Bartók's Polymodality

The Dasian and Other Affinity Spaces

José Oliveira Martins

Abstract The article proposes that a construct I call the *Dasian space* provides an effective framework to interpret harmonic aspects of scale relations in twentieth-century polymodality, particularly in the music of Bartók. Based on Bartók's intuition that the pitch space modeled after his notion of *polymodal chromaticism* retains integral "diatonic ingredients," the Dasian space (named after the medieval homonymous scale) establishes a system of relations between all potential diatonic segments, without relying upon traditional constraints, such as complete diatonic collections, harmonic functions, or pitch centricity. The Dasian space is a closed, nonoctave repeating scalar cycle, where each element is identified by a unique coordination of pitch class and modal quality. The dual description of each element enables both the specification of location in a given cycle and the emergence of a group structure, whose generators—named *transpositio* and *transformatio*—are also characteristic musical motions and relations. The proposed analytical methodology is probed in a couple of short pieces of Bartók's *Mikrokosmos* and in the third movement of his Piano Sonata. The article argues that, unlike other tonal and atonal classic approaches, the Dasian framework enables the analyst to reconcile the constructional character of a Bartókian idiomatic feature (the combination of distinct and integral scale strata) with the interpretation of harmonic space in terms of scale-segment interaction and formal processes. The article then contextualizes the structure of the Dasian space within a larger class of constructs, which I call *affinity spaces*, by generalizing some of its group-theoretical properties that model relations between nondiatonic scalar materials. The analytical pertinence of affinity spaces is probed in Bartók's "Divided Arpeggios," an intriguing posttonal piece appearing late in the *Mikrokosmos* set.

Keywords Béla Bartók, *Mikrokosmos*, Piano Sonata, polymodality, polytonality, diatonicism, chromaticism, scale theory, rotational form

THE COMBINATION OF DISTINCT DIATONIC LAYERS is a characteristic marker for early twentieth-century composers such as Béla Bartók, Igor Stravinsky, and Darius Milhaud. A variety of terms, such as *polymodality*, *polytonality*, *polychordality*, and *polyscalarity*, have been used to describe the layered construction of multidiatonic passages, and yet the harmonic implications for such

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preferential deployment remain poorly understood.[1] The prevailing analytical activity developed for this repertoire in the second part of the last century often downplays the scalar integrity of combined diatonic materials, thus questioning the syntactical pertinence of scale segments in the global organization of pitch space. In particular, tonal and atonal harmonic interpretations of Bartók's scalar layering often compromise, or simply override, the coherence of individual segments. Models sensitive to tonal associations propose extended tonal frameworks that fuse the combined diatonic strands to project prolongational and centric readings, whereas models concerned with progressive and atonal aspects of the style explore how layered diatonic elements engage with the syntactic possibilities of the twelve-tone chromatic space.[2] In short, tonal approaches reduce the resultant chromaticism of the combined strata into deeper-level diatonicism, whereas atonal approaches scatter surface diatonicism into deeper-level chromaticism.

While the combination of diatonic layers in the twentieth century has been understood as both providing a strong link to the past and signaling progressive harmonic innovations,[3] the rift between the compositional procedure and the established interpretative analytical activity invites a number of fundamental questions. If Bartók and other composers are using an expanded tonal framework, why are tonal routines often hard to detect or define? And if composers are using an atonal framework, why are tonal materials privileged in the combined deployment? Specifically, which resources and properties of the pitch system are explored by the interaction of layered diatonic materials, and how are such interactions best interpreted in light of local and global harmonic relations?

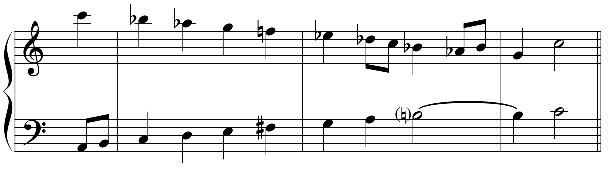
This article sketches an answer to these questions by proposing an analytical approach to some of the harmonic implications of Bartók's polymodal practice. A preliminary and useful interpretation of this practice is offered by the composer's own notion of "polymodal chromaticism."[4] Bartók coined the term using a short musical illustration (see Example 1), which superimposes two contrary-motion diatonic melodies (four flats over one sharp), wedging toward pitch class C. Bartók ([1943] 1992, 367) interprets the passage as the

1 These terms variously refer to a compositional technique and a stylistic label. A more general designation often used is *diatonic posttonality*.

2 For a selective sample of analytical approaches to Bartók's music that integrate the diatonic strata within models of expanded tonality, see Travis 1970, Waldbauer 1990, Morrison 1991, and Lerdahl 2001, 333–43. For approaches projecting the diatonic strata into chromatic (pitch and pitch-class) space, see Perle 1955, Forte 1960, Antokoletz 1984 and 2000, Cohn 1988 and 1991, Wilson 1992, and Bernard 2003. Some of these approaches are discussed in more detail below.

3 Fosler-Lussier (2001) discusses "conservative" versus "progressive" aspects of the "two Bartóks" in the context of the reception of his music during the Cold War in Europe.

4 The term *polymodal chromaticism*, coined by Bartók ([1943] 1992, 365–71), has been adopted and extended by some post-Schenkerian models of prolongational structure (see especially Waldbauer 1990; Morrison 1991; Lerdahl 2001). The concept is also explored in a nonprolongational manner by János Kárpáti (1994).



Example 1. Bartók's musical illustration of "polymodal chromaticism" (Bartók [1943] 1992, 367, example 4)

superimposition of integral modal segments or scales, which combine to exhaust the chromatic space and share a common final (and fifth):

> As the result of superposing a Lydian and Phrygian pentachord with a common fundamental tone, we get a diatonic pentachord filled out with all the possible flat and sharp degrees. These seemingly chromatic degrees, however, are totally different in their function from the altered chord degrees of the chromatic styles of the previous periods. A chromatically-altered note of a chord is in strict relation to its nonaltered form: it is a transition leading to the respective tone of the following chord. In our polymodal chromaticism, however, the flat and sharp tones are not altered degrees at all; they are diatonic ingredients of a diatonic modal scale.

Bartók's often-cited formulation for polymodal chromaticism attempts to reconcile two theoretical intuitions, namely, that (1) the global chromatic space combines diatonically coherent strata and that (2) an underlying harmonic principle ("a common fundamental tone") coordinates the strata.[5] While intended as a corrective to the view that his music uses multiple concurrent tonics, which result in a polytonal "maze of keys" (Bartók [1943] 1992, 366), the stipulation of a single "common fundamental" for combinations of strata often fails sustained analytical scrutiny for numerous polymodal passages: analyses of the repertoire reveal that projected pitch centers are often not common tones to the strata involved, available common tones are not deployed as pitch centers, or combined layers simply do not project salient pitch centers.

In contrast, this article explores Bartók's intuition that "all possible flat or sharp tones" in a polymodal deployment are not "altered" or "chromatic" but, rather, "diatonic ingredients" within their respective layers. I argue that Bartok's insight entails that the resulting harmonic space relies on the scalar

5 The insistence on a "single fundamental tone" for the combination of layers has caught the attention of the Schenkerian tradition, which led to the development of (abovementioned) pitch models that support prolongation in Bartók's posttonal environment. Such centric readings, however, not only rely on the fusion of "departure" or "referential" elements (Morrison 1991, 182–83) from distinct strands but also are frequently confronted with passages that fail to provide a common tonic to the combined layers, which in turn undermine a stable referent for large-scale progressions.

integrity and interaction of distinct diatonic segments.[6] Specifically, the article advances a framework for multidiatonic harmony, which interprets distinct diatonic strata in terms of harmonic distance within a single integral system that embeds all diatonic segments. This framework bypasses attributions of pitch centers, diatonic scale-degree functions, and the default invocation of complete diatonic collections.[7] (These aspects are still available if analytically useful but are not considered crucial for the understanding of a global polymodal space.)

The central feature of this inquiry is a cyclic model of diatonic scalar segments, as they interact within chromatic space. I refer to this construct as the *Dasian space*, since it generalizes some aspects of the medieval Dasian scale. Specifically, the space generalizes affinity relations at the perfect fifth of the four modal qualities that a given note can assume in any diatonic collection and explores the group-theoretical properties that result from the coordination (order pairs) of pitch classes and modal qualities.

After introducing the conceptual framework for the Dasian space and probing its analytical potential in a couple of brief pieces from the *Mikrokosmos*, this article examines how polymodal activity in the third movement of Bartók's Piano Sonata engages the exploration of pitch space. The patterns that emerge suggest that Bartók's preference for compositional deployments that preserve distinct diatonic strata do not merely explore "color" or texture but participate in local and large-scale tonal/pitch strategies, which in turn shape aspects of form.[8]

The article extends the principle of intervallic affinities underlying the Dasian space to a larger class of constructs, which I call affinity spaces, by generalizing some of its group-theoretical properties that model relations between nondiatonic scalar materials. The analytical pertinence of affinity spaces is probed in the modeling of pitch space for Bartók's "Divided Arpeggios" (*Mikrokosmos* no. 143). The article concludes by briefly inventorying some of the theoretical and analytical implications for the understanding of polymodality set forth by the proposed framework.

6 This insight is reinforced by Alfred Casella's (1924, 159–60) claim that "'polytonality' signifies, to be sure, the interpenetration of diverse scales; but, it likewise assumes—in the very nature of things, the survival of the original scales."

7 Traditional approaches consider complete diatonic scales as hypostasized entities. Those entities are thought to construct a tonal or modal space in which relationships obtain between entire scales. By invoking a part of the scalar entity, one inherently also invokes its entirety, and thus when distances between scale segments are measured, they are routed through distances between the entire scales with which the segments are associated. The traditional approach acknowledges that elements between those scales might be shared, but it insists that those elements remain conceptually distinct.

8 I am not suggesting that Bartók's music (or the theoretical apparatus I am proposing) is stylistically "neomedieval." However, I am proposing that a number of pitch space properties (such as interval affinities, modal qualities, and relations) relevant to medieval conceptions of pitch space are also appropriate and useful to understand the interaction of diatonic (and other scalar) layers that characterize twentieth-century polymodality. Several authors have developed approaches that study the interaction or combination of scalar spaces for the music of Bartók (Kárpáti 1994; Gollin 2007), Debussy (Hook 2008; Tymoczko 2004), Ives (Lambert 1990), Milhaud (Harrison 1997), Reich (Quinn 2002), Ravel (Kaminsky 2004), and Stravinsky (Tymoczko 2002) in ways that are variously resonant with the analytical approach proposed in this article.

Tetrachords	graves				finales				superiores				excellentes				(residui)	
Modal quality	*protus*	*deuterus*	*tritus*	*tetrardus*	*protus*	*deuterus*	*tritus*	*tetrardus*	*protus*	*deuterus*	*tritus*	*tetrardus*	*protus*	*deuterus*	*tritus*	*tetrardus*	*protus*	*deuterus*
Note names	G	A	B♭	c	d	e	f	g	a	b	c'	d'	e'	f♯'	g'	a'	b'	c♯''
Interval pattern		T	S	T	(T)	T	S	T	(T)	T	S	T	(T)	T	S	T	(T)	T
Octave-fifth regions																		

Figure 1. The medieval Dasian scale

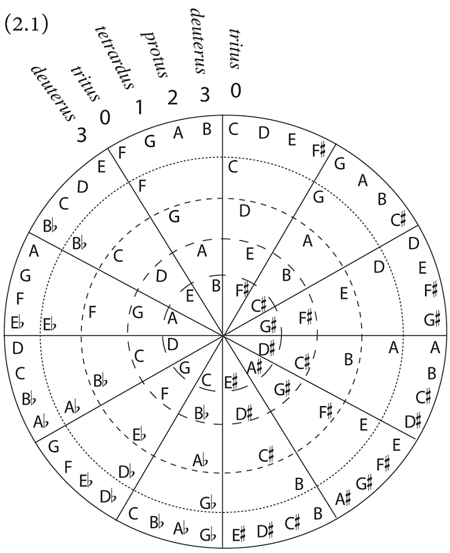
The Dasian space

The structure of the proposed Dasian space is built upon the pattern of intervallic affinities embedded in the medieval Dasian scale.[9] The scale, illustrated in Figure 1, is a stacking of four TST (tone–semitone–tone) tetrachords, consistently separated by a tone of disjunction, plus two "residual" notes. Each tetrachord embeds the four modal qualities—*protus*, *deuterus*, *tritus*, and *tetrardus*—which are defined by the relative position of a note with respect to the interval pattern of the tetrachord. The scalar arrangement of the Dasian scale privileges affinity relations at the perfect fifth, given that notes four steps apart share the same modal quality (i.e., the same pattern of neighboring tones and semitones). Affinity relations at the fifth, however, conflict with octave relations in the scale, since notes seven steps apart in the scale have different modal qualities. In other words, the consistency of modal replication at the fifth "modulates" or "curbs" a strict diatonic pattern, such that the Dasian and the diatonic scales intersect only in a span of a major tenth. Norman Carey and David Clampitt (1996) refer to this contiguous span as an octave-fifth region (see Figure 1, solid lines beneath the scale). As they explain, the relation between octaves and fifths in diatonic space is reversed in the Dasian scale: "Just as the periodicity at the octave in the usual diatonic pitch space allows two voices to move perpetually in parallel octaves, while voices moving in parallel fifths will encounter a diminished fifth, in the dasian scale, the reverse is the case: parallel fifths may be maintained perpetually, while motion in parallel octaves will eventually result in an augmented octave" (124).

The interval structure and associated affinity relations of the Dasian scale can be generalized into a Dasian space construct (Figure 2). The space

9 The Dasian notation and its associated scale are discussed in the medieval *Musica enchiriadis and Scolica enchiriadis* treatises (Erickson 1995). Interval affinities are discussed in Pesce 1987, Cohen 2002, and Atkinson 2008 in the context of medieval theory in the early middle ages.

(2.1)



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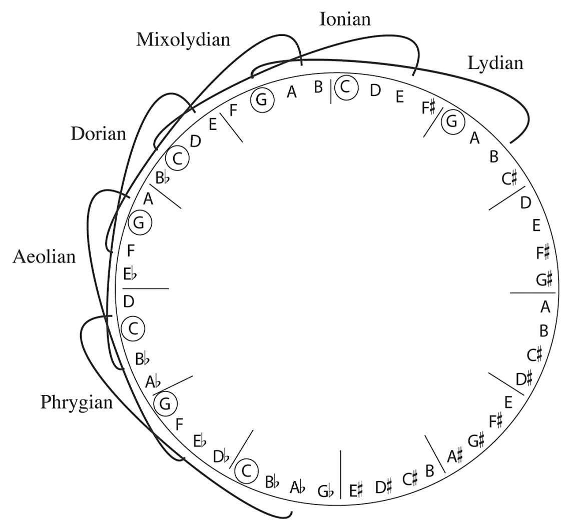

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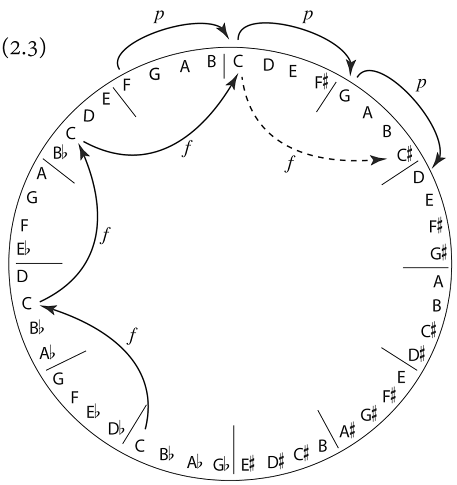


Figure 2. The Dasian space. (2.1) The four modal qualities—*protus*, *deuterus*, *tritus*, and *tetrardus*, reinterpreted as mq(2), mq(3), mq(0), and mq(1)—are structured by the four interlocked circles of fifths; the twelve pitch classes are structured by the twelve scalar cycles of modal qualities. (2.2) Extended region of six partially overlapping octave-fifth regions (reckoned from a C modal center) that contains all pitch classes C and G in Dasian space. (2.3) The operations transpositio (*p*) and transformatio (*f*).

is a closed structure in two ways. First, it stands as an ordered cycle of forty-eight space elements (outer circle in Figure 2.1), where each pitch class (pc) appears four times at different modal quality (mq) positions, and conversely, each modal quality appears twelve times at different pitch class equivalences. For generalization purposes, the four medieval modal qualities are relabeled as cyclic order positions, where *protus* corresponds to mq(2), *deuterus* to mq(3), *tritus* to mq(0), and *tetrardus* to mq(1). As a result, the ordering ⟨mq(0), mq(1), mq(2), mq(3)⟩ graphically privileges the continuity of whole-tone scalar adjacencies, where radii in the figure mark off the semitone between neighboring positions mq(3) and mq(0). Each space element is thus uniquely defined as an ordered pair coordinating a pitch class *x* and a modal quality *y*: [pc(*x*), mq(*y*)], where $0 \leq x \leq 11$ and $0 \leq y \leq 3$.[10] Second, the space exhausts several increasingly larger diatonic structures, which are related by perfect fifth (twelve times): semitones (which indicate unique scale-step locations in the space), "Dorian" tetrachords (TST), guidonian hexachords (TTSTT), and locally diatonic octave-fifth regions.[11] Thus conceived, the Dasian stands both as a scalar space (where the order of modal qualities interacts with register) and a pitch-class space (where each element assumes octave and enharmonic equivalence).

