CHAPTER One

Mapping the Triadic Universe

Three Ways to Calculate Triadic Distance

> It is self-evident that those keys whose scales have most notes in common are most closely related.
>
> —Johann Phillip Kirnberger, *The Art of Strict Musical Composition*, 1771

> It is as though a hidden, sympathetic bond often connected the most remotely separated keys, and as though under certain circumstances an insuperable idiosyncrasy separated even the most closely related keys.
>
> —E. T. A. Hoffmann, *Kreisleriana*, 1814

In the age of Mozart, distance between keys is linear and easily calculated. In the age of Beethoven, the matter is more complicated, although Hoffmann (writing as Kreisler) is unprepared to say why.

As key proximity became more complicated in the age of Beethoven, so too did the calculation of distances between the triadic harmonies of which keys are composed.[1] This is because for theorists of the time, triadic relations tracked those of their eponymous keys. Jean-Philippe Rameau proclaimed in 1722 that "every note that supports a perfect chord should be considered a tonic" (Dahlhaus 1990 [1967], 28). Adolph Bernhard Marx (1841–47) "understands every consonant triad to be 'borrowed' from the key in which it is the tonic, and he claims that these triads stand in the same relation to one another as the keys they represent" (Engebretsen 2002, 70). Hugo Riemann (1897, 86) wrote pithily that "key relation is nothing other than the relation of their two tonic triads." And Heinrich Schenker collapsed the distinction altogether, regarding "keys" as triads under prolongation (Schachter 1987).

1. Throughout this book, "triad," in its unmodified form, refers restrictively to the twenty-four consonant triads. Particular triads are identified in the standard manner, by root and mode. Roots for minor triads are sometimes in lower case. In the figures and tables, *major* and *minor* are often abbreviated as plus and minus signs, respectively; thus, C+ stands for C major, and c– for c minor.

Figure 1.1. Schubert, Sonata in B♭ major, D. 960, 1st mvt., mm. 217–56.

To introduce the porous boundary between chord and key, as well as the complicated proximity judgments at both levels of relation, consider figure 1.1, which opens the first-movement recapitulation of Schubert's B♭ major Piano Sonata (D. 960, 1828). The first theme, which ends at m. 233, and its counterstatement, which begins at m. 254, are separated by three spans that respectively prolong G♭ major, f♯ minor, and A major. Each span is locally diatonic. That is, within each local span's own context, the role of each note and chord is specified, consistently and without ambiguity, by any of the protocols (e.g., Roman numerals, Schenkerian graphs, Riemannian functions) that represent the detailed inner workings of diatonic tonality. From a global perspective, too, the passage is normatively tonal: it begins and ends in the tonic.

Everything that we have observed so far points to the conclusion that the music of figure 1.1 adheres to the syntactic principles of classical diatonic tonality. It would be premature, however, to conclude that the passage is determinately tonal in all of its aspects. We have yet to consider how the local keys (or, from a different perspective, the triads prolonged by the local spans) relate to one another and how they work together to express the global tonic of B♭ major. If we are unable to do so, we just have a bunch of tubs floating around on their own bottoms. Each vessel is internally coherent and occupies a space bounded by the B♭ shores. But in relation to one another, their relation is random, for all we know. And that is no way to express a tonality. We can't just go ⟨B♭ major, Cough, Wheeze, Honk, B♭ major⟩ and pretend that we have made coherent music in B♭ major (Straus 1987). If a tonal theory is to meet its claims of explanatory adequacy, it needs to be able to specify the role, with respect to tonic, of the harmonies that separate the bounding tonics. The fact that each harmony may also be a tonic of its own local context in no way relieves it of that responsibility, any more than my role in my own home relieves me of my role in the community.

Let us locate each triad, in turn, with respect to B♭ major:

[1] The opening triad, B♭ major, is rooted on the tonic axiomatically.

[2] G♭ major is rooted on the flatted 6th scale degree of B♭ major, as notated. We know this because we identify the cantus B♭ of m. 235 with the cantus at the previous cadence, which we identified as tonic in [1] above, and we hear the bass as a consonant third beneath it.

[3] "f♯ minor" is a notational surrogate for g♭ minor, rooted on the flatted 6th degree of B♭ major. We know this because we identify the bass pitch at m. 239 with the bass pitch in the measures just preceding, which we described as G♭ in [2] above.

[4] "A major" is a notational surrogate for B♭♭ major, rooted on the flatted 1st degree. We know this because we identify the bass pitch at m. 241 as octave-related to the cantus pitch in the preceding measure, which was a consonant third above the bass pitch that we identified as G♭ in [3] above, and because the bass proceeds from "F♯" to "A" through three steps of a scale in m. 240.

[5] "B♭ major" is a surrogate for C♭♭ major, rooted on the doubly flatted 2nd degree. We know this because we identify the cantus "D" at m. 255 as a notational surrogate for E♭♭, the proper tonic of the previous dominant seventh, which was rooted on B♭♭, as identified in [4] above; and we hear the bass of m. 255 as a consonant major third below that E♭♭.

