] The analyses chart the motives' principal harmonic progressions:

(a) the *Tarnhelm* progression – G ♯ minor → E minor → B minor/major
(b) the *Valhalla* progression – G ♭ major → B ♭ major → F major

Both progressions contain initial chromatic mediant progressions involving root motion of a major third, followed by an ascending fifth. The *Tarnhelm* mediant progression, between minor triads, moves down; the *Valhalla* mediant progression, between major triads, moves up. Cumulative harmonic motion for the complete *Tarnhelm* motive is an upward minor third; for the *Valhalla* motive, it is a downward semitone. Equivalence between these two progression series is possible because the system construes chromatic mediant relations as compound transformations.[29] The formula for both major-third chromatic mediant progressions is **LP**, which, as shown in Lewin's analysis, works oppositely from major and minor initiating triads: G ♭ $\xrightarrow{LP}$ B ♭, while g ♯ $\xrightarrow{LP}$ e.

Thus this analysis construes chromatic mediant relations as essentially dualistic. The notion that the reciprocally dualistic diatonic (relative-mode) third relations work in opposite ways from major and minor initiating chords is easy to accept.

[28] While Lewin's formal claim is that the two series of progressions are mathematically equivalent, he substantiates and fortifies his claim with his intuition of strong musical similarity between the two. "Some Notes on Analyzing Wagner," pp. 49–52.

[29] Both Lewin and Hyer do argue that these compound formulations act like a single transformation. However, in their analytic systems, they behave like compound processes.

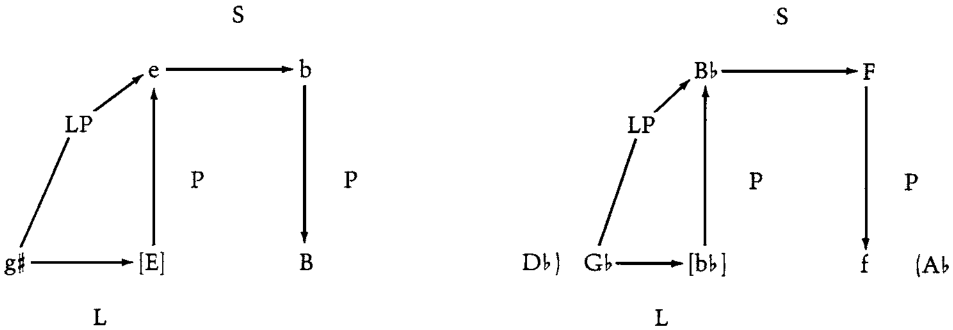


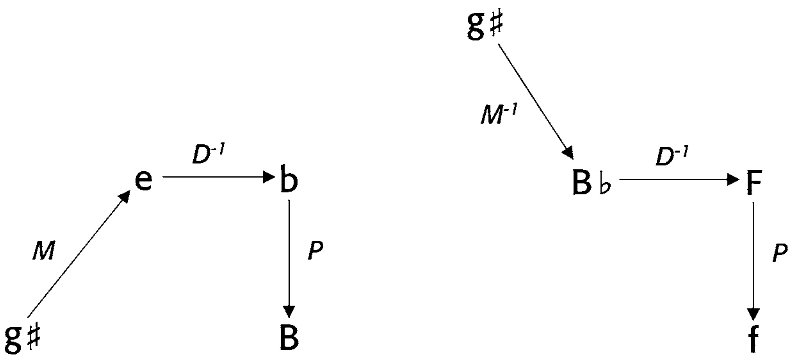
Plate 7.2 Lewin: transformationally equivalent dualistic analyses of themes from Wagner's *Das Rheingold*

Their nature as change relations has a good deal to do with this perception. But the conclusion of essential dualism for the mode-preserving chromatic third relations is less foregone. They are not change relations, and, as I have endeavored to show over the course of this essay, there is a long if informal theoretical tradition of conceiving chromatic third relations as direct progressions more harmonically analogous to fifth relations than to the relative mediants. In the chromatic system, the family of ***M*** transformations resembles the family of ***D*** transformations; both are plain, commutative, and non-dualistic. The ***M*** move that goes up a major third from a major triad would do the same from a minor triad. From this point of view, instead of equivalent **LP** transformations in the two Wagner motives, there are different transformations: ***M*** in the first case, $\boldsymbol{m}^{-1}$ in the second. Figures 7.7a and b show analyses based on unary ***M*** transformations without interpolations. While the first harmonic moves in the two graphs are not exactly the same, they do involve varieties of the same transformation, which may be seen to account for Lewin's strong perceived similarity between the two progressions. Chapter 1, after all, demonstrated the strong family resemblance between chromatic mediants, especially in relation to other types of progression. In a way, the equivalence demonstrated by Lewin is somewhat independent of harmonic content, and presages the voice-leading approach described later in this chapter.[30]