Figure 2.2 reframes within a Dasian framework Bartók's illustration of polymodal chromaticism discussed in reference to Example 1. The region of the circle that contains all pitch classes C and G is segmented into six overlapping diatonic regions. The modal labels, clockwise from Phrygian to Lydian, indicate the diatonic mode of each region's pitch-class context, reckoned from a C modal center (or tonic). As proposed in Bartók's formulation of polymodality, the Dasian modeling preserves the scalar integrity of modes but measures distance between them in terms of their chromatic mismatches rather than in terms of the distance between their pitch centers.[12] In addition, the partial overlap of scale segments in neighboring modes suggests a continuity and harmonic gradation in polymodal space rather than a discontinuity of distinct modal or diatonic affiliations, whereby relations between different segments are routed directly through a measurable distance between space elements.

The system of modal quality affinities and pitch-class replications in the Dasian space induces relations between nonadjacent (pc, mq) elements that can be conceived as privileged navigation modes or intraspace motions. Each modal quality affinity relation is structured by one of the four interlocked circles of fifths (internal circles in Figure 2.1). In a dynamic view of this property, the motion of (multiples of) four "steps" (or clockwise stations) in the space retains a given modal quality and induces a change of pitch class by (multiples of) T_7 (mod 12). Let us label this characteristic motion as *transpositio* (p), thereby adopting to the context of the Dasian space the medieval term that

10 While conceived in different terms and designed for a distinct repertoire, Steve Rings's (2011) formulation of a theoretical framework that combines scale-degree *qualia* and pitch-class *chroma* underlying a tonal generalized interval system (GIS) structure captures aspects akin to the relation between modal quality and pitch class proposed here. The theoretical framework is developed in his chapter 2.

11 Martins 2009 shows that seven-note diatonic segments embedded in Dasian space (bounded by the transformatio relation as described below) constitute a unique symmetrical set-class ("host set"), which establishes a privileged correspondence with the space based on the properties of consistency, completion, and locality. The article generalizes these properties in the context of affinity spaces, of which the Dasian space is a particular example.

12 Although not a primary focus of this article, the notion of pitch centricity, or referential element as privileged common tone to different modal segments, can be modeled by the operation of transformatio introduced below by "fusing" two or more occurrences of the same pitch class in the Dasian space. The notions of pitch centricity and set-referentiality are invoked in the analysis addressing the various contexts for pentatonic themes throughout the third movement of Bartók's Piano Sonata, as well as in the movement's closure.

refers to a transfer of an interval pattern to a different position (or location) related by affinity in the diatonic scale.[13] In Figure 2.3, solid transpositio arrows exemplify the recurrence of *p*, (F, 0) → (C, 0) → (G, 0) → (D, 0), or pitch-class transposition by T_7 that retains mq(0), or *tritus*.[14]

Conversely, pitch-class replication in the space is structured by a cycle of modal-quality order positions. The corresponding motion of (multiples of) seven steps retains the same pitch class and induces a change of modal quality by (multiples of) 1 descending order position (mod 4). Let us label this characteristic motion as *transformatio* (*f*), thereby also adopting the characteristic medieval term that refers to a change of a note's surrounding interval pattern, or *formation* (i.e., a change in its modal quality).[15] In Figure 2.3, solid transformatio arrows track the motion *f*, (C, 3) → (C, 2) → (C, 1) → (C, 0), which exemplify the retention of pitch class C over an incremental descent in modal quality order positions. However, the continuation of a seven-"step" motion, exemplified by the dotted *f* arrow from (C, 0) to (C♯, 3), enacts a change of pitch class by T_1 (mod 12) while continuing the descent in modal-quality order position from mq(0) to mq(3). Let us also label this motion *transformatio*, since it extends the change of modal quality to the (chromatic) alteration itself (which in turn effects the formation, or reconfiguration, of the surrounding interval pattern).[16] In short, there are two types of outcomes for a transformatio move: either a pitch-class retention when the move is applied to mq(3), mq(2), or mq(1), or a pitch-class change (by T_1, mod 12) when applied to mq(0).[17] A complete forty-eight-transformatio cycle results from the succession of twelve complete four-modal-quality cycles of descending order position (where each four-modal-quality cycle goes through the four pitch-class occurrences).

The structure of the complete forty-eight-transformatio cycle is isomorphic with the structure of the complete (clockwise) forty-eight-step cycle, which results from the succession of the twelve complete four-modal-quality cycles of ascending order position (the forty-eight-step cycle is the default cyclic representation of the Dasian space used in Figure 2). Let us define the two possible outcomes for a step move *s*: either *s* effects an "ascending whole-tone" (T_2, mod 12) when applied to mq(0), mq(1), and mq(2), or *s* effects an "ascending semitone" (T_1, mod 12) when applied to mq(3).

13 A historical and theoretical examination of the terms *transpositio* and *transformatio* in medieval writings appears in Pesce 1986 and 1987.

14 Rather than using an integer notation for the first element pc(*x*) of the ordered pair [pc(*x*), mq(*y*)], this article instead uses the traditional letter name designation for pitch classes: C for 0, C♯/D♭ for 1, and so on.

15 The mnemonics (later conceived as space generators) *p* and *f* stand for the initial letter in the words *positio* and *formatio*, respectively.

16 Using as framework the medieval Dasian scale, the writer of the *Enchiriadis* treatises labels changes in modal quality due to "chromatic" inflections of the given chant as *vitia*. See Atkinson 2008, 128–29.

17 The formula regulating the unique value of pitch-class change occurring in transformatio is generalized further below for all affinity spaces.

In short, the structure of the Dasian space forms a cyclic group of order 48 and can be expressed in the context of a generalized interval system (GIS). In the GIS triple (S, IVLS, int), S is the family of forty-eight [pc(x), mq(y)] pairs; IVLS forms a cyclic group of forty-eight intervals, of which p, f, and s are group generators; and int is a function mapping $S \times S \rightarrow$ IVLS, such that int([pc(x), mq(y)], [pc(w), mq(z)]) = int([pc($w - x$)], [mq($z - y$)]), where pitch class and modal quality intervals are computed in mod 12 and mod 4, respectively.[18]

The Dasian framework and associated analytical methodology are now probed in two short *Mikrokosmos* pieces, which illustrate the space's versatility for modeling polymodal relations.[19] Bartók's "Diminished Fifth" (*Mikrokosmos* no. 101) is frequently discussed as an instance of octatonic usage, but the piece also engages consistent polymodal diatonic relations. The piece can be organized in six phrases, which are paired up to form three larger sections, according to motivic and harmonic considerations: mm. 1–11, A A′ | mm. 12–25, B A′ | mm. 26–44, B′ A.[20] The pitch reduction in Figure 3.1 shows the piece systematically superimposes scale segments ("Dorian" (0235) tetrachords) related by T_6 (hence the piece's title), which combine into resultant octatonic collections. In the first section, the tetrachordal superimpositions of both phrases produce OCT23; in the second section, the first phrase starts with OCT01 and the second phrase returns to OCT23; and finally, the last section runs through the three different octatonic collections (starting with the remaining OCT12, followed by OCT01, and returning to OCT23 in the closing phrase).

While a traditional reading contextualizes each tetrachordal strand with respect to a global octatonic macroharmony for each phrase,[21] a linear modal component cuts across phrase boundaries and addresses the sequence of octatonic collections and tetrachordal partitions throughout the piece. The

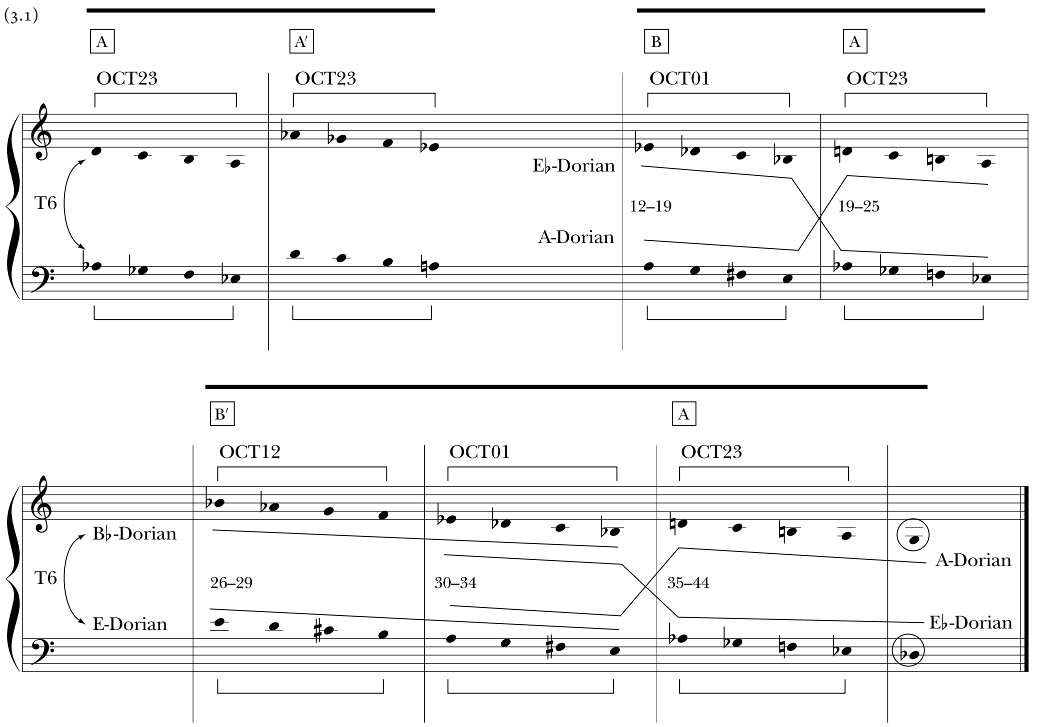
18 The concept of GIS is developed in Lewin 1987, chaps. 2–3. In the group structure of the Dasian space, the generators s and f yield the entire space, whereas p alone does not (iterations of p generate only twelve elements). Multiple (x) iterations of a given generator are expressed in the form p^x, f^x, and s^x. While focusing on a different set of pitch properties, Edward Gollin's (2007) notion of multi-aggregate cycles, structuring compound interval cycles, is amply consonant with the approach developed here; I address several aspects of Gollin's construct later in the article, when considering the larger framework of affinity spaces.

19 Bachmann and Bachmann (1983, 85–87) identify a pattern (referred to as the Lydian octachord scale cycle) that corresponds to a registrally disposed Dasian scale pattern. While relevant for mapping Bartók's musical materials, however, the pattern is used merely as a grid or background compendium for themes and motives drawn from different pieces of Bartók and not to explain matters of scale interaction or resultant harmony within a single piece. Martins (2006b) proposes a construct related to the Dasian space, called Guidonian space, which is used as a background scalar framework to model diatonic relations in Stravinsky's "Hymne" (Serenade in A). The Guidonian space is based on a pitch-class cycle that results from the stacking of a 2–2–1 modular unit, which efficiently embeds three diatonic modal qualities.

20 The formal arrangement proposed observes primarily thematic (motivic) and imitative relations. Other formal arrangements are naturally possible, such as a rondo-like scheme.

21 I use the term *macroharmony* in the sense proposed by Dmitri Tymoczko (2011, 4) as "the total collection of notes heard over moderate spans of musical time."

(3.1)



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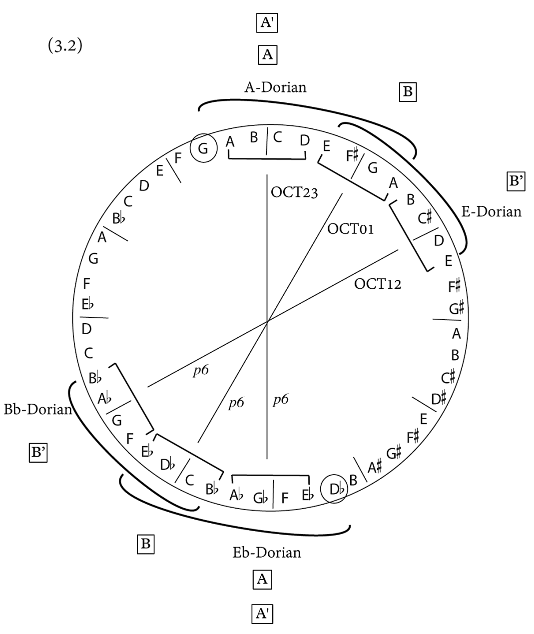


Figure 3. Bartók's "Diminished Fifth" (*Mikrokosmos* no. 101). (3.1) Pitch reduction of the piece based on the superimposition of "Dorian" (0235) tetrachords. (3.2) Octatonic and diatonic relations of Dorian tetrachords as projected in Dasian space.

crossed succession of tetrachords in the second section (from B to A′) yields the diatonic modal pair (E♭-Dorian, A-Dorian), unifying the entire section.[22] In the third section, the climatic phrase B′ moves by T_7 to the pair (B♭-Dorian, E-Dorian) before returning to the crossed succession of the previous pair of modal collections to finish the piece.[23]

The projection of Dorian tetrachords in Dasian space in Figure 3.2 shows that the piece explores two contiguous Dasian segments in opposite parts of the space. Adjacent Dorian tetrachords making up the modal collections are related by p, and octatonic tetrachordal pairs are related by p^6 occupying polar positions in the space. This reading privileges the diatonic continuity of the piece as it corresponds to contiguity in the space at the expense of octatonicism, which results from the superimposition of harmonically distant tetrachords.[24] The Dasian/modal reading also accounts for the two penultimate notes of the piece (G and D♭, circled in Figure 3), which are foreign to the octatonic reading but extend the three contiguous Dorian-tetrachordal segments.

Figure 4.1 is a pitch reduction for "Melody against Double Notes" (*Mikrokosmos* no. 70), which superimposes modally distinct scale segments in the left and right hands. Up to m. 16, both hands present (0257) tetrachords, suggesting two "gapped" scale segments A*l* (left hand), D–E–G–A, and A*r* (right hand), F♯–G♯–B–C♯. In the first phrase (mm. 1–9), the right hand plays a melody that emphasizes the note F♯ and the left hand plays a chordal accompaniment, whereas the second phrase (mm. 10–16) reverses the tetrachordal roles, with a left-hand inverted melody emphasizing the note A and a right-hand chordal accompaniment. After m. 17, gapped tetrachordal segments are filled by the notes F and A♯, respectively, resulting in inversionally related (02357) pentachords D–E–F–G–A and F♯–G♯–A♯–B–C♯.

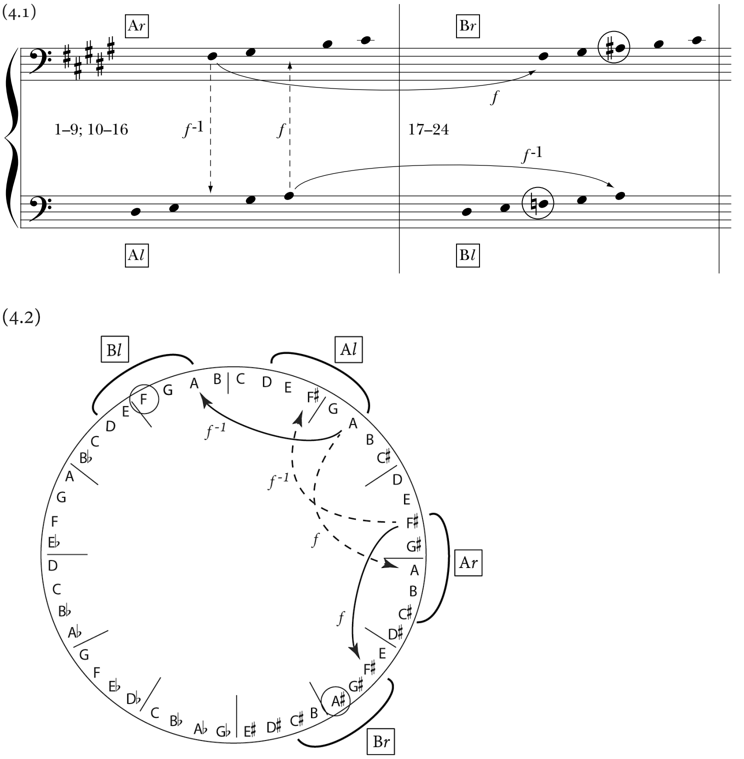
The ambiguity of diatonic affiliation of right- and left-hand tetrachords has interesting implications for polymodality and harmonic progression. Given the lack of a defining semitone, each gapped tetrachord can be projected at two alternative locations in Dasian space, marked A and B in Figure 4.2. As

22 The scalar arrangement of E♭-Dorian can be particularly heard across phrase boundaries in mm. 19–20. Notice the motivic imitation between hands and {E♭, C, B♭} as sharing aspects of both the B phrase (pitch content) and the ensuing A phrase (motive).

23 George Perle (1990) discusses the interaction of octatonic and diatonic modes in Bartók's "Song of the Harvest" (*44 Violin Duos* no. 33). The compositional design of the duet is similar to the piece analyzed here (except for the ending of the duet, which "resolves" the octatonic tension by turning to a single diatonic mode).

24 This reading argues for a perceptual as well as a theoretical distinction between what we could refer to as "diatonic" semitones C–B and F–G♭ that occur within strata and "chromatic" semitones A♭–A and D–E♭ that occur across strata. This distinction underlies the argument that this is a polymodal piece. The suggestion is that diatonic semitones occur "within" a given diatonic collection or, in Dasian terms, between contiguous modal qualities (*deuterus-tritus*), that is, [pc(x), mq(3)] → [pc(x+1), mq(0)], whereas chromatic semitones occur "between" distinct diatonic collections or, in Dasian terms, [pc(x), mq(y)] → [pc(x+1), mq(w)], where y and w are modal qualities other than mq(3) and mq(0), respectively. In the case of the opening octatonic collection, these chromatic relations are A♭–A, (A♭, 1) ↔ (A, 2); and D–E♭, (D, 1) ↔ (E♭, 2). Both "chromatic" semitones distance $p^5 + s$ (i.e., 5-times-transpositio + 1-step) in the space.