But our syllogisms have led us astray! No amount of logical sophistry can dislodge us from the conviction that the final chord of the progression represents the tonic degree, not the doubly flatted second. There must be an error to repair. Perhaps we can find it by retroengineering the analysis: [5] The final chord is B♭ major, axiomatically; and so [4] its immediate predecessor is rooted on its leading tone, qua dominant of its mediant, just as notated; and so [3] the chord just prior, a minor third below the leading tone, represents the fifth degree, F♯, again just as notated; and so [2] despite its notation, "G♭ major" represents F♯ major; and so [1] the cadence at m. 233 is on A♯ major.

The problem remains unrepaired. We have backed ourselves into another corner, on the opposite side of the room. Fortunately, there are still some options to explore. Taking them in reverse order: [4] Perhaps the roots of the last two chords are separated by a chromatic rather than a diatonic semitone? [3] Perhaps the root of the third chord lies an augmented second beneath that of its successor? (But then the bass of the latter represents a different scale degree than the soprano of the former; moreover, the stepwise approach in the bass signifies that the consecutive roots are not related by step.) [2] Perhaps the F♯ and the G♭ really do represent different degrees, just as Schubert notated it? (This is implausible prima facie: if you sing the bass while you perform the passage, nothing will persuade you to fracture the sustained pitch G♭2 into two noncommunicating entities.) [1] Perhaps the root of the second chord represents F♯ despite its notation? (But then, the soprano B♭4 represents a different scale degree than it did a moment ago.)

Are any of these perceptions plausible? What might motivate one to make a case for any one of them, other than the desire to preserve the initial premise, which is that the passage is determinately tonal in all of its aspects? If none of these questions can be answered in the affirmative, then we can only conclude that the passage is not entirely determined by the logic of classical diatonic tonality. This conclusion is independent of how we choose to regard the status of the entities that are progressing, that is, whether they are placed under the auspices of harmonic or modulatory or linear-prolongational syntax.

We can corroborate this conclusion by means of a simple measure of diatonic coherence: how many pairs of triads (not limiting to those presented in immediate succession) share membership in at least one diatonic collection?[2] In a typical diatonic passage in major mode, a randomly selected group of four distinct triads share membership in a single diatonic collection; ipso facto, so do the six pairs that they form. In a passage with a single applied dominant chord, four or five of the six pairs coexist in *some* diatonic collection (although not a single unified one). In figure 1.1, only a single pair, A major/f♯ minor, shares membership in some diatonic collection. This is, of course, very low on the spectrum of possibilities: of the 33,649 (= 23 choose 5) sextets of distinct triads that include B♭ major, only eight contain fewer (= zero) common diatonic memberships.[3] From a diatonic standpoint, this progression is among the most entropic. To the extent that Schubert is employing the logic of diatonicism here, it is in a negative sense: it is present in its absence.

We might then conclude that Schubert is being disjunctive, irrational, or arbitrary. To do so would place us in good company (Clark 2011a). Some critics of Schubert's time "described harmonic indirection as a kind of aimless wandering towards extraneous goals, which injected a quality of randomness and lack of plan into the music" (Shamgar 1989, 530–31). The more progressive of them placed high aesthetic value on tonal ruptures and disjunctions, connecting their inexplicability to the mysterious and sublime qualities so valued in the Romantic imagination (e.g., Hoffmann 1989 [1813–14], 131–36). A related view became the inheritance of historical musicology in its poststructural phase, for which ruptures constitute traces of ideological, sociocultural, and psychological formations that are otherwise occluded by the passage of historical time.[4]

2. I regard this measure as more suggestive than definitive. A more useful metric might additionally track the number of pairs that share membership in some harmonic minor scale, although that introduces other problems; for example, is {A♭, B, E♭} a diatonic chord in c minor?
3. These eight include B♭ major together with one chord from each of the following pairs: {G major, e minor}; {E major, c♯ minor}; {D♭ major, b♭ minor}.
4. Examples include Kramer 1986, 233; Subotnik 1987; Abbate 1991; and McClary 1994, 223. Carolyn Abbate's analysis of a scene from *Die Walküre* in her *Unsung Voices* (1991) is a particularly fertile garden for such tropic varietals; it refers to "harmonic irrationality and incongruence" (189), "unstructured harmonic improvisation" (192), "cannot be heard as a logical harmonic progression" (194), "disjunctive gap" (194), "no progress, no development . . . a repeated succession of discontinuous chords" (199).

But there is another available interpretation: perhaps diatonic distance is not the best metric for the situation at hand. In a treatise published in 1796, Francesco Galeazzi estimated the relationship between C major and d minor triads as "very irregular and poor" (*irregolarissimo e pessimo*), even though each has diatonic status when the other is tonic (Galeazzi 1796, 264).[5] Yet he values the relation between C major and E major as "regular and good" (*regolare e buono*), although there is no diatonic collection that includes them both. Why does he judge the diatonic progression less normative than the chromatic one? Because the latter has a common tone that the former lacks. He considers the relation between C major and e minor to be "even better" (*migliore*) than the previous two. Because the relation is diatonic? No—because the chords share two common tones (see Galeazzi 1796, 263–64).