Lewin cites a circle of alternating *leittonwechsel* and parallel progressions in another excerpt as an example of a basic harmonic process in the music of Wagner, an example of what Cohn would come to call a hexatonic system.[31] Showing the circle as the result of an additive process of alternating **L** and **P** moves implies that this is the fundamental form of the circle. I would like to suggest, though, that this

[30] Lewin revisits the *Tarnhelm* motive in "Some Ideas about Voiceleading between PC Sets," *Journal of Music Theory*, 42, 1 (Spring 1998), pp. 65–67, in order to demonstrate a relationship between its opening and that of the Forgetfulness motive from *Götterdämmerung*, which contains a similar alternation between two chords (minor triad and half-diminished $^4_3$) and a similar rhythmic profile. This discussion cites elements of voice-leading, transposition, and retrograde order; given that minor triads are contained in all four chords, dualism would not be relevant.

[31] Lewin, "Some Notes on Analyzing Wagner," pp. 56–57.



a) *Tarnhelm* progression

b) *Valhalla* progression

Figure 7.7 Lewin: Wagner analyses with unary chromatic mediant transformations

circle is derivative – a decomposed circle of (chromatic) major thirds. Consider the analogous case of the circle of fifths. For us, this circle is a fundamental system whose elements (each individual fifth) are basic, atomic progressions. This has not always been the common understanding: in the late seventeenth and early eighteenth centuries, for example, Werckmeister's circle was based exclusively on relative mediant relations, while Mattheson's and Heinichen's progression schemes showed circles in which fifths and relative chords intermingled.[32] In the nineteenth century, Dehn's and Hauptmann's common-tone theories, which recognized direct fifth relations, viewed them ultimately as progressions resulting from combinations of two relative mediant relations, which for them were closer and more basic than fifth relations. Our view rejects these constructs; we see plain fifth relations and the circle of fifths they generate as direct and fundamental, stronger than and prior to these component parts. Likewise, I argue, we may understand the circle of major thirds in chromatic music to contain the direct, fundamental relationships inherent in the ***M/m*** transformations representing chromatic mediants. While the chromatic mediant relation can be broken down into relative mediant and parallel-mode relations (e.g. either ***RP*** and ***rP***, using my transformation symbols), it, like the fifth, has an integrity which supersedes its component parts. The paradigmatic major-third circle is composed of ***M*** relations, just as the paradigmatic fifth circle is composed of ***D*** relations. And just as the fifth circle can be broken down into a series of alternating ***R*** and ***r*** relations, the major-third circle can be broken down into alternating ***R*** and ***P*** relations. Furthermore, both the ***R/r*** and ***R/P*** circles may be seen as derivative, demonstrably less fundamental than the ***D*** and ***M*** circles, for they incorporate more than one kind of transformation.[33]

[32] Shown in Lester, *Compositional Theory*, pp. 215–216.

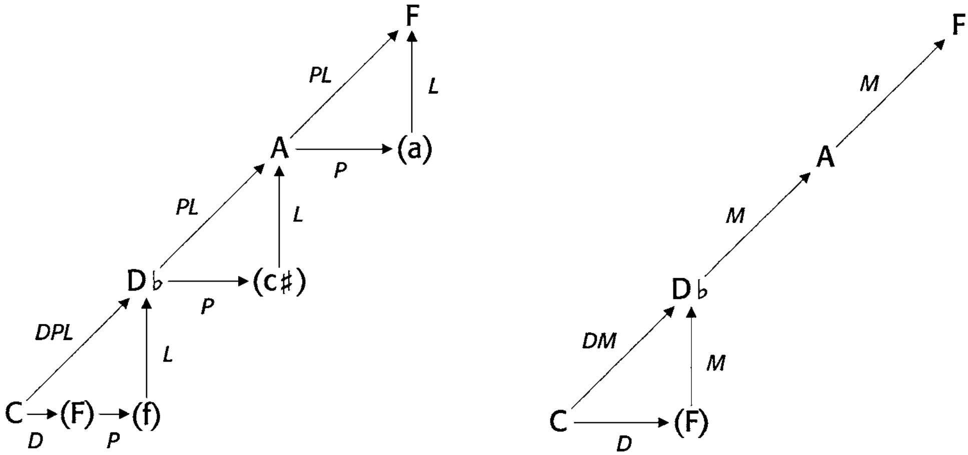
[33] Cohn's basic six-element hexatonic system (see Plate 6.6) is a formalization of Lewin's circle of **L** and **P** relations (***R*** and ***P*** in the chromatic system). Since in a hexatonic system a clockwise move, whether by **L** or by **P**, is a