(4.1)



(4.2)

Figure 4. Bartók's "Melody against Double Notes" (*Mikrokosmos* no. 70).
(4.1) Superimposition of "gapped" tetrachordal and "filled-in" pentachordal segments.
(4.2) Location and progression of superimposed segments in Dasian space.

suggested in Figure 4.1, the prominent F♯ in the right-hand melody (mm. 1–9) can be heard to "fill the gap" of the left-hand tetrachord, suggesting a "major" pentachord D–E–F♯–G–A; conversely, the left-hand A of the inverted melody (mm. 10–16) fills the gap of the right-hand tetrachord, suggesting the "minor" segment F♯–G♯–A–B–C♯. Both F♯ and A "major and minor third imprints" correspond to symmetrical transformatio moves (f and f^{-1}) in Dasian space. Figure 4.2 shows that a counterclockwise transformatio (dotted arrow) signals the change of modal quality or function from A*r* to A*l* for pitch class F♯, that is, f^{-1}, (F♯, 2) → (F♯, 3); conversely, a clockwise transformatio (dotted arrow) signals the reinterpretation from A*l* to A*r* for pitch class A, that is, f, (A, 1) → (A, 0). These mutual "imprints" suggest the necessary semitonal relations to define segmental locations in Dasian space.[25]

The filling in of tetrachordal segments also impacts the assessment of harmonic distance and tension. The segments at A are more closely located

25 Notice that these pentachordal scale segments are not uniquely affiliated to a single diatonic collection but are uniquely located in Dasian space.

in Dasian space, given their minimal step mismatch created by G/G♯ in each hand, that is, between (G, 0) and (G♯, 3). These segments are also symmetrically located in the space around C♯/D and G/G♯, reflecting the symmetry created by the combination of tetrachords in pitch space. The arrival of notes F and A♯ (m. 17), which are integral to each segment (located at [F, 0] and [A♯, 3], respectively), result in a symmetrical shift to locations at B, via f and f^{-1} (solid arrows in Figure 4). The increased mismatch between segments (F/F♯, G/G♯, A/A♯) is reflected on the increased harmonic distance of segments in the space, in which the inversional relation between pentachords is reflected in both the Dasian and pitch space.[26]

Polymodality in the third movement of Bartók's Piano Sonata

After drafting the ideas framing polymodal chromaticism, Bartók ([1943] 1992, 368) lists his Piano Sonata (1926) among works where "this principle is put into action—at least partially." While he did not specify which movements or passages in the Sonata might better "activate" the principles of polymodal chromaticism, the convocation of various combinations of diatonic materials in the third movement invites scrutiny. However, rather than relying on the composer's restrictive formulation of modal superposition projecting a single pitch center, I examine the gradual unfolding and cumulative effect of combinations (simultaneous and successive) of scale segments modeled by the Dasian system of relations. My general analytical goal is to understand how imprints of scalar layerings onto Dasian space might shape tonal trajectories (polymodal syntax), which participate in coherent formal processes. In particular, the analysis explores how both local and global trajectories of polymodal patterns, which create a kind of "inner" form, relate to and qualify rotational aspects of the movement's "outer" form, traditionally viewed as a rondo.[27]

A brief sketch of the movement's formal layout helps to frame the analytical argument. The rotational scheme summarized on Table 1 organizes the piece into four rotations, the last of which is condensed and functions as a coda. Each rotation results from the reiteration of a cyclic sequence of what we can refer to as blocks, which include time units of various lengths, from whole sections to brief passages, whose characteristic thematic materials, or prominent textural changes, result in significant formal markers.[28]

26 The superimposition of key signatures (zero and five sharps) along with the inversional relations between hands could be traditionally interpreted as the diatonic modes D-Dorian (lower pentachord) and C♯-Dorian (upper pentachord). While this (symmetrical and modal) reading is also captured in Dasian space, I suggest focusing not on whether we hear (poly)tonics for minor and major pentachords but, rather, on the interaction and contrapuntal deployment of mismatched and harmonically distant strata.

27 Analytical accounts of the movement as a rondo appear in Somfai 1990, Wilson 1992, Konoval 1996, and Susanni 2001. The historical scope and character of rotational form is proposed and discussed in Hepokoski and Darcy 2006 (see esp. app. 3, 611–14). For the aspects of rotational form in Bartók, see Keller 2011.

28 The notion of formal "block" is traditionally used in the context of Stravinsky's abrupt discontinuities of material and texture. The idea of block stratification was first pro-

Table 1. Rotational form and block sequence for the third movement of Bartók's Piano Sonata

Rotations	Rotation 1						Rotation 2					Rotation 3							Rotation 4 (Closing section)	
Blocks	A	B	C	D	E	F	A	B	C/D	E	F	A	B	C	D	E	F	G	A/B	C/D
mm.	1	20	38	40	53	84	92	111	137	143	156	157	175	192	194	205	222	227	247/254	264/265

Figure 5 sketches formal blocks in each rotation and aligns them according to thematic and textural materials. Blocks A initiate each rotation with a thematic refrain, characterized by a pentatonic pattern that descends from an anchoring pitch class F♯ ⟨−2, −3, −2, −2⟩ (which I refer to as the *fixed* refrain theme; see Example 2). Traditional attributions of the form as a rondo are based on the movement's cyclic alternation between fixed refrains in blocks A and episodic thematic variants (or *movable* episodic themes) in blocks E and in the additional block G. Given that episodic themes prominently feature transposed pentatonic variants of the refrain, there is considerable disagreement on what constitutes the contrast between refrains and episodes and which events mark the boundary between them.[29] Rather than weighing on the different analytical criteria for hearing contrast required in a rondo scheme, however, I propose to focus on the cyclic or rotational sequence of blocks A–F, which remains mostly invariant with respect to order and overall thematic content. In the rotational scheme, the main thematic statements (blocks A, E, and G) provide higher formal stability, while the remaining passages (blocks B, C, D, and F) project transitional, dynamic, and closing formal functions.

My argument unfolds three related analytical claims. First, the analysis shows that the various polymodal combinations and textures in the movement are efficiently modeled by the system of diatonic relations of the Dasian space and result in several tonal patterns that reinforce and establish internal relationships between blocks within the proposed rotational scheme. Focusing the analytical lenses on rotation 1, I propose that the pitch exploration in the first three blocks creates a larger continuous unit. In particular, the

posed in Cone 1962 and further developed in van den Toorn 1983. Because some of the blocks or "components" in the third movement of Bartók's Sonata (such as blocks C and D) are temporally brief, they are better understood as important changes of texture and harmonic material rather than constituting sections per se.

29 László Somfai (1990, 546, 547) sees the movement in a binary (ritornello-episode) scheme but describes it as "a monothematic rondo, for the episodes are, from the thematic standpoint, variants of the ritornello theme"—the basic contrast lies between the "agitated mixed meters of the ritornello" and the "smooth ostinato background" of episodes. As such, Somfai sees blocks C as the closing of an extended ritornello and locates the beginning of episodes at block D. For Paul Wilson (1992, 78), the contrast required for a sense of alternation between the rondo sections results from "changes in harmonic setting, rather than a true change of theme," such that the alternating binary scheme groups A and B, on the one hand, and C, D, E, and F, on the other. Yet other analytic studies (Konoval 1996; Susanni 2001) consider block B as providing the most prominent contrast to block A, and practically all sections other than A are considered as episodic material. Paolo Susanni (2001) considers block C as transition, whereas Michael Konoval (1996) does not consider block A (at m. 157) to be a true ritornello but, rather, a variant (episode) with transitional function.

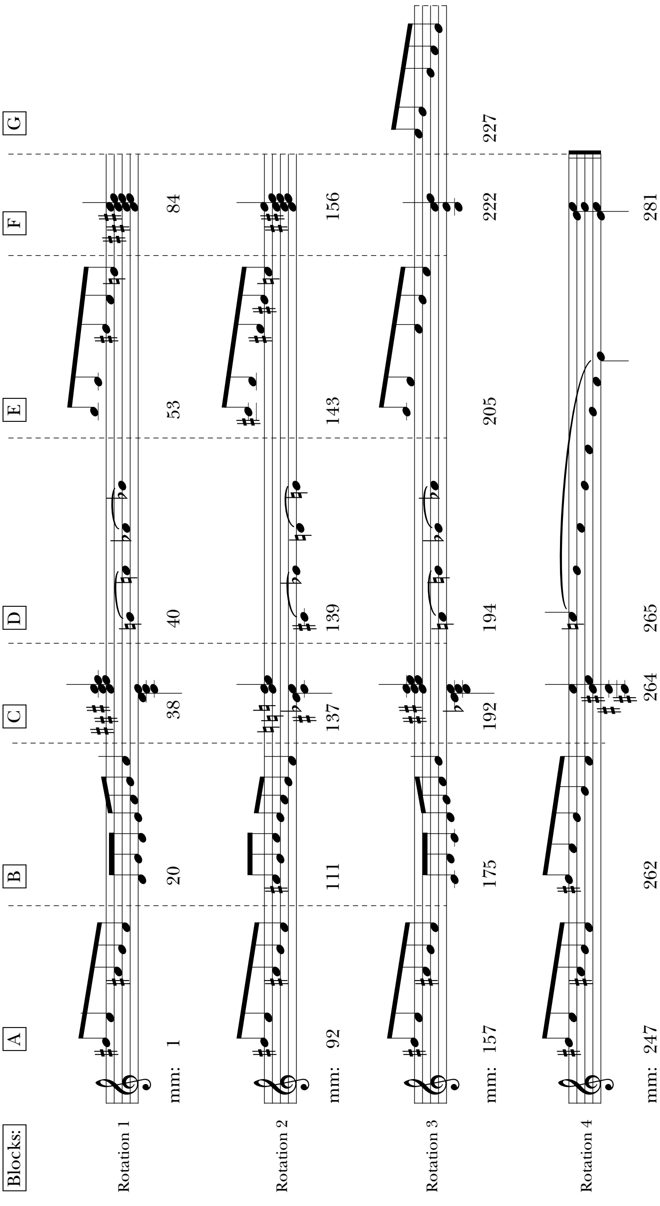
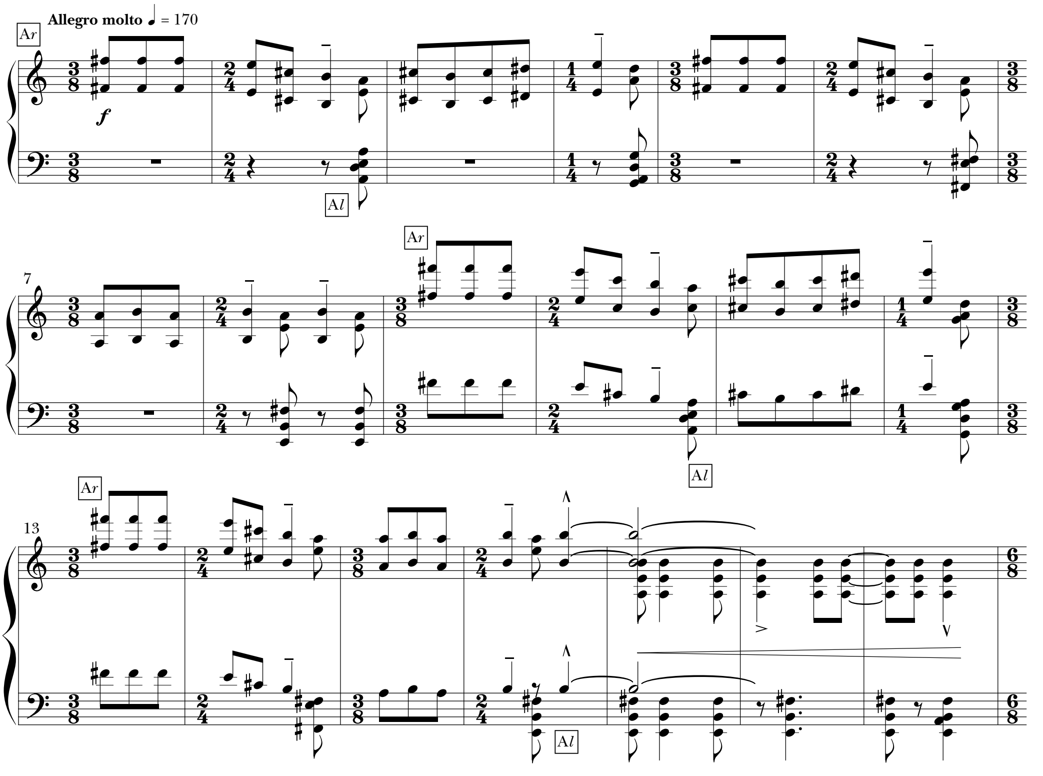


Figure 5. **Block sequence, alignment, and rotational scheme of prominent thematic material in the third movement of Bartók's Piano Sonata**



Example 2. Opening refrain (mm. 1–19), which is framed by a pentatonic pattern that descends from an anchoring pitch class F♯ ⟨–2, –3, –2, –2⟩

imitative counterpoint in block B sets up a dynamic Dasian trajectory, which dramatizes the static polymodal tension between the melody and accompaniment of the refrain in block A and progresses to the marked polychord of block C. The superimposition of diatonic segments in block C, in turn, neatly demarcates an extended Dasian region (explored in previous blocks) and culminates a process of symmetrical expansion for the first three blocks in the rotation. The last three blocks in the rotation also present a distinct pattern of continuity. The transitional block D and the concluding polychord in block F exhaust the Dasian regions not previously enacted, whereas the succession of phrases of the episodic theme in block E reenact the polymodal tension of superimposed layers in the opening refrain. The analysis thus shows that the structured exploration of the entire Dasian space is exactly completed within a full rotational cycle.

Second, while pentatonic patterns frame both fixed and movable themes, the specific Dasian regions explored depend on the distribution of semitones that "fill in" each pentatonic framework. As such, pentatonic frames function as potentialities for a variety of polymodal inflections in the piece. The analysis shows that the polymodal scope of fixed refrains is significantly expanded in later rotations, and the relation between superimposed layers of scale segments changes gradually at every rotation.

Third, in contrast to attributions of a "weak closure" for the piece (Wilson 1992, 84), the Dasian approach clarifies and reinforces the claim for a strong closure. I propose that closure in the piece entails two related large-scale gestures: first, a tentative closure is offered with the insertion of a new episodic variant or theme (in block G) at the end of rotation 3; a second, more emphatic and definitive closure is offered through the dissolution of the refrain's harmonic and thematic features in the contracted block C/D in rotation 4. I argue that the alteration of formal patterns at the end of the movement also provides a solution for the compositional problem posed by a cyclic form: how to articulate a closure without leading into a new rotation.

Polymodality in rotation 1

Perhaps the most suggestive passages of polymodal gradation in the movement are blocks B, where short scale segments overlap and modulate through different diatonic regions. These blocks function as transitional sections, providing some thematic and textural relief from the previous refrain of blocks A leading into the dense polychords of blocks C.[30] Example 3 illustrates the corresponding passage in rotation 1 (mm. 20–38), which builds up a forward-driving counterpoint of partially overlapping diatonic scale segments, culminating in the strident polychords at m. 38. Figure 6.1 presents a scalar reduction of the passage, segregating between right (*r*) and left (*l*) hands and temporally successive segments (B1 through B4).[31] There's an overall "modulation" from a "D-Mixolydian" region (B1*l*) toward an "E-Mixolydian" region (B4), accomplished through upward transposition by perfect fifth of certain scale segments on both hands.[32] In Figure 6.2, these two regions (B1*l* and B4) form a symmetrical layout in the space by overlapping the space elements (A, 1) and (B, 2). (These two elements assume a central role in the connection of block B to the polychord of block C, as discussed below.) In the overall harmonic progression of block B, the succession of segments shifts clockwise through contiguous space rather than signaling specific modal

30 Benjamin Suchoff, the editor of *Béla Bartók Essays*, chooses a passage from the third movement of Bartók's Piano Sonata (mm. 127–37) as an illustration of polymodal chromaticism from a list of possible pieces indicated in Bartók's notes for the lecture (Bartók [1943] 1992, 368). The passage chosen displays a counterpoint of distinct scalar segments culminating on a superposition of the two-flat diatonic collection and the white-note collection.

31 The analysis sets up the following labeling system: the first letter refers to the active block, integers distinguish temporally successive segments in order of appearance, and *r* and *l* refer to right- and left-hand components of temporally simultaneous strata (although at times this is not strictly the case, as in the vertical chords of the refrain, which are played by both the left and the right hands but are labeled as A*l*).

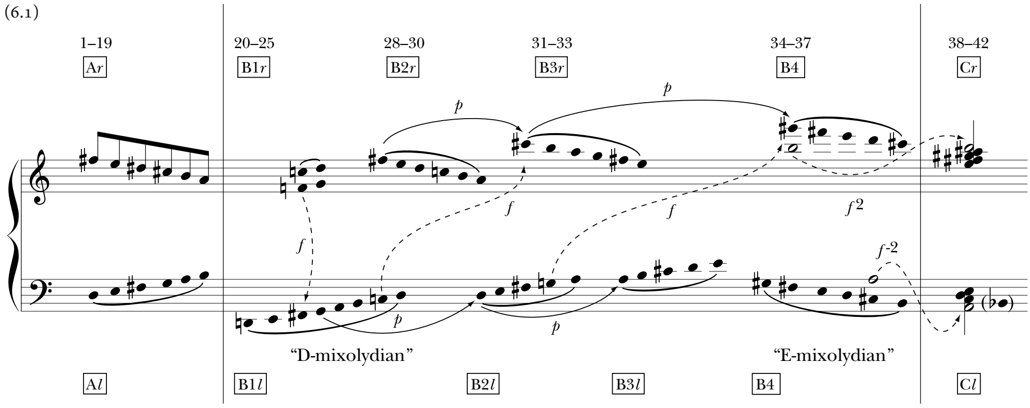
32 Although we might well hear pitch class D as a pitch center in B1*l* (mm. 20–25), modal finals are used here for labeling purposes only.