Diatonic collections play no role in the model of triadic proximity that underlies Galeazzi's judgments. Nor, for that matter, do harmonic roots have any role to play (although they are implicitly present to the extent that they furnish labels). Galeazzi's judgments are based on properties and relations that are independent of those identified by classical theory, such as acoustic consonance and diatonic inclusion. If we are willing to suffer the anachronism and the scientistic odor, we can express Galeazzi's implicit conception in the language of modern mathematical set theory: triadic proximity correlates with cardinality of pitch-class intersection.

Galeazzi's association of harmonic proximity with common-tone preservation recurs consistently in music theory treatises throughout the nineteenth century. K. C. F. Krause asserted in 1827 that "the most closely related consonant triads are those that have two notes in common with the given triad, then follow those with one note in common with the given chord" (qtd. in Engebretsen 2002, 69 n. 1). Nora Engebretsen notes that Krause "presents his view without any fanfare, in a manner suggesting that this is the standard approach" (69). Ten years later, Marx offered the opinion that co-occurrence of triads in a diatonic collection counted as a "superficial unity" but that "a more distinct tie exists in the connecting notes which each of our chords has in common with its neighbors" (Marx 1841–47, qtd. in Engebretsen 2002, 69). Two influential treatises from midcentury, by Moritz Hauptmann and Hermann Helmholtz, were equally dedicated to the common-tone basis of harmonic proximity, even though their epistemological bases (respectively, in idealist philosophy and scientific empiricism) were diametrically opposed (Hauptmann 1888 [1853], 45; Helmholtz 1885 [1877], 292). In the final decades of the century, the common-tone view of the harmonic *Verwandschaft* began to lose ground to a renewed interest in acoustic generation and consonant root relations (Engebretsen 2008). But the two methods are nonetheless frequently seated side by side. For example, Tchaikovsky's 1872 *Guide to the Practical Study of Harmony* distinguishes between "inner" relations, based on root distance on the circle of fifths, and "external" connections based on common tones (Tchaikovsky 1976 [1872], 11–13; compare Riemann 1897, 85ff.).

5. For an annotated translation of Galeazzi's treatise, see Burton and Harwood (forthcoming).

Table 1.1(a). Number of common tones between each triadic pair in figure 1.1

	G♭ major	f♯ minor	A major
B♭ major	1	0	0
G♭ major		2	1
f♯ minor			2

Applying this criterion to the Schubert passage gives a rather different picture of its coherence. For example, the f♯ minor triad, which is the most difficult of the four to integrate into a B♭ major tonal framework, shares two common tones, the maximum possible, with both its predecessor, G♭ major, and its successor, A major. Table 1.1(a) counts the number of common tones between each pair of triads in the progression, disregarding their order of presentation. The total of six common tones positions the progression toward the upper end of the range for quartets of triads, which extends from zero to nine, and well above the average, which is just below four.[6]

In counting common-tone connections in a particular passage, we have implicitly assumed that voice leading is *idealized*.[7] In most compositions, tones freely transfer registers, and multioctave tone doublings liberally appear and disappear. We say that two triads have a common tone even when, in a particular setting, those tones appear one or more octaves apart. Identity of tones, then, is independent of the particular register in which those tones appear. When we speak of common tones, then, we are adopting a conception of *tone* that is allied with *pitch class* rather than pitch. There is nothing special about idealized voice leading; music theory teachers and scholars assume it every day of their working lives. It is so familiar, indeed, that it takes a special effort to acknowledge it.

Idealized voice leading is also assumed by a related method for calculating the distance between triads, which attends not only to the number of moving voices but also to the absolute distance of motion.[8] We define a unit of *voice-leading work* as the motion of one voice by one semitone. The initial Schubert progression, B♭ major → G♭ major, requires two units of work: the voices containing F and D both move by semitone (up and down, respectively), while the voice containing B♭ stays put. The progression G♭ major → f♯ minor involves only a single unit of work, B♭ to A (assuming no surcharge for enharmonic exchanges). And the progression from f♯ minor to A major involves two units of work, F♯ → E. Table 1.1(b) calculates the work for the six pairs of triads in the Schubert progression. Summing the values in the table, the progression as a whole involves fourteen units of work.

6. An example of the maximum is {C major, a minor, e minor, c minor}. An example of the minimum is {G major, e♭ minor, D♭ major, a minor}; see figure 5.25(b) in chapter 5.
7. Proctor 1978 attributes the term to Godfrey Winham.
8. Not all theorists agree that voice leading should be idealized when voice-leading measurements are assessed. Tymoczko (2005, 2009c, 2011b) presents an argument in favor of measuring voice leading along paths in circular pitch class space, distinguishing between upward and downward motions. My own views are flexible on this matter, in accordance with the position taken in Rings 2011, 51–54.