Example 7.1 Brahms, clarinet sonata op. 120, no. 1, IV, mm. 158–175

7.9 THREE SHORT TRANSFORMATIONAL ANALYSES

Three further musical examples will show some advantages of thinking in terms of the ***M*/*m*** and ***R*/*r*** transformations. Two examples involve elaborations of the chain of major thirds. The first of these, from the final movement of the first Brahms clarinet sonata, op. 120/1, also contains an ***M*** transformation which forms part of a compound transformation (Ex. 7.1). The harmonic structure of the excerpt is simple,

transposition by one interval, calling different harmonic progressions by the same name, T_1, gives the system a semblance of mathematical equality and symmetry which is not reflected in the progressions themselves. As two clockwise moves, i.e. T_2, always yield a chromatic third relation, whether **LP** or **PL**, or **P** then **L**, this aspect, corresponding to the chromatic system's unary ***M*** transformation, does reflect the mathematical symmetry.



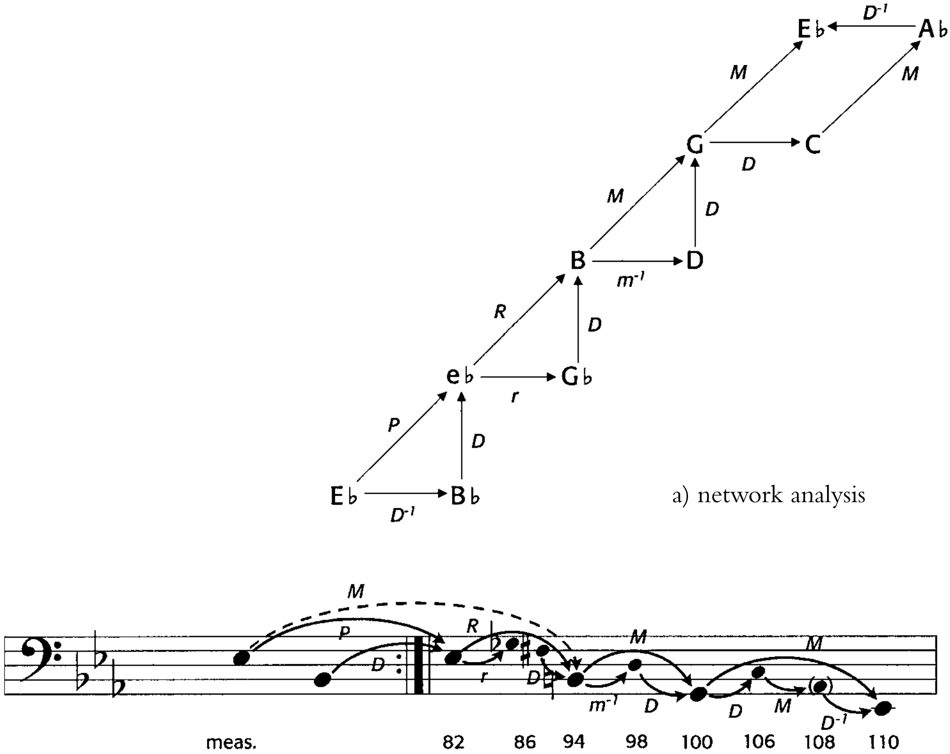
a) with compound **PL** transformations b) with unary ***M*** transformations