Example 3. Block B (mm. 20–37), block C (mm. 38–39), and beginning of block D (m. 40) in rotation 1

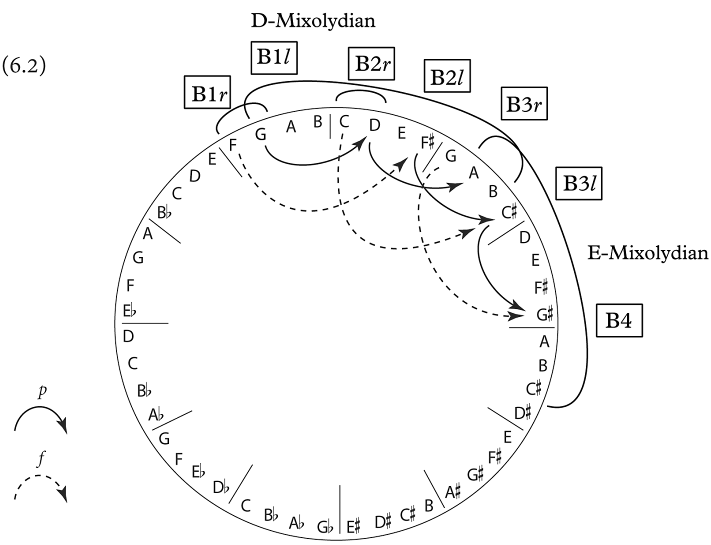
affiliations. The motion through melodic peaks in the right-hand F♯–C♯–G♯ and the motion of bass notes G–D–A in the left hand is modeled by transpositio (*p*), (F♯, 3) → (C♯, 3) → (G♯, 3), and (*p*), (G, 1) → (D, 1) → (A, 1).[33] The succession of scale segments also creates occasional chromatic mismatches, as indicated by the dotted arrows in Figure 6. These mismatches (F/F♯, C/C♯, G/G♯) are modeled by transformatio (*f*), (F, 0) → (F♯, 3), (C, 0) → (C♯, 3), and (G, 0) → (G♯, 3).

33 Wilson (1992, 81) also notes the T_7 upward motion in both hands as part of the transformation of set-class 7–35 (i.e., from D-Mixolydian to E-Mixolydian).

(6.1)



(6.2)



(6.3)

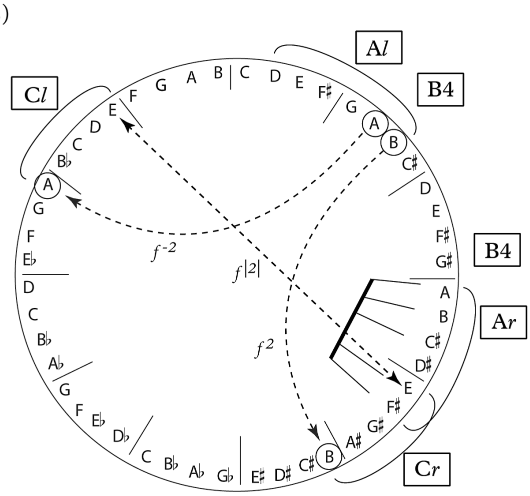


Figure 6. Activity of scale segments in blocks A, B, and C of rotation 1. (6.1) Pitch reduction and contrapuntal deployment of scale segments. (6.2) Progression of segments within block B in Dasian space. (6.3) Dasian location for block A and symmetrical split for the arrival of block C.

The Dasian trajectory unfolded by the transitional block B dramatizes and amplifies the polymodal relation and harmonic distance of combined layers in block A, that is, between the melodic theme (A*r*, fixed theme) and chordal accompaniment (A*l*). The theme is articulated in two phrases, mm. 1–8 and 9–16, the second of which repeats and reinforces the first phrase (see Example 2).[34] The scalar reduction of block A layers in Figure 6.1 superimposes the diatonic segment A–B–C♯–D♯–E (built on the fixed pentatonic frame, bracketed off in the figure) and a chordal accompaniment, which uses the collection D–E–F♯–G–A–B. (While the registral deployment of the accompaniment privileges stacks of "fourths" and "fifths," the bass descending motion A–G–F♯–E emphasizes the scalar aspects of the collection.) A simple way to conceptualize the harmonic distance between accompaniment and melody is to extend their respective pitch-class materials upon the line of fifths G–D–**A–E–B–F♯–C♯**–(G♯)–D♯. The fixed pentatonic frame takes up the continuous segment A–E–B–F♯–C♯ (in bold), the accompaniment takes up the segment between G and F♯, and the melodic theme takes up the segment between A and D♯, overpassing G♯ (more on this below). The two layers overlap the segment A–E–B–F♯ and are mismatched by D/D♯. Figure 6.3 maps scale segments for the melody (A*r*) and accompaniment (A*l*) in Dasian space, assigning them precise locations due to their respective embedded semitones. Therefore, while the layers (melody and accompaniment) share some common pitch classes, the Dasian projection clarifies their distinct, albeit harmonically close, affiliations. We can thus interpret the Dasian trajectory in block B to first expand to the counterclockwise region of A*l* (notice the semitone B–C in B1*l* and the whole tone F–G of B1*r*)[35] and then to gradually unfold and bridge the distance between the segments that correspond to A*l* and A*r*. Furthermore, the mentioned transpositio relations between right-hand melodic peaks (F♯, 3) → (C♯, 3) → (G♯, 3) explicitly bridge that distance.[36]

The strategy of region expansion from block A to block B culminates in block C, thus suggesting a formal continuity across these three sections. The rhythmic culmination of block B is also a sectional overlap with the beginning of block C (see Example 3, mm. 38, 39, and 42), which articulates the collection G♯–A–A♯–B–C–D–E–F♯, the most dissonant simultaneity thus far in the piece, the set-class (0123468t).[37] The collection is registrally split into a

34 In later rotations, the second phrase of refrains and episodes is varied and expanded.

35 While there are three potential locations in Dasian space to accommodate B1*r*, F–G–C–D, its chosen location provides both the lower boundary F and the gap C for block A. This analytic choice can be understood as the fulfillment of an analytic opportunity.

36 G♯, the melodic peak of the passage, or (G♯, 3) in Dasian space, also fills in the gap between C♯ and D♯ in the line of fifths connecting the scalar materials of A*l* and A*r*.

37 Both Somfai (1990) and Wilson (1992) consider that block C articulates the boundary between ritornello and episodes (chords in block C are included in the ritornello for Somfai and excluded for Wilson). As pivotal overlap between large-scale sections, block C signals both the arrival of the previous block B and sets up the (major seconds) material that characterizes the rest of the rotation.

quasi-symmetrical polychord, superimposing the segments C*l*, A–(B♭)–C–D–E, and C*r*, E–F♯–G♯–A♯–B.[38] The rhythmic continuity between blocks B and C is emphasized by the scalar goals of the outer voices, arriving on pitch classes A and B on the downbeat of m. 38 (marked by open note heads on Figure 6.1). The analysis suggests a reinterpretation of these notes from the scalar context of B4 to melodic prominence in the extreme registers of the split polychord. This reinterpretation is modeled in Dasian space via a symmetrical split of transformatio relations, which take pitch classes A and B from B4 to C*l* and C*r*, respectively (see connected open note heads in Figure 6.1). Solid arrows in Figure 6.3 capture the transformatio reinterpretation f^{-2},(A, 1) → (A, 3), and f^{2},(B, 2) → (B, 0). The split polychord of block C borders the entire space covered by the previous blocks A and B, and in turn C*r* and C*l* are themselves related by transformatio through the common pitch class E ($f^{|3|}$: (E, 0) ↔ (E, 3), shown by the dotted line in Figure 6.3).[39] This transformatio relation (and axis of symmetry) through the common pitch class E between superimposed segments in opposite parts of the space also plays a crucial role for the movement's closure.

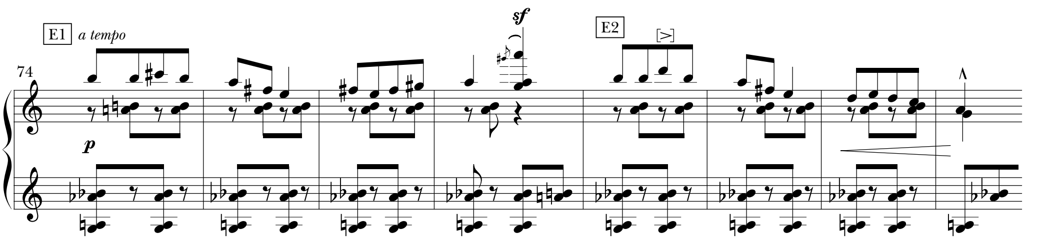
From block D up to the closing block F of rotation 1, whole-tone dyads combine in various ways to produce different textures and assume several harmonic roles. Therefore, the proposed formal and Dasian (extended diatonic) continuity of blocks A, B, and C finds, to some extent, a counterpart in the continuity across episodic blocks D, E, and F, although the Dasian modeling of the latter set reveals distinct polymodal patterns. Figure 7.1 registers pitch reductions of all (simultaneous and successive) scalar activity in these three blocks, and Figure 7.2 interprets such activity in terms of Dasian locations and resultant segment relations.

The episodic theme in block E is framed by a pentatonic pattern, which is transposed by T_5 or p^{-1} from the refrain to B–A–F♯–E–D (bracketed off in Figure 7).[40] But unlike the refrain, the second phrase (mm. 74–83) expands the polymodal scope of the theme. The first phrase (mm. 53–72) and the

38 The registrally reinforced subset D–E–F♯ is also drawn symmetrically from both segments. The cluster at m. 38, although (abstractly) symmetrical about the axis E/A♯, is not literally split symmetrically by the left and right hands. Instead, B♭ is absent in the left hand (hence the parenthesis in C*l*). In the absence of a defining semitone for the left-hand pentachord—which would be either A–B♭ or B–C—my analytical choice falls on A–B♭, not only because it allows for the symmetrical transformatio but also because it is reinforced by the left hand at m. 43. This is in line with the compositional logic of "gap filling" demonstrated in the previous sections. Also, in the reduction of block C, I am assigning F♯ to the right hand, thus considering the F♯3 in the left hand to double the F♯6 of the right hand. I justify this somewhat arbitrarily analytical move because F♯3 disappears from the left hand at the subito forte (m. 38). Furthermore, this subito forte supplies E3, which was absent from the C*l* downbeat. While we can consider the polychord at m. 38 both as partially segmented and partially fused, it nonetheless results from the superimposition of the Dasian segments C*r* and C*l*.

39 The set D–E–F♯ articulated in fortisimo in mm. 38, 39, and 42 reinforces the symmetrical layout of the split in a number of ways: it highlights the pitch-class symmetry by fifths A–D in the bottom and B–F♯ on the top registers and reinforces pitch class E as the pitch-class axis of the collection and note of transformatio.

40 Reflecting the actual deployment of pentatonic mottos in the musical surface, the analytical Dasian projections privilege minimally gapped scale segments.



Example 4. The second phrase of the episodic theme, mm. 74–83

beginning of the second use the segment E1, D–E–F♯–G♯–A–B–C♯, while in the course of the second phrase, there's an inflection to E2, C–D–E–F♯–G–A–B (see Example 4, mm. 74–81). As the two segments present different scalar contexts upon the same pentatonic framework, they also refer to different harmonic regions and Dasian locations. The syncopated vocal "whoop" in m. 77, standing at the boundary between E1 and E2, seems to dramatize the shift between segments: the note A is associated (in different octaves) to G and G♯, suggesting the connection of different occurrences of pitch class A in the space via transformatio f^{-1}: (A, 0) → (A, 1).[41] Compared with the polymodal state of the refrain (of block A), the expansion at the end of the second phrase of the episodic theme retains the region that corresponds to the refrain's accompaniment but shifts (by p^{-1}) the region of the first phrase. In other words, the polymodal expansion now occurs in successive rather than simultaneous phrases, as is the case between melody and accompaniment in the opening refrain.[42]

As a formal process, block D rarefies the rhythmic outburst of block C and serves as a transitional antechamber for the arrival of the episodic theme in block E (see Example 5). In this process, whole-tone dyads first combine to create the two-flat diatonic collection C–D–E♭–F–G–A–B♭ (D1, mm. 40–46), while later the dyads D♭–E♭ and A♭–B♭ modulate to a corresponding four-flat diatonic collection (D2, mm. 47–48). As shown in Figure 7.2, the modeling of scalar activity in block D mostly maps into uncharted Dasian territory,[43] bordered by the elements (E, 3) and (F♯, 0).

41 Somfai (1990, 547) refers to the sforzando syncopation in m. 77 as imitating a vocal "whoop." The registral dissociation between G–A and G♯–A helps project both associations for pitch class A.

42 There is further reason for associating blocks E and A: the episodic theme is bookended by material that maps into similar Dasian locations as those covered by C*r* and C*l*. The vertical sonority at m. 53 G–A♭–A–B♭–B is transposed by T–1 (mod 12) from the chromatic partitioning of the cluster at m. 38 G♯–A–A♯–B–C (the vertical sonority at m. 53 obtains from combining the whole-tone dyads from block D with the pitch class B from block E1). Furthermore, these moments are registrally associated because in both cases the note B5 is the highest note. Depending on the analyst's temporal focus, one could say that m. 38 anticipates m. 53 or that the m. 38 re-elaborates m. 53. At m. 53, however, the pitch class B stands as the crux of superimposition of two Dasian spaces, corresponding to the chordal and linear contexts of the note B at that point. When pitch class B is understood as part of the segment B–B♭–A♭, it reenacts the same location as in m. 38, since the determining dyad is B–B♭, while as the initiator of the thematic melodic segment B–A–G♯–F♯–E, pitch class B is "channeled" (by transformatio) one counterclockwise station to E1, and the determining dyad becomes A–G♯.

43 The note B♭ (in D1) fills the gap (B♭, 0) left open at C*l*.

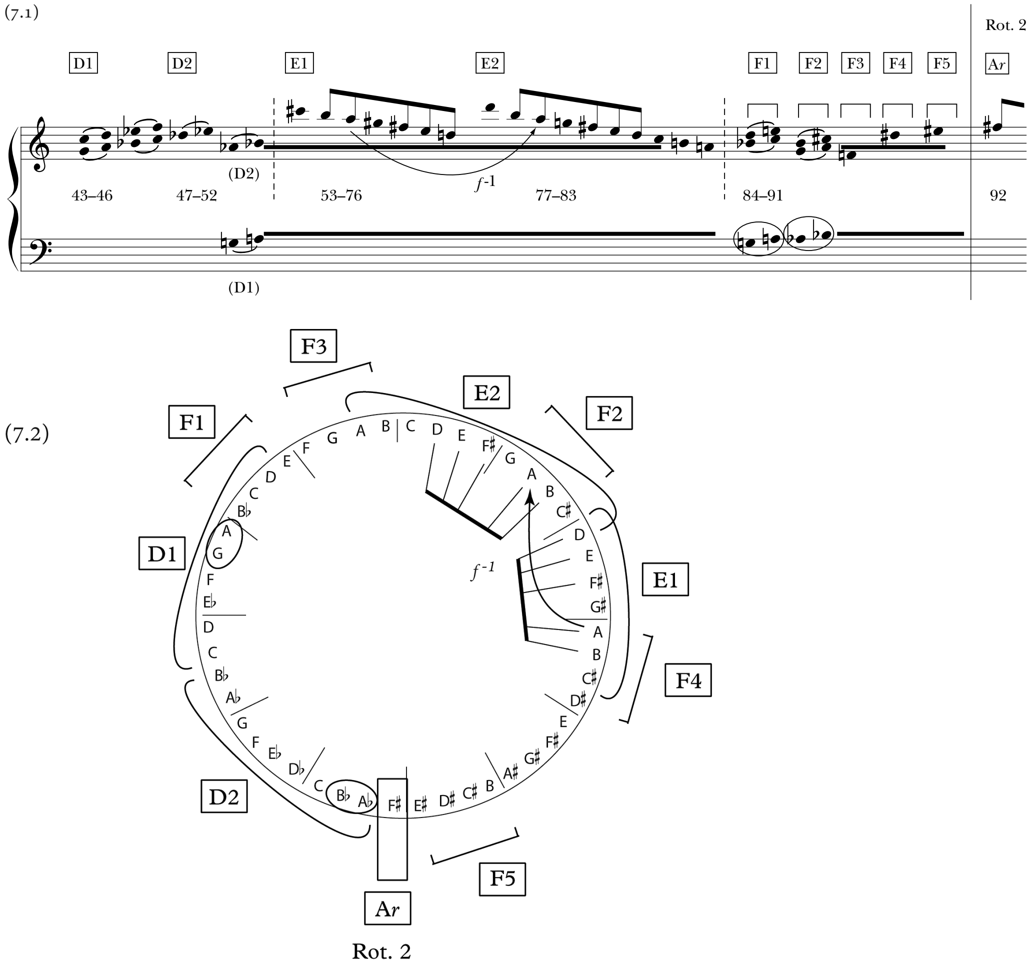
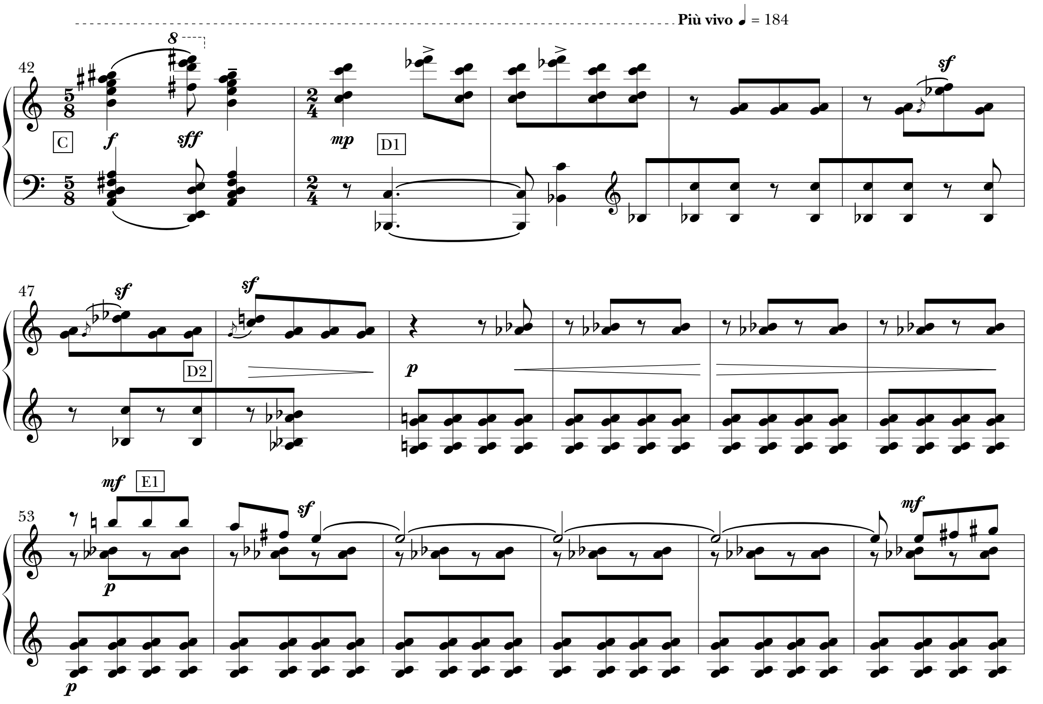


Figure 7. Activity of scale segments and collections in blocks D, E, and F of rotation 1. (7.1) Scalar and chordal reductions. (7.2) Dasian locations and relations between scale segments.