Table 1.1(b). Number of semitonal displacements ("voice-leading work") between each triadic pair in figure 1.1

	G♭ major	**f♯ minor**	**A major**
B♭ major	2	3	3
G♭ major		1	3
f♯ minor			2

This is on the lower end: for a set of four triads, the minimal total work is ten, the maximum twenty-eight.[9]

The assumptions underlying this method of calculating triadic proximity are even more venerable than Galeazzi's. Already in the early fourteenth century, Marchettus of Padua was articulating a "closest approach" preference for semitonal voice leading (Schubert 2002, 506).[10] Gioseffo Zarlino wrote that "when from the third we wish to arrive at the unison . . . the third should always be minor—this being closer" (1968 [1558], 79). Early-nineteenth-century theorists cultivated melodic fluency as an alternative to fundamental bass progression (Engebretsen 2002), and at the turn of the twentieth century, Georg Capellen proposed that triadic connections are based on a combination of common tones and semitonal motions (Bernstein 1986, 142).

Maximum common tone retention and minimal voice-leading work are so closely related to each other that one might be tempted to think of them as equivalent. They are conflated, for example, in the "law of least motion," which decrees that voices should move by minimal intervals, holding common tones in the same voice.[11] This principle has the status of a robust prescription if one takes the classical view that voice leading is secondary to harmony. If one first selects a pair of chords and then considers how most economically to join them, maximum common-tone preservation entails minimal voice-leading work. If, however, these metrics serve as a primary determinant for selecting harmonies, rather than as a criterion invoked only after the harmonies have been selected, then they do not yield identical judgments about triadic proximity (Cook 2005, Tymoczko 2009b). In some cases, voice-leading work makes a finer set of distinctions than does common-tone retention, since the former spreads its results across six distinct values whereas the latter returns only three (see figure 4.7 in chapter 4). For example, in figure 1.1, f♯ minor shares two common tones with both the preceding G♭ major and the subsequent A major. Yet the moving voice travels by semitone in the first case, whole tone in the second, a distinction that disappears when one is merely counting common tones. In other cases the two metrics make

9. An example of the minimum is {B♭ major, b♭ minor, G♭ major, f♯ minor} (see chapter 2). The maximum is fulfilled by {G major, e♭ minor, D♭ major, a minor}, which is the minimum of note 6.

10. Dahlhaus (1990 [1967], 335 n. 7) speculates on even earlier origin.

11. The law of least motion was erroneously deposited in Arnold Schoenberg's theoretical account, but, like so many of the other treasures banked there (the chart of regions from Weber, the emancipation of dissonance from Weitzmann), it was siphoned from the accounts of predecessors. The "law" was a staple of thoroughbass theory and debuted no later than Charles Masson's 1694 treatise.

contradictory claims about relative proximity. Is C major closer to g minor or to g♯ minor? The former preserves a common tone where the latter has none, and it is thus closer on one criterion. But the latter involves three units of voice-leading work ({C, B} = 1; {E, D♯} = 1; {G, G♯} = 1), whereas the former involves four ({C, B♭ = 2; {E, D} = 2; {G, G} = 0), producing a proximity judgment that contradicts the previous case.

In summary, we have reviewed three distinct metrics, each of which formalizes a different set of intuitions about triadic proximity. The classical metric evaluates triadic proximity in terms of mutual membership in diatonic collections and interprets figure 1.1 as very disjunct. The same passage is interpreted by Galeazzi's common-tone metric to be fairly conjunct, and by the voice-leading metric to be very conjunct. The diatonic collection, which plays a central role in the first metric, has no privilege whatsoever in the remaining two. In the voice-leading metric, which most successfully captures the intuition that the triads in Schubert's progression inhabit a similar neighborhood, it is the chromatic collection that explicitly comes forward as the template against which distance is assessed.

Triads in Chromatic Space

To view consonant triads against the background of chromatic space is to decline to interpret them in terms of the number of diatonic degrees that separate their root from some tonic. This choice cuts against the multiple denominations of classical tonal theory and their pedagogical offshoots, which all teach that chromatic harmonies are primarily to be understood as transformations of some underlying diatonic one. The idea that the diatonic collection conceptually precedes and regulates the interpretation of the chromatic one, already implicit in the names of notes, their position on the staff, and the system of key signatures, became canonized with respect to classical tonality in the early nineteenth century, at roughly the same historical moment that musical education became institutionalized in conservatories, analysis evolved into its own discipline, a theory of tonality began to congeal under that name, and Roman numerals became the default first-level descriptors for triads (Wason 1985, 53). That idea has proven hardy indeed, as can be confirmed with reference to any English-language harmony textbook.

The diatonic view of chromaticism has prevailed for good reason. Triads and diatonic scales together constitute the foundational organizing materials of classical tonality. Although the diverse traditions of classical theory assign consonant triads and diatonic scales different values in relation to each other (Dahlhaus 1990 [1967]), they all agree that it is through their coordination that major and minor keys are established. For an acculturated listener, a major or minor triad, sounded in isolation and without prior context, signals the tonic status of its root by default. In a process first described by Gottfried Weber (1846 [1817–21]), a listener spontaneously imagines an isolated triad housed within a diatonic collection, signifying a tonic that bears its name.