Figure 7.8 Two network analyses for the Brahms op. 120/2, IV excerpt

consisting only of four direct chord relations. From tonic F major, a dominant pedal on C culminates in an abrupt move to a D♭ major triad at m. 163. From there, a series of pure chromatic mediant progressions, or ***M*** transformations, with the common tone quite prominent, completes the cycle of major thirds, moving through an A major triad at m. 167 back to tonic F major at m. 170. The initial progression from C major to D♭ major is the most interesting one here. One could say that it is a deceptive progression from the dominant to the major triad on the sixth degree of the parallel minor – in other words, to a borrowed chord. But the subsequent move to A major shows this frame of reference to be temporary at best. The diatonic transformation formula for the progression from C to D♭, ***DPL***, conveys the sense of an intricate process. The chromatic system assigns this cadence the simpler formula ***DM***.[34] This latter expression captures the directness of the move and the immediate displacement of the goal by mode-preserving major third. At the same time, it shows explicitly how it comes to form the first element in the complete cycle of major thirds which it initiates. Figure 7.8 shows two network diagrams depicting the entire excerpt.

The first diagram, which employs the compound transformations, requires eight nodes, half of which are interpolated steps, including two interpolations in a row for the deceptive progression. The second uses the unary ***M*** transformations. This requires only five nodes, with a single interpolation for the deceptive progression. This diagram is more concise, directly reflecting the circle-of-thirds structure of the

[34] The standard deceptive cadence V–vi has the transformation formula ***Dr***. Thus these two deceptive cadences, one diatonic, one chromatic, may receive analogous formulas, ***Dr*** and ***DM***, which express their underlying harmonic and syntactic similarities. The second term in each formula specifies the exact "deceptive" displacement.



b) bass sketch showing circle of third and component transformations

Figure 7.9 Transformational analyses of the development section of Beethoven's *Les Adieux* sonata, op. 81a, I

passage. It also shows the value of the ***M*** transformation for describing harmonic relations beyond chromatic mediants.[35]

The second example consists of the development section of the last movement of Beethoven's *Les Adieux* piano sonata, op. 81a. This involves a series of brief tonicizations of key areas whose structure resists easy characterization either in terms of conventional harmonic analysis, or in Schenkerian terms as the long-range arpeggiation of a single triad. The section begins at m. 81 in E♭ minor, the parallel key to the tonic. The arrival to this key from the dominant, B♭, at the end of the exposition, along with its formal status as the opening harmony of the section, gives it a structural importance. A similar, sequential phrase begins on G♭/F♯ major, which in turn becomes the dominant of a tonicized B major at m. 94. The interrelation of these harmonies and their relative mode connection is shown at the beginning of the network diagram of Figure 7.9a and the bass sketch of Figure 7.9b, in which the local relative mediant relation between E♭ and G♭, ***r***, and the dominant relation

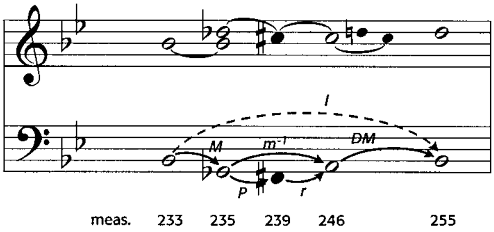
[35] In this and the following network diagram, the surface progressions form the jagged trace along the right-hand side, while the structural chromatic mediant progressions form the smooth trace along the left-hand side.

between F♯ and B, ***D***, combine to form ***R***, the controlling *leittonwechsel* relation between E♭ and B. From B major, the music moves next to tonicize G major at m. 100 by way of its dominant, D, which is reached from B by a direct chromatic third relation. Figures 7.9a and 7.9b again show the dual relations between initiating chord and dominant and tonic of the newly tonicized area, this time with the character of chromatic mediants: B to D involves $\boldsymbol{m}^{-1}$, D to G involves ***D***, and the overall motion from B to G involves ***M***.