Taken together, blocks A–D exhaust almost the entire Dasian space except for the segment (C♯, 1)–(D♯, 2)–(E♯, 3)–(F♯, 0), which assumes an important role in the closing of block F leading into rotation 2. Block F (mm. 84–91) alternates two polychords that superimpose each of the whole-tone dyads retained from block D (through block E) to the opposing whole-tone collection (or its subset), resulting in F1, G–A/B♭–C–D–E (mm. 84 and 86),[44] and F2–F5, A♭–B♭/F–G–A–B–C♯–D♯–E♯ (mm. 85 and 87–91). The buildup of a complete whole-tone collection in block F neutralizes or disrupts the

44 B♭–C–D–E (which is a transformatio echo of m. 80, with B♭ instead of B) is superposed to the dyad G–A (circled in Figure 7.2), thus reinforcing the position of C*I* (where B♭ was absent).



Example 5. End of block C, block D, and beginning of block E

diatonic/Dasian pattern beyond a 2–2–2 pattern, which is uncharacteristic of the scalar fabric suggested in all previous sections of rotation 1; it is thus fitting for assuming a liquidating function. The modeling of whole-tone collections onto the Dasian space implies the partial intersection of every other whole-tone tetrachord (2–2–2). Therefore, "opposite" whole-tone tetrachords are reached in the cycle as a result of building the scale up to the octave completion (2–2–2–2–2–2) (see F2–F5 in Figure 7.2).

As F5 reaches the last tetrachord of the whole-tone collection B–C♯–D♯–E♯, it occupies the only remaining segment in the Dasian cycle not yet covered in rotation 1, with the exception of (F♯, 0) associated by semitone to (E♯, 3). However, the semitone E♯–F♯ can be heard via the voice-leading connection between E♯, the top note of the whole-tone collection, and F♯, beginning rotation 2 (mm. 91–92). The exhaustion of Dasian space in rotation 1, achieved only by the arrival of F♯ initiating rotation 2, is suggestive of a rotational reading in which the beginning of a new rotation also concludes the previous rotation.[45]

45 The scalar context provided by the refrain in rotation 2 (block A) reinterprets the note F♯ (ending rotation 1) via transformatio f^{-1}: (F♯, 0) → (F♯, 1) to its original space location.

Polymodal expansion in refrains of rotations 2, 3, and 4

The analysis of the episodic theme in block E showed that pentatonic frames are pliable to different scalar contexts and thus adaptable to distinct (Dasian or diatonic) harmonic regions. Figure 8 registers the various scalar contexts for blocks A over the four rotations, accommodated by the fixed pentatonic framework F♯–E–C♯–B–A (bracketed in the figure). Compared with the segment of the opening right-hand melody (A*r*, rotation 1), the theme's polymodal scope is expanded considerably in rotations 2–4 through the use of various registers and scalar patterns.

The modeling of polymodal relations in Figure 8 exposes the double nature of the Dasian space as both a scalar space (where the order of modal qualities interacts with register) and a pitch-class space (where each element

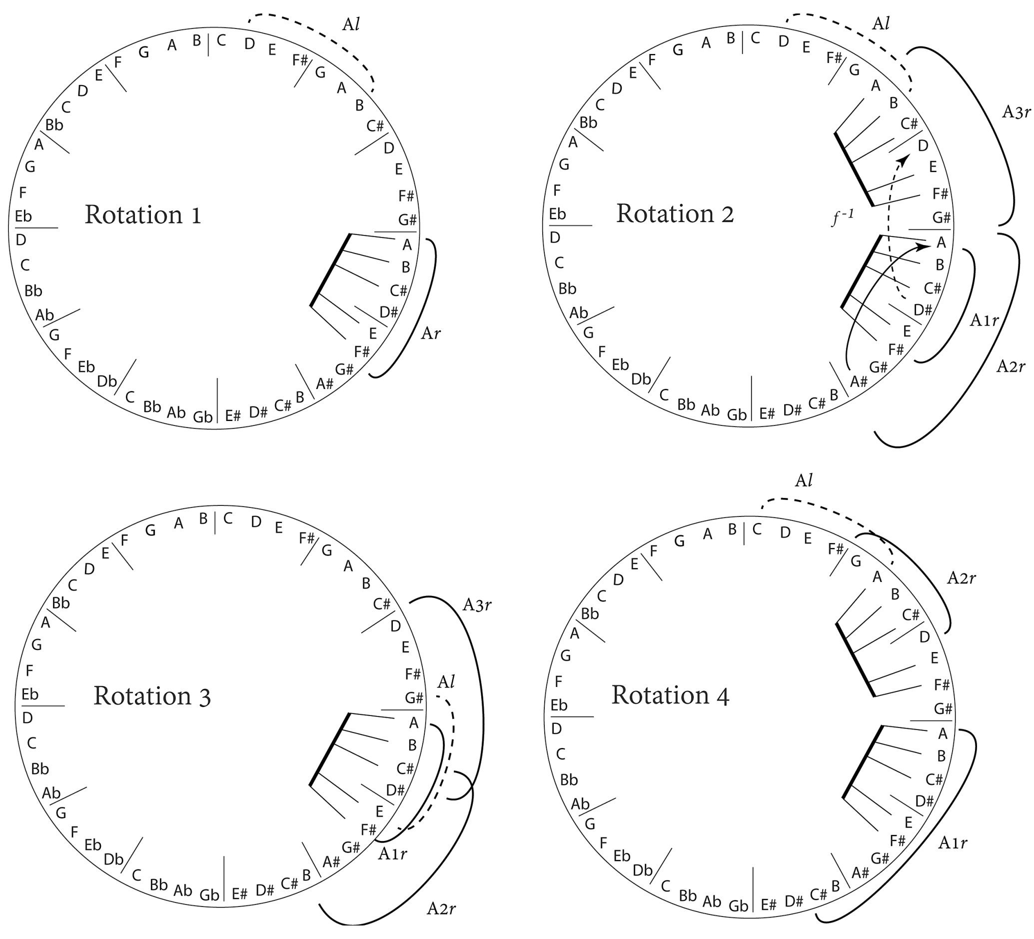


Figure 8. Mapping of blocks A (A*r* and A*l*) throughout the four rotations

Example 6. Second phrase of the refrain (block A) in rotation 2, mm. 100–110.

assumes octave and enharmonic equivalence). At times, the polymodal expansion results from the assignment of different chromatic inflections to distinct registers in the same melodic statement, thus differentiating by register two Dasian regions. For instance, the beginning of the theme's second phrase expansion in rotation 2 (see Example 6) projects the extended space A2*r*, A–B–C♯–D♯–E–F♯–(G♯)–A♯ in mm. 100–103, thus reaching A♯ in the upward expansion (m. 100) and retaining A of the pentatonic frame in the melodic descent (m. 102). (This relation is modeled by f^{-1}: [A♯, 3] → [A, 0].) In contrast, the polymodal expansion later in the second phrase results from the chromatic inflection of D♯ (m. 102) to D (m. 106). While the inflection D♯/D occurs in the same register, their respective integral segments correspond to distinct Dasian regions (A2*r* to A3*r*). The modeling of the passage as involving distinct Dasian locations thus rests on considering elements as pitch classes, where the mismatch D♯/D is modeled by f^{-1}: (D♯, 3) → (D, 0).[46] In this analytical case, hearing the passage in terms of pitch-class equivalence is actually facilitated by the three- or four-octave registral reinforcement of the theme.

Figure 8 also shows that, in refrains of rotations 2 and 4, the expanded polymodal region of the melodic theme (A*r*) extends to the Dasian location

46 The analysis of Figure 7, which refers to changes in the scalar context occurring in the same musical register (block E), also rests on the notion of space elements as pitch classes.

(A*l*) of the accompaniment, which is retained in its original location. In contrast, the Dasian location A*l* for the chordal accompaniment in rotation 3 projects the location G♯–A–B–C♯–D♯–E, which is shifted significantly from its initial location in rotation 1 and now overlaps with the theme's A1*r*. One of the consequences of this drastic shift in space location in rotation 3 is that A*l* "vacates" a Dasian region, which is later occupied by the episodic themes in block E (and the additional block G) as part of the strategy for the movement's closure.

Pentatonic frames and the movement's closure

To ultimately assert harmonic and thematic stability over the episodes, the formal scheme for a classical rondo or baroque ritornello would typically grant a final statement of the refrain's theme in the home key to close the movement. The ending of rotation 4, however, features a rhythmically driven passage built on the textures typical of blocks C/D (and also F), lacking most of the thematic and harmonic features that characterize the refrain. This strategy is at odds with a typical global scheme for a rondo and has been interpreted as providing a weak and unprepared closure to the movement (Wilson 1992, 84).[47] Here, however, I argue that this strategy is effective, given that formal patterns at the end of the movement provide a two-phase articulation of the piece's closure, substituting for the refrain's role of culminating a rotation and launching a new one. First, a tentative closure is sketched by the interpolation of the episodic theme (block G, mm. 227–47), which features a pentatonic frame anchored on pitch class E, "impersonating" the formal location of the fixed refrain that would initiate rotation 4; a second, more emphatic closure is offered by the final E-Phrygian collection, which dissolves the refrain's harmonic and thematic features in block D (mm. 268–81) at the end of rotation 4. The following transformational and polymodal approaches show that the ending section not only completes a process for the entire rotation but also reinforces the completion of a pattern that structures pentatonic frames in both refrains and episodes throughout the movement.

In Figure 9, each of the five pentatonic patterns used in the piece (the fixed refrain, F♯–E–C♯–B–A, and the four movable episodes, B–A–F♯–E–D, C♯–B–G♯–F♯–E, A–G–E–D–C, and E–D–B–A–G) is represented as a continuous five-note segment embedded in the extended line of fifths from C to G♯, bordered by F and D♯ (more on this below). Two theoretical observations are

47 Wilson (1992, 84) argues that the "the concluding events [of the movement] contain some element of the arbitrary or the unprepared" (referred here to blocks C/D of rotation 4) and thus provide only a "weak closure." In Wilson's analysis (combining extended Schenkerian and pitch-class set techniques), the final E♮ in the upper voice cannot be connected with the large-scale structural beam that links other structural events "because of its weakness in context and obvious difference in function."

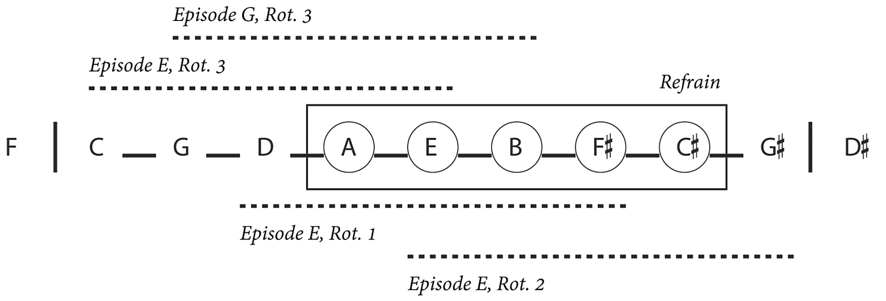


Figure 9. Line of fifths (from C to G♯) embedding all pentatonic frames used in the movement (refrain and episodes)

relevant for the movement's large-scale organization: these are the only five pentatonic patterns that contain pitch class E, and in addition to the refrain's anchoring F♯, the anchoring notes for each of the four movable pentatonic frames (namely, B, C♯, A, and E, circled in Figure 9) also constitute the notes of the refrain's complete pentatonic pattern (boxed in Figure 9). As such, the interpolation of the "impersonator" episode anchored on pitch class E (block G) at the end of rotation 3 not only breaks the large-scale alternation between refrains and episodic themes but also completes a large-scale pentatonic pattern formed by the five anchoring notes.[48] However, despite the episode's role completing the large-scale pentatonic pattern, and its formal position "impersonating" a refrain, it provides only a partial closure for the movement as the theme's consonant stability is undermined by a "dominant" pedal and a chordal collection that suggests B-aeolian (the bass note B is retained from mm. 222–26 in the previous block F and kept throughout the section).[49]

The movement's tentative closure in rotation 3 sets up a transpositional scheme that is reinforced and re-elaborated in the rotation 4. Figure 10.1 presents a transformational scheme between pentatonic frames structuring refrains and episodic themes throughout the four rotations.[50] The additional episode at the end of rotation 3 sets up a large-scale T_{10} transpositional gesture between frames anchored on pitch class F♯ (refrain, block A) and pitch class E (episode, block G). This T_{10} closing gesture is retaken in rotation 4,

48 Wilson (1992, 83) notices that the initiating notes make up the pentatonic collection of the initiating F♯ fixed theme, although not venturing into the implications (for form and closure) of such relation.

49 The voice-leading connection between the closing block G in rotation 3 and the beginning of rotation 4 also informs the E-anchored theme's attempt to close the movement. While the voice-leading connection at corresponding formal places from rotation 1 into 2 and rotation 2 into 3 was done through the semitone E♯–F♯, linking the closing whole-tone collection with the opening F♯-theme, the voice-leading connection from rotation 3 into 4 is done by a whole-step E–F♯ (and after a breath mark), thus somewhat weakening the urge for a rotational recycling. This observation is further supported by the mapping of rotation 3 in Dasian space, where only (F, 3) fails to be covered in the Dasian space. Given the unique locations of semitones in the space, this note's absence undermines the possibility for the semitonal voice leading into F♯.

50 Transposition (T) is here used conventionally as operating in (chromatic) modulo 12 pitch-class space.

Example 7. Ending of rotation 4: blocks B, C, and D

from the pentatonic refrain anchored on F♯ to the closing pentatonic frame E–D–B–A–G, embedded in the E-Phrygian sonority (see Example 7, right hand and thumb of left hand in mm. 273–81).