Yet there is a difference between a default interpretation and a necessary one. The tonic status of a triad requires confirmation, weakly through the remaining tones of its associated diatonic collection; more strongly by arranging those tones into a local cadence; more strongly yet by repeating that cadence, perhaps with supplementary rhetorical packaging, at the end of the movement or composition. Such a confirmation is by no means inevitable. Sometimes an initial triad comes to be understood as representing a nontonic degree (Kirnberger 1982 [1771–76], 45; Aldwell and Schachter 1989, 135). And sometimes it initiates a succession of triads, any of which could claim tonic status under appropriate conditions (Kirnberger 1982, 114; Schenker 1954 [1906], 254). The more tones put in play, the less likely their alignment with respect to a diatonic collection that organizes their position and role with respect to some tonic. Until such a collection emerges and is cadentially crowned, the triadic progressions are diatonically indeterminate.

Once essential enharmonic relations arise, indeterminacy evolves into contradiction. By essential enharmonicism, I am excluding those notated enharmonic conversions that arise as artifacts of notational pragmatics, as might happen if a composer presumes that a performer is more comfortable reading a signature of four sharps than eight flats. What distinguishes essential enharmonicism is that the composer has no choice but to convert between sharps and flats in order to retain global diatonic logic (Schenker 1954 [1906], 333–34). In such cases, the exact point where the composer notates the conversion is a pragmatic matter without significance; in a phenomenological sense, such a conversion happens everywhere and nowhere, which is tantamount to saying that it is distributed evenly across all of the possible moments when it could occur (Proctor 1978, 177; Telesco 1998; Harrison 2002a). When enharmonically paired pitch classes are juxtaposed directly, the ear cannot avoid identifying them as the product of a single tone. As we saw with reference to figure 1.1, the splitting of a tone's scale-degree constituency can have a ripple effect, destabilizing the diatonic collection and the tonic that is claimed to anchor it.

The recognition that triadic music is not always fully determined by the principles of diatonic tonality is by no means a new one. As already noted, early-nineteenth-century critics intuited that contemporary music defied familiar logic (although they disagreed as to whether this was a good thing). The impulse to systematize these intuitions was first acted on in the writings of François-Joseph Fétis, a Belgian music critic working in Paris, initially in a series of articles from 1832, eventually in his 1844 harmony treatise. Fétis divided tonality into four stylistic species, each representing a stage of historical development, and each defined by its own syntactic principles and affective properties. The most progressive of the four species, omnitonality, is distinguished by a proliferation of enharmonic relations that indicate a "multiplicity, or even the universality of the keys" (Fétis 2008 [1844], 190), a process that Fétis predicted would lead to "the total destruction of the scale in certain cases, and the beginnings of an acoustic division of the musical scale into twelve equal semitones" (Berry 2004, 257, quoting Fétis 1832). For Fétis, the objects of omnitonality are chromatically intensified dissonant harmonies, rather than the consonant triads that concern us in the present study. It is rather in the historically earliest of Fétis's four species, unitonality, that one finds tonally indeterminate chromatic successions of triads, as in some music of Marenzio and Gesualdo from the turn of the seventeenth century. Such successions fail to define a key because their constituent triads do not communicate with each other: "No attraction is evident, because every perfect chord is a harmony of repose" (Fétis 2008 [1844], 163). If every chord is a potential tonic, then no chord can fulfill that potential by functioning as one. Each tub is on its own bottom, bobbing around the sea independently of the others.

Although each of Fétis's four tonal species arises at a particular historical moment, the later species do not supplant the earlier ones. According to his historical model, they are cumulative; once available, the best composers know how to combine them in a single work of art (Berry 2004, 255). It is evident, then, that Fétis conceives of classical tonality ("transitonality") as a category whose constituent elements are not integral "pieces"—compositions or complete movements—but rather musical moments. On Fétis's view, the faculty of (transi)tonal listening is capable of spontaneous suspension and reengagement without notice or fuss, like a carpenter exchanging a screwdriver for a hammer. He recognizes a similar dynamic in a purely diatonic environment, as when a sequence arises midphrase. At the moment that the sequence is recognized, the "law of tonality" is placed in abeyance, as our cognition is submitted to a "law of uniformity." "The mind, absorbed in the contemplation of the progressive series, momentarily loses the feeling of tonality, and regains it only at the final cadence, where the normal order is reestablished" (Fétis 2008 [1844], 27).