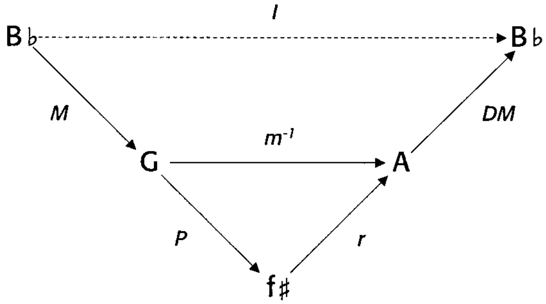
The return to tonic E♭ major from G major takes place by means of a more complex process again involving a direct chromatic mediant along with dominant transformations. First, G major moves away at m. 106 to C major, its subdominant, by way of ***D***. At m. 108, C major is transformed directly into an A♭ major $^6_3$, another instance of ***M***. A♭ then gives way as subdominant to tonic E♭ major at m. 110, with a $\boldsymbol{D}^{-1}$ transformation balancing out the previous dominant move.

Thus in this last link of the chain of thirds, the chromatic mediant relation takes place on two levels: the immediate level of the ***M*** relation of the subdominants C and A♭, and the principal level of the analogous ***M*** relation between tonicized G and tonic E♭. This is shown best in the parallel left- and right-side traces in the upper part of Figure 7.9a. It is noteworthy that, while the earlier ***M*** relation between B and D is facilitated in the conventional manner by the agency of the common tone in the principal melodic register, the effect of this ***M*** relation between subdominants is enhanced by featuring the chromatic semitone in the upper register and relegating the common tone to the bass, and by revealing itself gradually, first as an ambiguous dyad, and only later as a complete triad.

Transformation formulas are useful in describing chromatic mediant relations when they occur between chords which are neither tonic nor dominant, or in sections of harmonic flux in which a tonic is not evident. In these situations the function terms LFM, UFM, etc., have attenuated meaning, since they are predicated on direct relations with a tonic. Transformations may also help to indicate the influence of mediant relations which may not be present on the surface. A case in point is the modulating passage from the recapitulation of the first movement of Schubert's piano sonata D960, whose parallel in the exposition was described above in section 2.4. While the single chromatic mediant progression to G♭ in the earlier passage takes place completely within tonic B♭, the recapitulation progression goes farther afield, as shown in Figures 7.10a and b. The first chromatic mediant progression (***M***) at m. 235 moves directly from B♭ major to G♭ major as before, with common tone B♭ changing meaning from $\hat{1}$ to local $\hat{3}$. The next progression, a compound one on the surface ($\boldsymbol{Pr} = \boldsymbol{m}^{-1}$) culminating at m. 242, takes the music into A major, well outside the purview of the tonic key, where it lingers. A new common tone D♭ = C♯, changing local meaning from $\hat{5}$ to $\hat{3}$, becomes prominent. Both common tones, although beginning with different meanings, arrive to the third of the new chord, providing an aural connection between the two chromatic mediant progressions. Eventually a dominant seventh is added to A major at m. 253. Whereas the parallel chord in the exposition resolved as an augmented sixth, this one introduces a



a) Transformations in succession



b) Network diagram from the excerpt

Figure 7.10 Mediant transformations acting in different ways in Schubert sonata D960, I, recapitulation

deceptive cadence in the next measure, leading to the chord a major third below the expected one (***DM***). C♯, up until now heard as common tone, resolves up as leading tone to D, third of B♭, the true tonic. This ascending semitone produces a heightened dramatic effect, since at the parallel place in the exposition, the common tone persisted unchanged into the cadential arrival. According to this analysis, chromatic mediants figure in the transformation formulas in three different ways. In the first progression, a direct chromatic mediant results in a unary transformation. In the second, a compound progression with a relative mediant results in the cumulative value of a chromatic mediant. In the third, an unexpected twist is explained in the formula as displacement by the value of a chromatic mediant. Thus chromatic mediants inform the sound of this passage in a variety of ways.

The analytical bass sketches and musical reductions which appear here and in succeeding chapters should not be understood as modifications of Schenker-style analyses. Upper-part slurs for the most part are ties which indicate common-tone relationships between actual pitches. Curved lines with arrowheads in the bass are analytical, indicating directed harmonic (chordal) motion. It may seem that slurs and lines are showing two quite different things. But, since I maintain that common tones also embody directed musical motion through change of meaning, the upper-part slurs are also intended as analytical notation working together with the lower-part curves, which indicate transformations, not prolongational processes, to show the combined effect of directed root-interval and common-tone connections. Some diagrams, such as Figure 7.9, are only bass sketches; others, such as Figure 7.10, show other voices or four-part reductions of the musical texture in order to show common-tone connections and other surface features.