The T_{10} closing gestures of rotations 3 and 4, however, are differently composed. In rotation 3, T_{10} is composed by $\langle T_3, T_7\rangle$ via the episode anchored on pitch class A (m. 205, block E), whereas in rotation 4, T_{10} is composed by $\langle T_5, T_5\rangle$ via a "rotated" pentatonic frame F♯–E–D–B–A in the contracted block B (mm. 262–63), which retains F♯ in the top register. These two transpositional pairs have a number of manifestations at the musical surface and middle-ground levels throughout the movement. In rotation 4, the marked

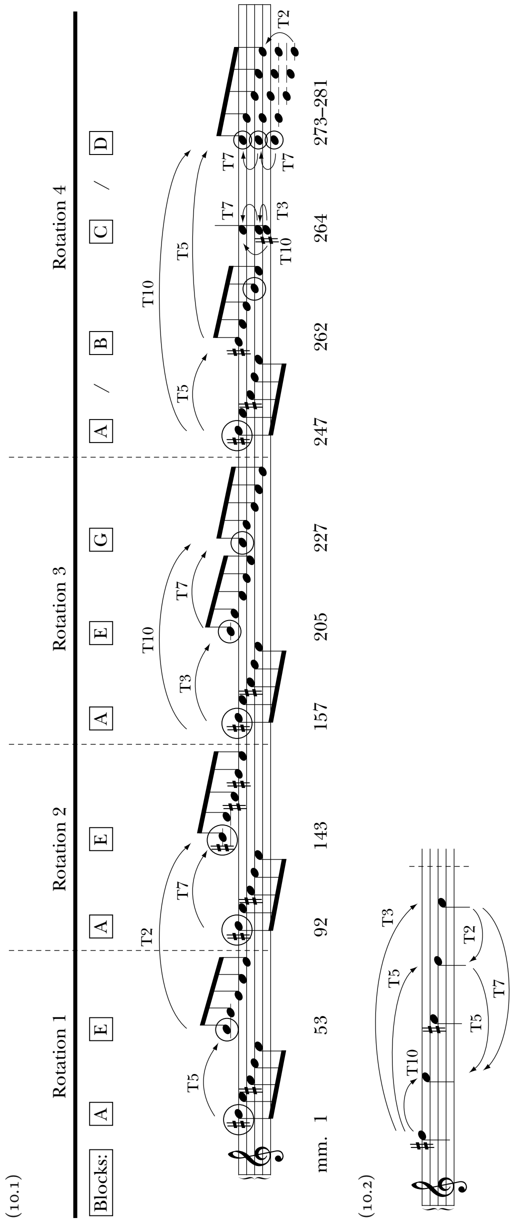


Figure 10. Deployment of pentatonic thematic frames in the movement. (10.1) Transpositional scheme between fixed pentatonic frames of refrains and movable episodes throughout the four rotations. (10.2) Structural melodic intervals used in the refrain throughout the four rotations.

polychord (block C, mm. 264–67) that initiates the final gesture is registrally disposed into ⟨T_3, T_7⟩, where the upper register dyad A–E (superimposed to the left-hand sharp notes F♯–C♯–G♯) partially anticipates the T_{10} arrival of the final E-Phrygian collection.[51] The T_{10} relation between pentatonic frames anchored on F♯ and E at the beginning and ending of the rotation goes through a pliable process: the T_5 from block A to B retains F♯ as the "head note" (but a different pentatonic "body"); then the arrival of block C polychord shifts the F♯ to the bass, while the top register moves to the "head" note E (without a pentatonic "body"); and finally, block D brings the "head" note E and the pentatonic "body" together, concluding the remaining T_5. The final E-Phrygian collection is articulated by three pentatonic frames ⟨–2, –3, –2, –2⟩ (anchored by D, A, and E), which are superimposed as ⟨T_7, T_7⟩, thus offering a counterpart and creating a convergence point to the ⟨T_5, T_5⟩ spanning the rotation. Similarly, the spanning of T_{10} in rotation 3 is counteracted by T_2 between pentatonic episodes from rotation 1 to 2. Figure 10.2 diagrams the main intervals underlying the melodic gestures in the refrain, which are precisely those that are unfolded by the transpositional scheme between pentatonic frames throughout the movement just discussed.[52]

Given that pitch class E is included on all pentatonic frames discussed for the piece and is the ultimate goal (as pentatonic anchor, pitch center, and transpositional arrival) for the movement, we now consider how the succession of episodes interacts with the four modal qualities for pitch class E. Figure 11.1 presents the various scalar contexts for pentatonic frames (bracketed) and the common tone pitch class E (circled) in all movable episodes (blocks E and G) in rotations 1–3. Figure 11.2 situates these segments with respect to their Dasian locations and the four modal qualities that pitch class E assumes in the space (circled). In rotation 1, the movable theme anchored on pitch class B at first defines (E, 1) and later expands to (E, 2), given the change in scalar context (as discussed above; see Figure 7). In rotation 2, the movable C♯-anchored theme significantly expands the polymodal scope of its constituting segments, by using both (E, 1) and (E, 0).[53] In rotation 3, the movable A-anchored theme is split into two segments, each of which includes one of the two remaining modal qualities for pitch class E, (E, 2) and (E, 3), although the latter mq(3) is only weakly asserted at the theme's conclusion (m. 212), given the lack of an explicit semitone E–F in the segment.[54] The tentative closure provided by the E-anchored episode (block G) actually fails to reinforce the position (E, 3) and is instead situated in the context of (E, 1), a position already amply covered in previous episodes.

51 Both Wilson (1992, 79) and Somfai (1990, 548) consider m. 264 as initiating the movement's coda. Konoval (1996) notices that the F♯ below E at m. 264 delays the large-scale descending bass line A–G–F♯–E.

52 The transpositional scheme ⟨T_3, T_7⟩ is particularly active in the refrain's melodic profile F♯–A–E in rotations 2–4.

53 (E, 0) is also characteristic of the refrain given the embedded semitone D♯–E.

54 (E, 3) is also touched upon very briefly in the context of E–F (block F, rotation 3, m. 219).

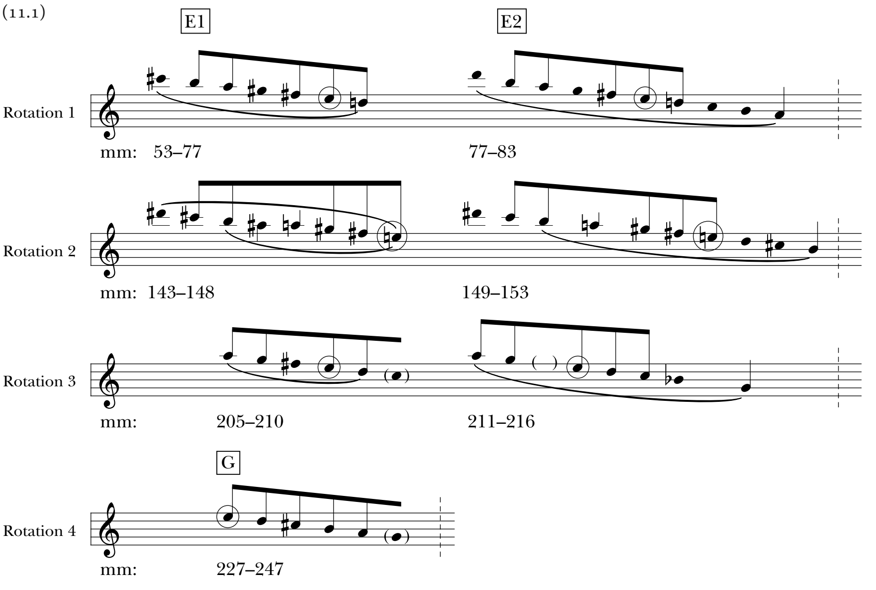


Figure 11. Modal qualities for pitch class E in episodes. (11.1) Scalar contexts for pentatonic frames (bracketed) of movable episodes (blocks E and G) and changing scalar context for pitch class E (circled). (11.2) Dasian locations of thematic episodes (blocks E and G) and changing modal qualities for pitch class E (circled).

In contrast with the formal process where rotations are launched by the initiating power of refrains, the movement's closure avoids a new rotational beginning by asserting a clear pitch space differentiation from the refrain. Figure 12 shows that scalar materials in the contracted rotation 4 explore an extended Dasian continuity encompassing the four modal qualities for pitch class E.[55] The ending brought about by the "E-Phrygian" collection (block D) not only camouflages the thematic pentatonic frame within the open-fifths ostinato but also provides a differentiated Dasian location from the opening refrain at the farthest end of the extended region.

The formal role for the marked polychord (block C, m. 264) as a boundary event is interpreted a superposition of layers that bridges the opening and closing regions. The polychord's wide registral ambitus (F♯–C♯–G♯/A–E), which entangles F♯ in the bass and E on the upper register as discussed above, corresponds to a distant relation in Dasian space, connecting the most counterclockwise pitch class F♯ (F♯, 3) with the most clockwise pitch class E (E, 0).[56]

55 The thematic material corresponding to block B starts around m. 254. The "dramatic" moment at m. 264 can easily be related to the beginning of the pivotal block C because of its change of texture and point of overlap between sections. What follows m. 264 recalls the rhythmic ostinato of block D (left hand) and the wide register (right hand) of block C.

56 Alternatively, the analysis could choose to view the "polychord" as a scalar segment E–F♯–G♯–A–C♯, but this interpretation would miss the superposition ("poly") of layers and its wide registral distribution.

(11.2)

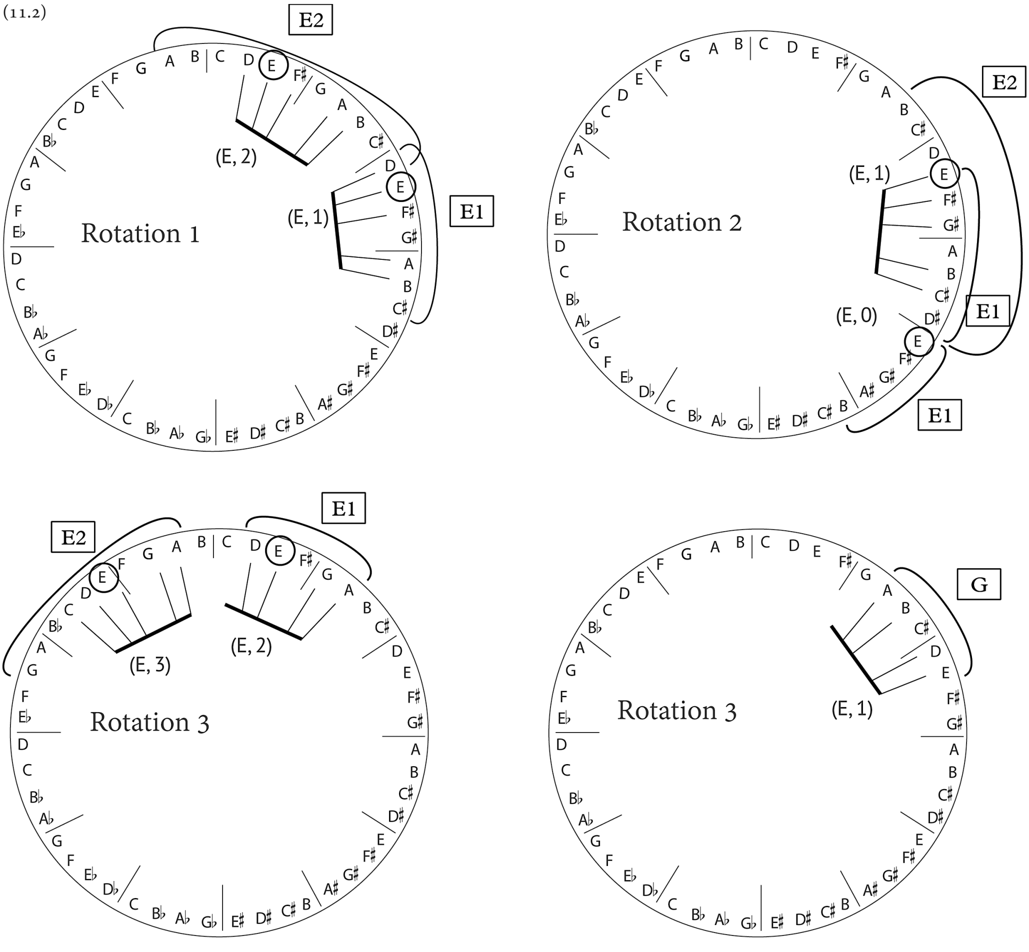


Figure 11 (continued). Modal qualities for pitch class E in episodes. (11.1) Scalar contexts for pentatonic frames (bracketed) of movable episodes (blocks E and G) and changing scalar context for pitch class E (circled). (11.2) Dasian locations of thematic episodes (blocks E and G) and changing modal qualities for pitch class E (circled).

After m. 264, the pitch material gradually abandons the polychord and centers upon the diatonic collection E–F–G–A–B–C–D–E, which is precisely mapped into the contiguous counterclockwise space to (F♯, 3), the bass of block C. The fifth A–E of the E-Phrygian collection is retained from and related to the polychord by transformatio f^{-2} (Figure 12, solid arrows).

The closing function of the white-key diatonic collection is further emphasized through its large-scale symmetrical opposition to the refrain's melodic theme. The axis of symmetry inverts the four-sharp diatonic collection that characterizes the refrains (A–B–C♯–D♯–E–F♯–G♯) to the final E-Phrygian collection (C–D–E–F–G–A–B) via transformatio ($f^{|3|}$) between (E, 0) and (E, 3). This large-scale polymodal juxtaposition reverses the semitonal asso-

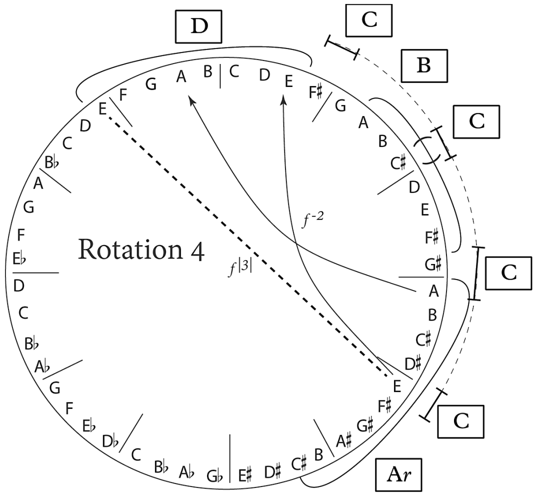


Figure 12. Dasian modeling of symmetry and closure in rotation 4

ciation for pitch class E (E–D♯ vs. F–E), helping to reinforce its projection as a pitch center.[57] The common tones in the polymodal juxtaposition include pitch classes E, A, and B, which are projected as vertical chords precisely at the end of the opening refrain (rotation 1, mm. 17–19) and the beginning of the closing E-Phrygian collection (rotation 4, mm. 268, 271, and 273).[58] Furthermore, the final chord for the piece (E–F–B–D–E) reaches pitch class E, in both top and lower registers, reinforcing octave-bounding centricity on pitch class E and condensing marked elements of the three previous rotations: the notes E, F, and D correspond to axes (related by transformatio) that connect distinct scale segments within polychords and mark the boundaries between the extended areas between refrains and episodes in rotations 1, 2, and 3, respectively. In short, the closure for the movement, far from "arbitrary" or "weak," is, rather, effective and powerful.

Affinity spaces and Bartók's "Divided Arpeggios"

Having considered a model of polymodality that correlates the combination of distinct diatonic strata with the exploration of the Dasian space, we now

57 The diatonic juxtaposition of the four-sharp diatonic to white-note diatonic reaches exactly one station beyond both boundaries in the line of fifths activated for all pentatonic frames, as discussed above (see Figure 9). The explicit diatonic ending is thus an extension of the pentatonic procedures of the piece and also completes the double leading-tone figure to the common note E. Bartók's compositional procedure of suggesting a pitch center via double leading-tone attraction has been addressed by the notions of "encirclement" (Gilles 1989) and "disposition pairs" (Morrison 1991). Lerdahl 2001, 333–41, examines the prolongational potential of double leading-tone formations in Bartók.

58 As discussed above (see Figure 6), the notes E, A, and B fill a crucial role at the arrival of the polychord at m. 38 in rotation 1 (as axis of symmetry and as bass and top notes, respectively).

turn to a more flexible notion of polymodality, which models combinations of intervallically consistent, nondiatonic scale strata. I develop the analytical framework of affinity spaces for this enlarged notion of polymodality by generalizing the central properties of the Dasian space, namely, nonoctave affinity relations and space closure. These properties are structured by a general modular-unit formula that gives rise to diverse configurations of pc-mq cycles. The analytical potential of such an enlarged framework is probed by considering pitch relations in Bartók's "Divided Arpeggios" (*Mikrokosmos* no. 143). In particular, the analysis shows that motivic and contrapuntal relations of the piece's arpeggios are structured by two distinct affinity spaces, where the respective operations of transpositio and transformatio model salient relations between different types of arpeggiated tetrachords.[59]

Sections A (mm. 6–25) and A′ (mm. 50–80) of "Divided Arpeggios" focus on combinations of "gamma-chord" (3–5–3) segments, characteristic of Bartók's music; the contrasting B section (mm. 30–46) turns to minor-seventh chord (3–4–3) arrangements and other stacks of thirds.[60] Tetrachordal relations throughout the piece suggest the construction of two closed pitch-class spaces illustrated in Figure 13. Space A assumes two concurrent cyclic forms (or cocycles 0 and 1) correlated to sections A and A′; space B assumes a single cyclic form correlated to the contrasting B section. Each of the spaces stacks its respective "modular" tetrachord (bracketed in the cycles) separated by pitch-class interval 3. This arrangement results in modular patterns of 3–3–3–5 (or its rotations) for space A and 3–3–3–4 (or its rotations) for space B. The analysis shows that cocycle 0 of space A is completely exhausted at the opening of the exposition in mm. 6–11 by arpeggios of adjacent gamma chords, whereas cocycle 1 is exhausted at the corresponding opening of the re-exposition in mm. 50–54. The B section does not exhaust its corresponding space: the opening pair of juxtaposed minor-seventh chords (bracketed in space B) gradually expands by various stacks of (over and under) "thirds," resulting into an overall symmetrical exploration of space B.

59 Although the main ideas concerning the theoretical framework and analytical applicability of "affinity spaces" have been conceived and developed independently within the scope of my Ph.D. dissertation (Martins 2006a), several aspects of its conceptual underpinning and analytical relevance intersect with the work of Gollin (2007, 2008) on multiaggregate cycles. In particular, Gollin (2007, 157–63) frames the analysis of Bartók's "Divided Arpeggios" in reference to multiaggregate cycles whose graphic representations coincide with those proposed here. That our approaches were developed independently only attests, I think, to the strength of the ideas involved. However, our approaches differ significantly in a number of important aspects concerning the conception of the cycles/spaces, focus and relevance of certain properties, and analytical modeling of musical relations. Below, I point to some of these distinctions in reference to both the theoretical and analytical focus. Seminal work on the relevance of single interval cycles for structural relations in the music of Bartók includes Perle 1977 and Antokoletz 1984. The notion of compound interval cycles is first explored by Philip Lambert (1990) regarding the relevance of cyclic alternation of two intervals (what he calls "combination cycles") for the compositional language of Charles Ives.