The idea of simultaneously accessible tonal schemata was developed specifically with relation to pan-triadic progressions seventy-five years later by Ernst Kurth, who was raised in an era of rampant, fully ramified omnitonal chromaticism that Fétis could only divine. Kurth's *Romantische Harmonik und ihre Krise in Wagners "Tristan,"* initially published in 1920, proposed that many chromatic progressions, particularly those that involved root relations by third, introduced rifts, wedges, and fissures into the fabric of tonality. The identity and function of these chords are found in their internal structure and in their local connections to their immediate antecedents and successors. When concatenated with sufficient intensity and persistence, such absolute progressions bring about "the total disruption of the original embracing tonal unity" (Kurth 1991, 120). Kurth discovered an agent of tonal disruption in chromatic sequences, which, like Fétis's diatonic ones, are governed by the logic of repetition. Such progressions are "extratonal" in the sense that their relation to the tonal pillars that bound them on either side is not tonally determined.

After Kurth's 1920 treatise, it became a commonplace of German musicology that neither the appearance of consonant triads nor their framing by occasional cadential progressions was sufficient to justify the judgment that their syntax was governed by the principles of classical tonality; other factors were necessary in addition (Adorno 1964; Kunze 1970; Dahlhaus 1980a [1974]; Motte 1976). Among the adherents of this view were Theodor Adorno and Carl Dahlhaus, both of whom eventually acquired a significant readership in North America, one result of which was that Kurth's views immigrated into the arena of American musicology (e.g., Newcomb 1981; Meyer 1989, 302; Agawu 1989, 27; Abbate 1991, 192). Similar views can also be found, perhaps surprisingly, in the writings of American Schenkerians, who otherwise are committed to the vision of the masterwork as organically unified by *Ursatz* emanations that function uniformly at all compositional levels. These included Adele Katz, for whom the Magic Sleep music from *Die Walküre* "lack[s] . . . tonal implication" (1945, 213); William J. Mitchell, who noted that a triadic circle of fifths "can be arrested at any point or it can just as easily go on in perpetuity" (1962, 9); and Felix Salzer and Carl Schachter, who wrote that "we register the equal intervallic progressions without referring them to a supposed diatonic original. This temporary lack of a diatonic frame of reference creates, as it were, a suspension of tonal gravity" (1969, 215).

The dissemination of this view has not, however, dislodged a broadly shared commitment to the notion that the chromatic triadic progressions characteristic of the nineteenth century are determined by their position with respect to some tonal center. This commitment is evident not just in the profusion of inflected Roman numerals or function symbols that dominate the textbook teaching of nineteenth-century harmony on both sides of the Atlantic. It also dominates various branches of research, whether based in Roman-numeral/fundamental bass traditions (Lerdahl 2001), Schenkerian/linear approaches (Darcy 1993; Brown 2005), Riemannian functions (Harrison 1994), or Lewinian transformations (Kopp 2002). Although these denominations interpret triadic harmony according to quite different sets of assumptions, and express those interpretations using distinct modes of representation, they all share a base in the late-eighteenth-century classical harmonium, from which they reach out to lay claim to the chromatic triadic music of the nineteenth century.

I can think of three reasons that analysts of nineteenth-century triadic music have continued to dance to a modified eighteenth-century beat, despite the many stumbles induced by the terrain. First is the promiscuity of triadic descriptive categories, combined with the illusion that to describe is to explain. Roman numerals are flexible enough to furnish a first-level description of almost any triad in almost any key (Dahlhaus 1980a [1974], 68; Hyer 1989, 229–30). Many Roman numeral practices are satisfied, moreover, with finding a local tonic for each harmony, without any demand that local tonics be reconciled to each other and to a global tonic. Riemannian functions likewise are catchall categories, such that "a student of Riemann's system can analyze virtually any chord into any one of the three functions should the occasion demand" (Harrison 1994, 284). Schenkerian approaches allow chromatic triads to degrade into coordinated linear spans (Benjamin 1976; Smith 1986), which serve as carpets under which to sweep enharmonic paradoxes.

A second reason for the continued resistance to alternative views of triadic chromaticism is that it requires an embrace of some form of double syntax. Most nineteenth-century passages that can be seen to juxtapose triads according to nonclassical principles exist in close proximity to other behaviors that are normal under classical diatonic tonality. The Schubert excerpt with which we began (figure 1.1) is not atypical: while the local spans are classically tonal, the middleground tonics adhere to a different logic. To analyze such a composition requires not only that we navigate, sometimes in rapid alternation, between two or more syntaxes, as Fétis imagined listeners moving between his four kinds of tonality. It also requires the capacity to simultaneously process two distinct sets of syntactic principles that unscroll at different speeds. Can music of high aesthetic value really partake of two systemic modes of organization, shuttle between them quasi instantaneously, and even overlay them? Are our musical brains wired in such a way that we have the capacity to shift between these syntaxes as if at the click of a switch, or to multitask between them? If the responses of several prominent music scholars are representative, it seems that there is a strong motivation to reject any such idea on a priori grounds, which is to say, independently of the details of the proposal under which a double syntax program might be carried out (Dahlhaus 1990 [1967], 111; Smith 1986, 109; Lerdahl 2001, 85). Chapter 9 considers and responds to this line of objection; readers who share this prima facie skepticism may wish to teleport there before proceeding with the linear exposition.

The final reason pertains to the absence of a fully ramified alternative. We are inclined to come out from under familiar technologies only when we are prepared to substitute for them an alternative that is plausible, coherent, and productive. To acknowledge that chromatic progressions of triads might be based in some syntactic principles other than those of diatonic tonality is to clear a space, but that is not the same thing as building a house. One needs to be able to say something about what that syntax is, not just what it is not. To say that "Beethoven's third period seemed destined to shake the absolutist regime of the main tonality for the first time" (Draeseke 1987 [1861], 315) or that some Wagnerian progressions "stand . . . in certain opposition to tonal unity" (Kurth 1923, 249; my translation) constitutes a necessary first step. To allow for the existence of a "countersyntax" that stands in "dialectical" relation to classical tonality (Kramer 1986) constitutes a significant second one. But to posit the terms of that countersyntax, it is necessary to do more than substitute a Latin adjective for its Greek equivalent (as occurs whenever a writer feels that they have scratched an explanatory itch when they have attributed a chromatic harmony to a "coloristic" effect),[12] or refer to linear processes without being prepared to specify anything beyond pointing to lots of semitones (e.g., Dahlhaus 1980a [1974], Agawu 1989). In addition, one wants to know what principles underlie the syntax, how it operates, how its analyses are represented. Are its claims consistent, well formed, and free of internal contradiction? How is the syntax motivated by the lexicon; that is, what properties do triads possess that qualify them for the job that (the syntax claims) nineteenth-century composers put them up to perform? What sorts of problems does this syntax help solve? Does it generate analyses that reflect some aspect, however obliquely and abstractly, of a musician's or listener's experience? Does it lead us to notice interesting things about a score, or about its relationship to other scores, that would have otherwise escaped attention? Does it help us think differently about historical problems of genre, style, evolution,

12. See Tischler 1964, 233; Rosen 1980, 245; Kramer 1986, 203; Todd 1988, 94; Meyer 1989, 299; Ratner 1992, 113; Somer 1995, 219; and Taruskin 2005, 69. "This chromaticism has a coloristic effect" has roughly the propositional status and explanatory value of "this box is so heavy because it weighs a lot." David Kopp (1995, 345) makes a similar point.

and the like, or about the relationship between music and the historical conditions of the individual, society, or culture that produced it?

This book responds to these questions by adapting a conceptual framework erected between 1955 and 1980 by the field of atonal pitch-class theory, whose great achievement was to develop a systematic approach for exploring the properties, potentials, and interrelations of chords ("sets") within the chromatic universe. Atonal theorists of that era were not much interested in consonant triads, as their analytic interests were focused on a repertory whose principal phonological constraint was, on some accounts, their absence (Boulez 1971 [1963]; Forte 1972; but see Straus 1990). Reciprocally, music scholars of that era who were open to the cultivation of alternative approaches to nineteenth-century triadic music were alienated from American atonal theory because of geography, the serendipities of disciplinary configuration, or the low priority that cold-war theorists placed on disciplinary outreach. In exploring the properties and potentials of consonant triads using a method adapted from atonal theory, I hope to defuse the suspicion that "applying analytical techniques derived from contemporary music" to late-Romantic repertory is "menial and easily accomplished" (Dahlhaus 1989 [1980], 381–82), or an act of desperation (Harrison 1994, 2).

Remarks on Syntax and Maps

Syntax is a central term in the study of natural language, and not all of the meanings that it accumulates there can be transferred into music. Syntax is that branch of linguistics that studies how words and their constituent particles combine to form coherent sentences, independently (in principle) of how those sentences represent concepts and states, or motivate actions, in the world. I use *syntax* in this book in three different ways. First, syntax contrasts with phonology and lexicon, which respectively treat the internal structure of atomistic units and their first-level bundling into units of signification or reference. Because music, under ordinary conditions, lacks the referential dimension of language, phonology and lexicon come close to fusing: a lexicon is a list of available sounds (chord, scales, sets), and phonology provides a principled account of what properties make those sounds available for use. Second, syntax is the study of the ordering of events as they sequentially unfold in time: how triads "progress" in a moment-by-moment sense, and perhaps also in a middleground sense where such interpretations are appropriate. Third, and most important for present purposes, I use syntax in the same sense that the Greeks used *harmonia*, the "means of codifying the relationship between those notes that constituted the framework of the tonal system" (Dahlhaus 1980b, 175). This broader domain is roughly equivalent to what Roger Sessions (1950, 33) designated as "the relationships between tones, and . . . the organization which the ear deduces . . . from those relationships" and what David Lewin (1969, 61) characterized as the way that "sound [is] conceptually structured, categorically prior to any one specific piece." It is at this most abstract level that we can also refer to Fétis's four types of tonality as evincing distinct syntaxes, or think of his laws of tonality and uniformity as manifesting distinct syntactic principles. They are distinct in the sense that they generate different orderings of the harmonies, doublings of chords, and expectations about dissonance treatment. But they are also distinct in the sense that they evoke different modes of musical cognition.