60 Ernö Lendvai (1971, 44–47) constructs "gamma chords" as the partial superimposition of two (tonic and dominant) of his "axes systems" (1–16). In a post-Schenkerian reading of the piece projecting a large-scale tonal framework, Ivan Waldbauer (1982) sees the small-scale juxtaposition of gamma chords as a somewhat "mechanical" procedure, which is balanced and overcome by the developmental and contrasting B section.

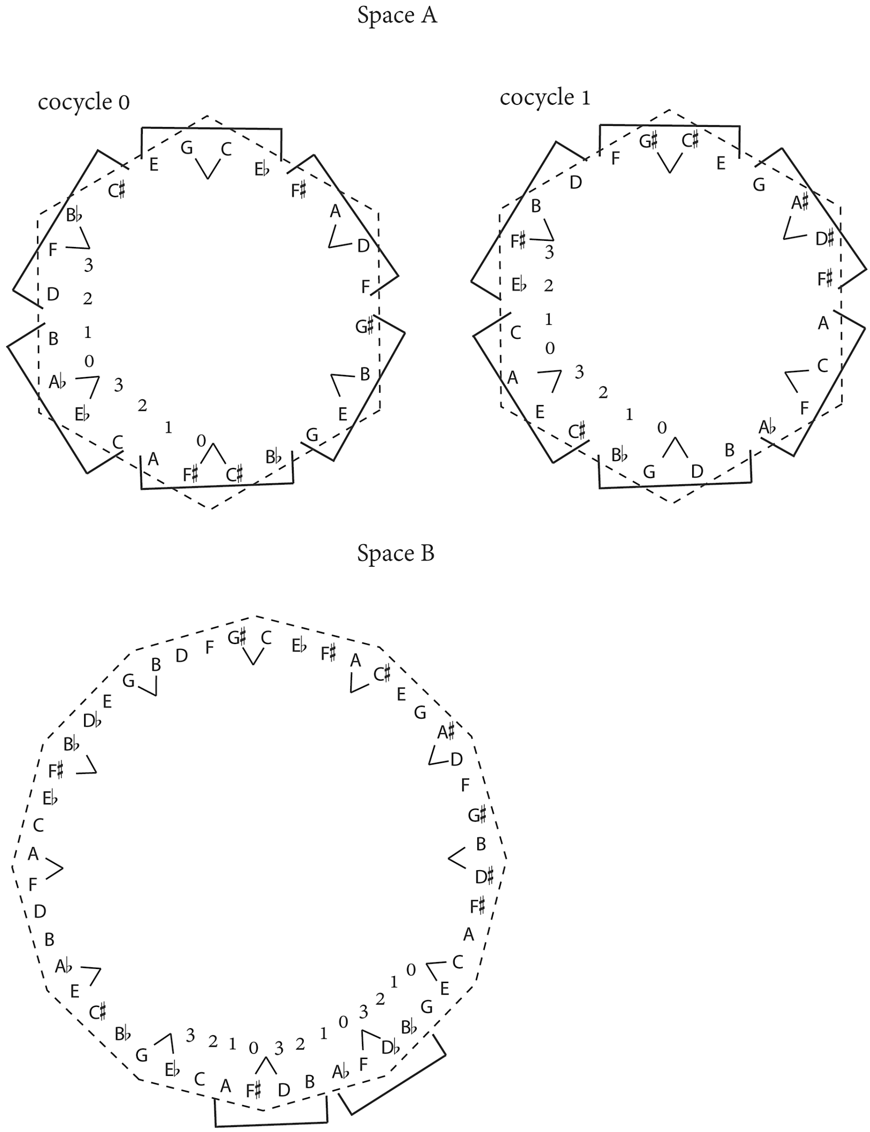


Figure 13. Two affinity spaces, 3–3–3–5 (A) and 3–3–3–4 (B), suggested in Bartók's "Divided Arpeggios," displaying interval affinities at pitch-class intervals 2 and 1, respectively

The modular construction of both spaces resembles the Dasian's stack of "Dorian" tetrachords separated by a whole tone of disjunction (2–2–2–1 modular unit, or its rotations). The unique pitch-class interval in a given modular unit (1 for the Dasian, 5 for space A, and 4 for space B) induces twelve unique locations in each of these cyclic spaces, whereas a recurrent interval normalizes the remaining "steps" within the modular units. Also, just as the Dasian space creates interval affinities every four steps at pitch-class interval 7, so spaces A and B create affinities every four steps at pitch-class interval 2 and pitch-class interval 1.

Let us now generalize the structure of *affinity spaces* by lifting value constraints of pitch-class intervals ("step" sizes) within a modular unit.[61] The formula $p = nr + q$ (congruence mod 12) generalizes the modular unit of affinity spaces, where p, r, and q are pitch-class intervals (mod 12) and n is a positive integer: p corresponds to the interval of transpositio, measuring the affinity in the space, n is the number of recurrent normalized steps r, and q is a unique step interval in the modular unit. Given this formula, the Dasian space and the two affinity spaces A and B can be expressed, respectively, as $7 = 3{\times}2 + 1$, $2 = 3{\times}3 + 5$, and $1 = 3{\times}3 + 4$, (congruence mod 12).[62]

For a given affinity-space formula, the number of distinct, transpositionally related cycles produced (cocycles) is determined by the interval value of transpositio p (the interval of affinities in each cycle). If p is 1, 5, 7, or 11 (i.e., if p is coprime with 12), the result is a single closed cycle, as is the case of the Dasian and space B. If, however, p is 2, 3, 4, 6, 8, 9, 10 (i.e., if p is a divisor of or has a common multiple with 12), the result is concurrent cocycles, whose number equals the value of p (or $12 - p$, when p is 8, 9, or 10). In the case of space A (where $p = 2$), it comprises two cocycles (0 and 1). In any affinity space, there are always $n + 1$ interlocked p-cycles.[63]

As explored for the Dasian space, the pattern of (nonoctave) affinities leads to a unique correspondence between pitch class and modal quality. In a given affinity space, each of the various occurrences of a pitch class is uniquely associated to a distinct position in the modular unit (and consequently to a different interval pattern surrounding that position). The range of modal qualities in a space corresponds to the cardinality of the modular unit ($n + 1$) and is ordered from 0 to n, such that the unique interval q always spans between positions n and 0 in adjacent modular units. The four modal

61 *Step* (or *s*), the directed (clockwise) interval (congruence mod 12) between adjacent (pc, mq) positions in a cycle, is also defined as a group generator below.

62 The formula for the modular unit positions the unique interval q at the "end" of the unit. We can, of course, state a less restrictive formula for a modular unit by allowing other intervals to sum up to the interval of transpositio. However, this design ensures the homogeneity of recurrent steps, on the one hand, and the presence of a unique interval that results in (twelve) unique locations in any affinity space, on the other. As is the case of semitones in the Dasian space, unique intervals are markers of locality and harmonic differentiation in the affinity space. Gollin (2007, 146) uses a less restrictive formula for the generation of what he calls "compound interval cycles," which are "generated by a repeated pattern of two or more distinct intervals" and takes the form (x, y, z, . . .)-cycle, where x, y, z, . . . , are ordered pitch-class intervals. As such, the Dasian space would take the form of a (2, 1, 2, 2)-cycle. Special cases of compound interval cycles are what he calls "multiaggregate cycles," which are compound interval cycles that "run through the tones of more than one aggregate" (143). Gollin's approach is particularly interested in investigating the conditions and analytical uses of the property of maximally even distribution of occurrences of the same pitch class in multiaggregate cycles (measured by the number of steps in a distribution vector). What I refer to as *affinity spaces* A and B used in the analysis of Bartók's "Divided Arpeggios" thus take the form of (3, 5, 3, 3)-cycles and (3, 4, 3, 3)-cycles in Gollin's formulation (the analysis of the piece appears at 157–63).

63 Gollin (2007, 147) points out the "sufficient, but not necessary condition": if the "sum of the component generating intervals is co-prime with 12," the result is a multiaggregate cycle.

qualities of spaces A and B are marked from 0 to 3 inside the cycles of Figure 13. Generalizing the procedure developed for the Dasian space, we can thus conceive of affinity space elements as ordered pairs (pc, mq), where each pitch class is assigned to every available modal quality and each modal quality is assigned to every pitch class.

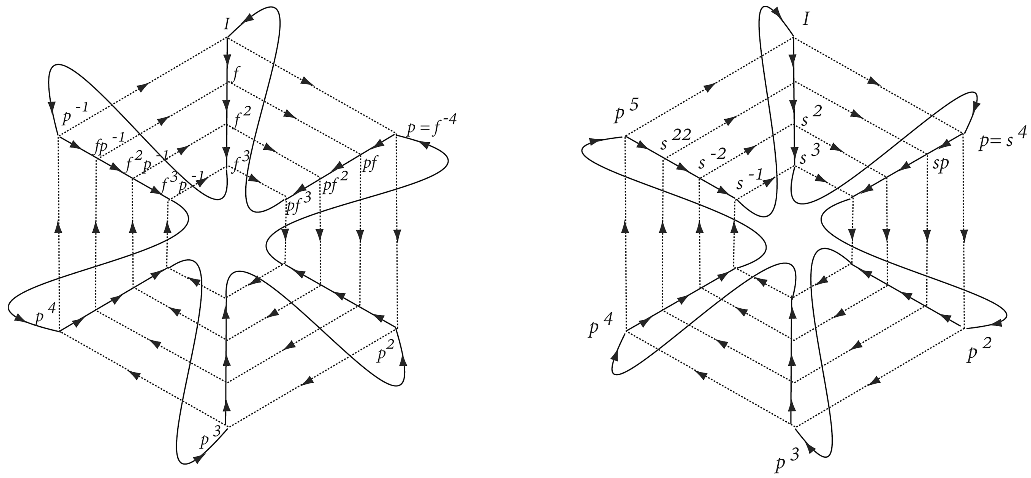
The ordered pair (pc, mq) assignment of affinity spaces induces a closed group structure, in which the operations transpositio (p), transformatio (f), and step (s) act as generators of the space, specifying three kinds of motion between ordered pairs.[64] Group structures for spaces A and B are captured in Figure 14 by two graph representations for each of the spaces: one of the graphs combines the generators p/f, and the other combines the generators p/s. All graphs have four closed paths corresponding to four (interlocked) p-cycles, resulting from the recurrence of p, and thus $p^6 = I$ (identity) in space A (in both cocycles 0 and 1), while $p^{12} = I$ in space B.[65] In space A (Figure 14.1) generator increments of either f or s run through twenty-four group elements only (i.e., half of the total forty-eight elements), while in space B (Figure 14.2) generator increments produce all forty-eight group elements. Any ordered pair (pc, mq) can serve as a reference point for the origin of motion on the corresponding graph, and a given overall motion might follow alternative (but group-theoretically equivalent) paths using different generators. For instance, consider three alternative paths for the overall motion from (C, 3) to (A♯, 3) in space A (both ordered pairs are hosted in cocycle 1). One path moves by increments of the transpositio generator in either the p/f or the p/s graph: p–p^2–p^3–p^4–p^5 (or p^{-1}) yields (C, 3)–(D, 3)–(E, 3)–(F♯, 3)–(G♯, 3)–(A♯, 3). Another path follows increments of the transformatio generator in the p/f graph: f–f^2–f^3–f^4 yields an alternation between cocycles that runs through (C, 3)–(C, 2)–(C, 1)–(C, 0)–(A♯, 3). Finally, a third

64 As proposed for the Dasian space, we can describe the structure of any affinity space as a GIS (Lewin 1987). In the triple (S, IVLS, int), S corresponds to the set of all ordered pairs (pc, mq) in a given affinity space, IVLS is the group of intervals composed of the generators transpositio (p), transformatio (f), and step (s), and int is a function mapping S × S into IVLS, such that int([pc(x), mq(y)], [pc(w), mq(z)]) = int[pc(w) – pc(x), mq(z) – mq(y)], where pitch class and modal quality intervals are computed in mod 12 and mod n + 1. While the set of generators p, f, and s might be analytically useful, reiterations of a single generator are not all required to produce the entire space. As such, the exhaustion of a complete affinity space might require the composition of generators. The generators p, f, and s can be described as p = int([pc(x), mq(y)], [pc($x + p$), mq(y)]), that is, the interval spanning from a given pc(x) to pc($x + p$) (or pc[$x + (n \times r + q)$]) of the same modal quality in the (clockwise) adjacent modular unit. (Notice the double usage of p as a generator in IVLS and pitch-class interval in int.) The generator transformatio (f) has two possible outcomes depending on whether it is applied to mq(0) (modal quality in position zero) or to some other modal quality mq($n \geq 1$). In other words, applying transformatio to a group element either retains the pitch class descending its modal quality by one, that is, f: [pc(x), mq(y)] → [pc(x), mq(y – 1)], for $0 \leq x \leq 11$ and $1 \leq y \leq n$, or (if the element is in modal quality 0) substitutes for another pitch class in modal quality n. The interval producing a pitch-class substitution under f is always equal to r – q (mod 12), that is, f: [pc(x), mq(0)] → [pc($x + r – s$), mq(n)], for $0 \leq x \leq 11$, and r, s, and n defined by the modular unit formula. It is interesting to note that while the value of p (transpositio) determines the existence of single cycles or cocycles in any given affinity space, it does not affect the value for the interval of pitch-class substitution under f. For a more detailed discussion of affinity space group-theoretical properties, see Martins 2009, 505–7.

65 The p-cycles correspond to four 2-cycles in space A and to four 1-cycles in space B.

(14.1) Space A

generators *p/f* generators *p/s*



$I = p^6 = f^{24} = fpf^{-1}p^{-1}$

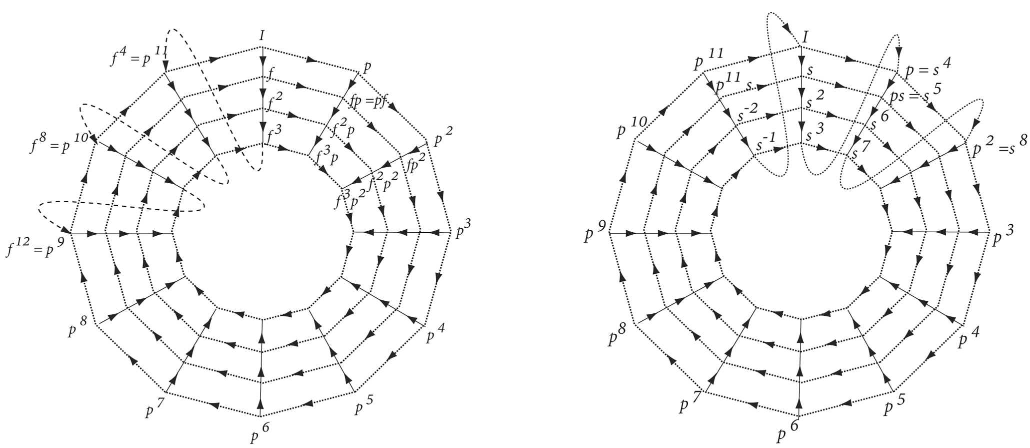
$f^{22} = fp = pf$

$I = p^6 = s^{24} = sps^{-1}p^{-1}$

$s^5 = sp = ps$

(14.2) Space B

generators *p/f* generators *p/s*



$I = p^{12} = f^{48} = fpf^{-1}p^{-1}$

$fp = pf$

$p = f^{44}$

$I = p^{12} = s^{48} = sps^{-1}t^{-1}$

$s^5 = sp = ps$

Figure 14. Group structure (graph representation) for spaces A and B using generators *p*, *f*, and *s*. (14.1) Space A. (14.2) Space B.

path follows increments of the step generator in the p/s graph: s^{-1}–s^{-2}–s^{-3}–s^{-4} (or s^{20}) yields (C, 3)–(A, 2)–(F♯, 1)–(E♭, 0)–(A♯, 3). In short, while the operations p^5, f^4, s^{-4} suggest different paths, they yield the same overall motion in space A and are group-theoretically equivalent.[66]

Some linear and contrapuntal combinations of tetrachords in Bartók's piece are modeled by strong recurrences of affinity space generators. In addition, the somewhat surprising polymodal superimpositions of tetrachords in the closing passages of sections A and A′ give rise to consistent generator relations. Figure 15.1 sketches gamma-chord activity in section A (mm. 6–22) and traces some relations modeled by transpositio and transformatio. The overall gesture of the passage is shaped by an arched bass movement C–E–G♯–F♯–E–D–B♭, which encompasses three phases: in phase 1 (mm. 6–11), pairs of stacked gamma chords ascend by a major third (bass: C–E–G♯); in phase 2 (mm. 11–13), a similar pairing of tetrachords descends by a whole tone (bass: G♯–F♯–E–D), which reverses and fills in the previous ascending motion; and in phase 3 (mm. 14–22), a closing gesture prolongs the superimposition of two nonadjacent gamma chords.