In this third sense, musical syntax has long benefited from geometric and graphical representation. Geometric models of pitch space have been in use for some 1,500 years (Popovic 1992; Westergaard 1996). During the eighteenth and nineteenth centuries, they were frequently applied to relations among keys, and later among chords. American music theory of the postwar era generally favored algebraic models, for their compactness (a significant consideration in print media) and their strong generalizing capacity. A surge in geometric models began in the 1980s, and has intensified in the last decade, in part due to the increasing accessibility of graphics software and the economies afforded by electronic space. Geometric models can thrive as effective modes of exploration and communication only if the phenomenon being modeled meets certain structural and psychological conditions. Structural problems arise if there are more conceptual dimensions than are available in the physical medium. In linear space, one dimension is great, two's fine, three's the limit, and four blows the mind. In cyclic space, even a second dimension introduces falsifications and distortions, like the Bering Strait problem familiar from Eurocentric world maps. Moreover, Euclid's logic often collides with that of the psyche. The symmetry of spatial distance may lack psychological salience for someone walking uphill, and the triangle inequality prohibition is violated whenever two miles walked in intense conversation feels shorter than one mile alone on a sore ankle. Fortunately for my project, in the case of triadic distance measurements these problems are kept to a minimum. The cyclic structure of chromatic space will create some Bering Strait problems, but we will find that these are easily negotiable with the help of a supplementary "legend" that guides interpretation of the map.

The supreme advantage afforded by musical maps is their capacity to reflect judgments about the psychological proximity of musical objects or states (Popovic 1992). Elementally, such judgments come in binary form ("*these* two notes sound close, *those* two sound distant") that lead naturally to comparison ("'*these* notes sound closer together than *those*"). When structural and psychological conditions align, a map has the capacity to draw together a family of pairwise distance assessments. Such a map then acquires the capacity to capture syntactic judgments, which might take the form of conjunct versus disjunct, normal versus unusual, or acceptable versus unacceptable. Moreover, it earns the potential to aid in the exploration of semantic predicates, such as "betweenness" (there is a gap that one expects will be filled), "orientation" (we can chart our distance from and direction with respect to some "home"), or "momentum" (there is a pattern whose continuation we anticipate).

Like a geographical map, a good representation of musical space does not merely sit there, as a static structure. It acts as a stage upon which imaginative performances are mounted, thus serving the same function as a geographical map for a child with a toy car, or for a medieval monk tracking a crusade (Connolly 1999).

A musical map can illuminate compositional decisions as selections from a finite menu. It can move composers to ask, "How many ways are there to connect these two chords?" "What chord stands halfway between these two chords?" "How can I form a cycle, with some desired number of elements, that begins and ends at the same element?" or "If I'm at A, what state B should ensue, if I want to mimic the gesture that carried Q to R?" It invites analysts to ask, "Is this a step or a leap?" "Is this connection the most direct one?" "Are these two paths parallel?" or "Is this path an embellishment of that one?" Questions of this type are concerned with corpuswide ideals, norms, and limitations, as well as with "motivic" elements that shape and individuate a particular composition in dialogue with those norms and ideals. These are the musical equivalents of *langue* and *parole*, language and utterance, the topics most central to the syntactic study of natural language.

Steven Rings (2006) suspects me of using *coherence*, in related contexts, as a fourth-order stalking horse for the universalization of nineteenth-century German aesthetic ideology, by way of the intermediate terms "unity," "autonomy of the artwork," and so forth, and he and others may suspect that *syntax* just heaps another shell or two of derivatives on top. I am not committed to either italicized term and would by happy for readers simply to substitute some other, or perhaps some neutral, term (e.g., *X-factor*) in their place. I do think that there is some profit in acknowledging that, among communities, some musical phenomena "go down easy" and some "go down hard." *Asyntactic* and *incoherent* signify the neighborhood of aesthetic responses that might alternatively take the form "doesn't make sense," "sounds weird," "sounds erratic," "doesn't fit," "sounds random," "sounds awful," "I don't get it," "I wasn't expecting that," "that's not normal," "not immediately intelligible." Someone with a historical sense might posit those same responses through comparison, similar to the way that Forkel responded to that odd passage from C. P. E. Bach's f minor Piano Sonata (Kramer 2008, 11), or that some Viennese critics heard Schubert's modulations (Shamgar 1989), or that the first European heard South Asian music or the first Indian heard European music. Although I don't care what term is used to make the distinction, I am pretty sure that there is a distinction to be made.

I would even go so far as to suggest that that distinction is universal, on the hypothesis that, for individuals or communities or cultures, there are things that make sense and things that don't, things that go down easy and things that go down hard, things that are familiar and things that are foreign, and so forth. An anthropologist might make this distinction with the term *emic* (fits the world-view of the folks who live *there*), an intellectual historian with *episteme* (fits the world-view of the folks who lived *then*), and a linguist with *syntactic* (has potential meaning within that linguistic community). What I mean by "syntactic," then, is the musical equivalent of all of those.