Figures 15.2–15.4 capture tetrachordal activity via affinity-space relations. Figure 15.2 shows that pairs of stacked gamma chords (adjacently located and related by p in cocycle 0) move by p^2 (major third bass ascent) to exhaust the entire cocycle in phase 1. Figure 15.3 models the whole-tone descent of tetrachordal pairs in phase 2 by three consecutive moves $\langle p^{-1}, p^{-1}, p^{-1}\rangle$, which fall short of undoing the $\langle p^2, p^2\rangle$ of phase 1. This descent is also characterized by a two-pitch-class overlap between top and lower notes of the arpeggiated figures. This overlap connects different instances of the same pair of pitch classes in different parts of cocycle 0 and is modeled by f^{-2}. For instance, the two top notes (F♯, 0) and (A, 1) that finish phase 1 in m. 11 (see Figure 15.1 and 15.3) are related via f^{-2} to the two lower notes (F♯, 2) and (A, 3) that initiate phase 2 in the same measure. Throughout phase 2, pitch-class overlap between arpeggios continues to be modeled by f^2, but the pattern of overlap is broken at the end of m. 13, where the top notes (C, 0) and (E♭, 1) do not find corresponding lower counterparts at (C, 2) and (E♭, 3) (signaled by X over an arrow in Figures 15.1 and 15.3).[67] This moment corresponds to a break in both transformatio and transpositio patterns and avoids the premature closure that the opening gamma chord C–E♭–A♭–B would bring to the passage. Instead, phase 3 restates in the left hand the gamma chord B♭–C♯–F♯–A that finished phase 1, and later superimposes the new gamma chord C♯–E–A–C in the right hand. This suggests that phase 3 provides a sense of relative closure by filling in a harmonic distance opened

66 All affinity-space groups are commutative, allow for paths formed by the combination (words) of generators, and yield the following relations: if $f^a = s^b$ and $p^a = s^c$, then $f^c = p^b$, where a, b, and c are integers modulo m for $f^m = s^m = I$ (identity), and modulo n for $p^n = I$.

67 Gollin (2007, 159) makes a similar claim, remarking that "Bartók exploits the ordered, pairwise distribution of pitch classes of the (3, 5, 3, 3)-cycle in the stretti that follow the cyclic unfoldings in the exposition and reprise, using the cycle's invariant dyads to align new cycle segments."

(15.1)

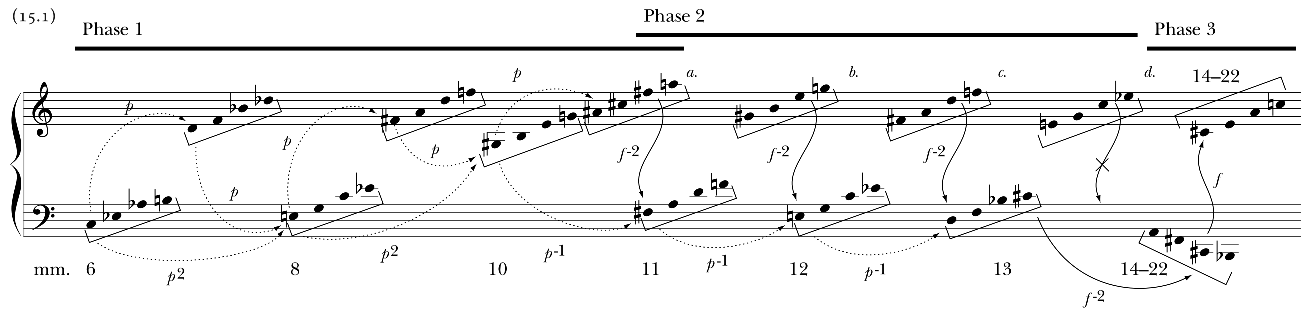
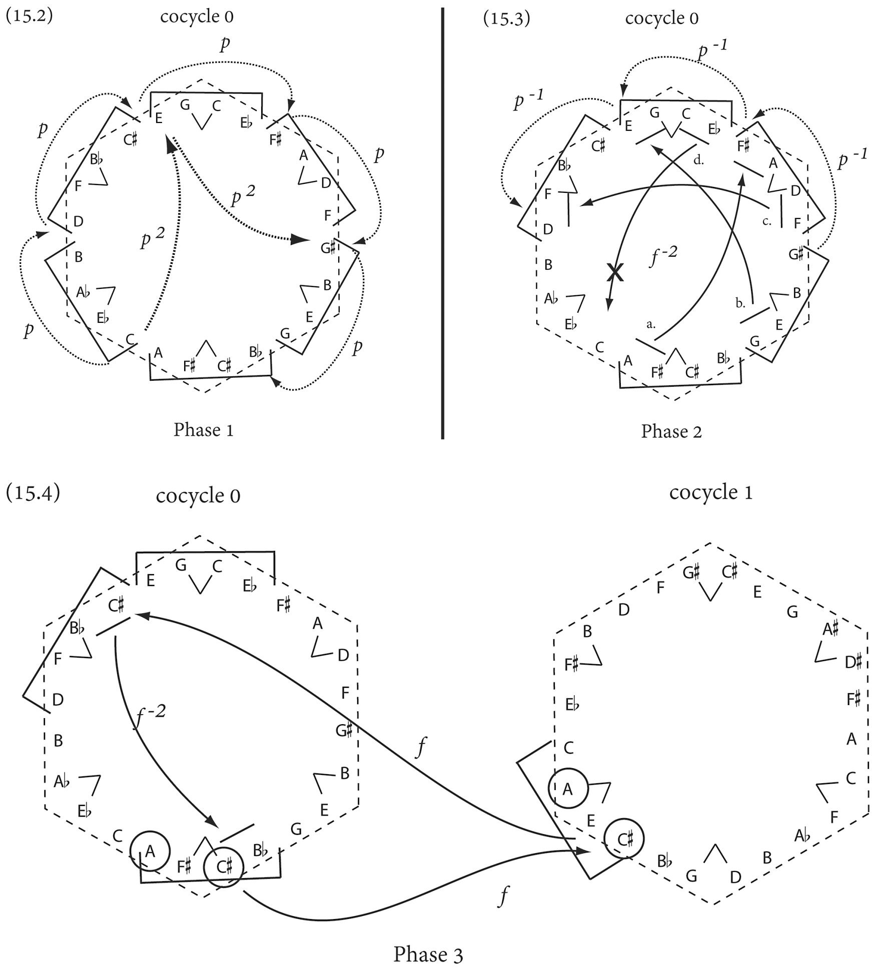


Figure 15. Gamma-chord activity in section A (mm. 6–22). (15.1) Gamma-chord relations in phases 1, 2, and 3 modeled by transpositio and transformatio. (15.2) Phase 1: transpositio relations traced in affinity space A. (15.3) Phase 2: transpositio and transformatio relations. (15.4) Phase 3: transformatio relations between gamma-chords in different cocycles 0 and 1. (15.5) Phase 3: relation of materials displayed on the *s/f* pitch lattice alternating cocycles 0 and 1.



(15.5)

$f^{|2|}$ $f^{|1|}$ $f^{|1|}$ f^{-2} f

F	G♯	C♯	E	G	A#	D#	cocycle 1
F	B♭	C♯	E	G	C	E♭	cocycle 0
G	B♭	C♯	E	A	C	E♭	cocycle 1
G	B♭	C♯	F♯	A	C	E♭	cocycle 0
G	B♭	D♯	F♯	A	C	F	cocycle 1

Figure 15 (continued). Gamma-chord activity in section A (mm. 6–22). (15.1) Gamma-chord relations in phases 1, 2, and 3 modeled by transpositio and transformatio. (15.2) Phase 1: transpositio relations traced in affinity space A. (15.3) Phase 2: transpositio and transformatio relations. (15.4) Phase 3: transformatio relations between gamma chords in different cocycles 0 and 1. (15.5) Phase 3: relation of materials displayed on the *s*/*f* pitch lattice alternating cocycles 0 and 1.

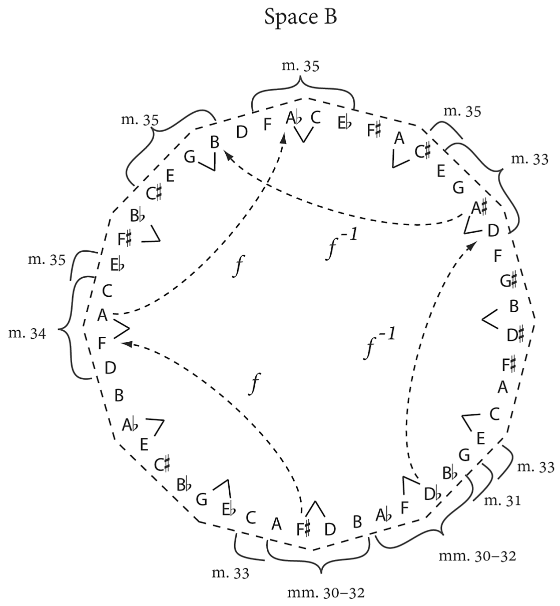


Figure 16. Symmetrical exploration of materials and transformatio relations in section B

at the passage from phase 2 to phase 3. Figure 15.4 shows that the pair (B♭, 2) and (C♯, 3) of B♭–C♯–F♯–A (phase 3) distances f^{-2} from (B♭, 0) and (C♯, 1) of the arpeggio at the end of phase 2, thus retaining the transformatio relation that characterizes phase 2. This f^{-2} distance is filled in by the superimposed gamma chord C♯–E–A–C of phase 3, which finds its space location in cocycle 1. This harmonic relation is better captured in Figure 15.5 by an *s*/*f* lattice coordinating step relations on the rows with transformatio relations on the columns.[68] As the vertical dimension in the lattice alternates between cocycles, the right-hand C♯–E–A–C in cocycle 1 mediates left-hand gamma chords of cocycle 0 by filling in the harmonic gap between those events. Common notes C♯ and A between left and right hands in phase 3 are related by *f* and become the melodic goals of the section in m. 25.[69] The contrasting section B turns to an overall symmetrical exploration of affinity space B, as shown in Figure 16, where the transformatio arrows signal the change of modal quality for some pitch classes.

Phase 1 in the re-exposition (mm. 50–55) completely exhausts cocycle 1, but the direction of the exploration now runs counterclockwise $\langle p^{-2}, p^{-2}\rangle$, matching the reversal of the melodic major-third descent in the arpeggios. The notion of harmonic mediation is also useful to approach gamma-chord

68 Oblique lines traversing rows and columns of space A mark off the unique perfect-fourth intervals on the rows and the note change produced by transformatio on the columns.

69 Superimposed gamma chords B♭–C♯–F♯–A and C♯–E–A–C stand at ⟨*f*, *s*⟩.

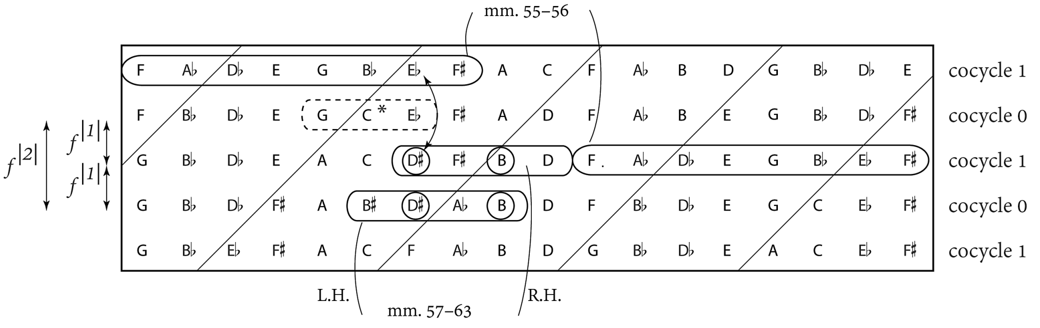


Figure 17. ***s/f*** **lattice structuring gamma-chord relations in re-expository phases 2 and 3 (mm. 55–63, section A′)**

relations in corresponding phases 2 and 3 (mm. 55–63) of section A′ and the coda (mm. 68–80). However, I am arguing that, unlike events in the exposition, re-expository events spanning from phase 2 to phase 3 fail to be harmonically mediated. This mediation, however, is ultimately achieved by the final gamma chord in the coda. Figure 17 presents the *s/f* lattice structuring gamma-chord relations in re-expository phases 2 and 3. Phase 2 is abridged on a single arpeggio (F–A♭–D♭–E–G–B♭–E♭–F♯, mm. 55–56), whose top notes (E♭, 0)–(F♯, 1) stand at f^{-2} from (D♯, 2)–(F♯, 3), which both end phase 1 (in E♭–F♯–B–D, m. 55) and initiate phase 3 (in D♯–F♯–B–C𝄪, m. 57).[70] Having explored exclusively cocycle 1 in phases 1 and 2 of the re-exposition, phase 3 brings a new gamma chord on the left hand (B♯–D♯–G♯–B, mm. 57–63) located in cocycle 0, thereby also reversing the expository exploration of cocycles. However, while the concluding phase 3 reenacts the superimposition of gamma chords,[71] the left-hand tetrachord (B♯–D♯–G♯–B) does not mediate the harmonic space spanned from phase 2 to phase 3 but, rather, stands harmonically further. This harmonic distance is fleetingly mediated by the slight pattern deformation at the end of m. 57. The fleeting note C at the top of the arpeggio (the last note of m. 56) forms a perfect fourth with the previous note G and thus alters the location of the unique perfect-fourth interval that characterizes gamma chords. Such deformation induces a (C, 0) location in cocycle 0 (Figure 17, asterisk).[72]

The coda presents isolated gamma chords interspaced with "appoggiatura" figures, thereby creating a gestural fragmentation appropriate for the

70 In the exposition, the left-hand gamma chord B♭–C♯–F♯–A (which initiates phase 3) returns to the same tetrachord that concludes phase 1, thereby bookending the entire phase 2. In m. 11, however, {F♯, A} initiates the transformatio overlap, while in m. 14, {B♭, C♯} (the remaining dyad of the gamma chord) is engaged in the relationship.

71 As in the exposition, superimposed gamma chords (D♯–F♯–B–C𝄪 and B♯–D♯–G♯–B) also stand at ⟨*f*, *s*⟩ in the re-exposition (mm. 57–63), and common tones E♭ and B also become melodic goals of the passage in m. 66.

72 The (C, 0) location for the last note in m. 56 is confirmed by the ensuing D♯ on m. 57, which juxtaposed with C is heard as (D♯, 1).

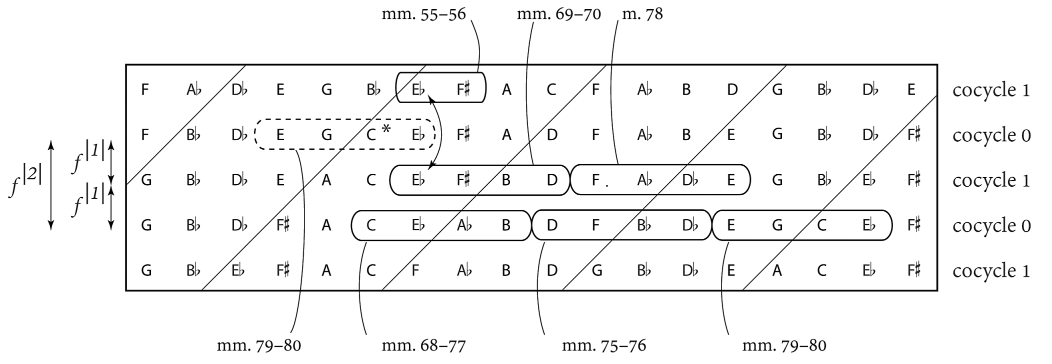


Figure 18. ***s/f*** **lattice for gamma-chord activity in the coda (mm. 68–80): The last gamma-chord E–G–C–E♭ finally fills in the harmonic gap.**

piece's conclusion. The material revisits and condenses procedures developed on section A of the piece and ultimately provides the harmonic gap mediation created in mm. 55–57 (fleetingly covered by the note C in m. 56). Figure 18 maps gamma-chord activity of the coda into the *s/f* lattice, showing that the four gamma chords articulated in mm. 68–78 (C–E♭–A♭–B, D–F–B♭–D♭, E♭–F♯–B–D, and F–A♭–D♭–E), while gesturally fragmented, stand at adjacent rows and columns in the lattice (i.e., they create scalar and harmonic adjacencies in the affinity space A). The last gamma chord of the piece (E–G–C–E♭, mm. 79–80) fulfills two roles: it continues the melodic adjacency of gamma chords set forth in the coda (bounded by a solid line in the figure), and it touches upon (C, 0)–(E♭, 1), finally arriving upon the harmonic mediation thus far unfulfilled in section A′ (mm. 55–56; dotted line).

In conclusion, the analytical approach developed in the article to examine the combination of scalar layers in early twentieth-century music, particularly in Bartók, suggests a fertile interaction between diatonicism and chromaticism. Based on the intuition (proposed by Bartók and others) that scalar layers are not fused but retain their diatonic integrity when combined in certain (polymodal) contrapuntal textures, the article proposes conceptual constructs (the Dasian and other affinity spaces) that share properties of both scalar and pitch-class spaces in order to better understand harmonic relations in multidiatonic passages. This contrasts with the prevailing tonal and atonal analytical models developed in the second half of the century for this music, which espouse either a deeper level diatonicism or chromaticism. The analytical argument developed in the article suggests that composers such as Bartók have explored the combination of layers not only to create new textures or characteristic harmonic states but also to shape and contribute to large-scale tonal strategies. In addition, the morphologies of affinity spaces sustaining the analytical framework have deep historical roots as models for the understanding of the relation between melodies and scales. The central properties of those spaces (in particular interval affinities, and the interaction of pitch class and modal quality) are awakened from dormancy in our familiar materials and put to new analytical uses, while bypassing some (at times) limiting features usually associated with those materials (such as the specification of pitch centers, enharmonic spellings, and labels for and affiliations to complete scales). The generalization of those properties to non-diatonic contexts proposed in the article also allows us to better understand how compositional strategies in the twentieth-century scalar tradition created new harmonic relations that contributed to the overall exploration of chromatic space.

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