§1b. Chesterton above is wrong in one respect. Or at least imprecise. The danger he's trying to name is not logic. Logic is just a method, and methods can't unhinge people. What Chesterton's really trying to talk about is one of logic's main characteristics—and mathematics'. Abstractness. Abstraction.

It is worth getting straight on the meaning of *abstraction.* It's maybe the single most important word for appreciating Cantor's work and the contexts that made it possible. Grammatically, the root form is the adjectival, from the L. *abstractus* = 'drawn away'. The *O.E.D.* has nine major definitions of the adjective, of which the most apposite is 4.a.: "Withdrawn or separated from matter, from material embodiment, from practice, or from particular examples. Opposed to *concrete.*" Also of interest are the *O.E.D.*'s 4.b., "Ideal, distilled to its essence," and 4.c., "Abstruse."

Here is a quotation from Carl B. Boyer, who is more or less the Gibbon of math history[4]: "But what, after all, are the integers? Everyone thinks that he or she knows, for example, what the number three is—until he or she tries to define or explain it." W/r/t which it is instructive to talk to 1st- and 2nd-grade math teachers and find out how children are actually taught about integers. About what, for example, the number five is. First they are given, say, five oranges. Something they can touch or hold. Are asked to count them. Then they

[4] **IYI** Boyer is joined at the top of the math-history food chain only by Prof. Morris Kline. Boyer's and Kline's major works are respectively *A History of Mathematics* and *Mathematical Thought from Ancient to Modern Times.* Both books are extraordinarily comprehensive and good and will be liberally cribbed from.

are given a picture of five oranges. Then a picture that combines the five oranges with the numeral '5' so they associate the two. Then a picture of just the numeral '5' with the oranges removed. The children are then engaged in verbal exercises in which they start talking about the integer 5 per se, as an object in itself, apart from five oranges. In other words they are systematically fooled, or awakened, into treating numbers as things instead of as symbols for things. Then they can be taught arithmetic, which comprises elementary relations between numbers. (You will note how this parallels the ways we are taught to use language. We learn early on that the noun 'five' means, symbolizes, the integer 5. And so on.)

Sometimes a kid will have trouble, the teachers say. Some children understand that the word 'five' stands for 5, but they keep wanting to know 5 *what?* 5 oranges, 5 pennies, 5 points? These children, who have no problem adding or subtracting oranges or coins, will nevertheless perform poorly on arithmetic tests. They cannot treat 5 as an object per se. They are often then remanded to Special Ed Math, where everything is taught in terms of groups or sets of actual objects rather than as numbers "withdrawn from particular examples."[5]

[5] B. Russell has an interesting ¶ in this regard about high-school math, which is usually the next big jump in abstraction after arithmetic:

> In the beginning of algebra, even the most intelligent child finds, as a rule, very great difficulty. The use of letters is a mystery, which seems to have no purpose except mystification. It is almost impossible, at first, not to think that every letter stands for some particular number, if only the teacher would reveal *what* number it stands for. The fact is, that in algebra the mind is first taught to consider general truths, truths which are not asserted to hold only of this or that particular thing, but of any one of a whole group of things. It is in

The point: The basic def. of 'abstract' for our purposes is going to be the somewhat concatenated 'removed from or transcending concrete particularity, sensuous experience'. Used in just this way, 'abstract' is a term from metaphysics. Implicit in all mathematical theories, in fact, is some sort of metaphysical position. The father of abstraction in mathematics: Pythagoras. The father of abstraction in metaphysics: Plato.

The *O.E.D.*'s other defs. are not irrelevant, though. Not just because modern math is abstract in the sense of being extremely abstruse and arcane and often hard to even look at on the page. Also essential to math is the sense in which abstracting something can mean reducing it to its absolute skeletal essence, as in the abstract of an article or book. As such, it can mean thinking hard about things that for the most part people can't think hard about—because it drives them crazy.

All this is just sort of warming up; the whole thing won't be like this. Here are two more quotations from towering figures. M. Kline: "One of the great Greek contributions to the very concept of mathematics was the conscious recognition and emphasis of the fact that mathematical entities are abstractions, ideas entertained by the mind and sharply distinguished from physical objects or pictures." F.d.l. Saussure: "What has escaped philosophers and logicians is that from the moment a system of symbols becomes independent

the power of understanding and discovering such truths that the mastery of the intellect over the whole world of things actual and possible resides; and ability to deal with the general as such is one of the gifts that a mathematical education should bestow.

of the objects designated it is itself subject to undergoing displacements that are incalculable for the logician."

Abstraction has all kinds of problems and headaches built in, we all know. Part of the hazard is how we use nouns. We think of nouns' meanings in terms of denotations. Nouns stand for things—*man, desk, pen, David, head, aspirin.* A special kind of comedy results when there's confusion about what's a real noun, as in 'Who's on first?' or those *Alice in Wonderland* routines—'What can you see on the road?' 'Nothing.' 'What great eyesight! What does nothing look like?' The comedy tends to vanish, though, when the nouns denote abstractions, meaning general concepts divorced from particular instances. Many of these abstraction-nouns come from root verbs. 'Motion' is a noun, and 'existence'; we use words like this all the time. The confusion comes when we try to consider what exactly they mean. It's like Boyer's point about integers. What exactly do 'motion' and 'existence' denote? We know that concrete particular things exist, and that sometimes they move. Does motion per se exist? In what way? In what way do abstractions exist?

Of course, that last question is itself very abstract. Now you can probably feel the headache starting. There's a special sort of unease or impatience with stuff like this. Like 'What exactly is existence?' or 'What exactly do we mean when we talk about motion?' The unease is very distinctive and sets in only at a certain level in the abstraction process—because abstraction proceeds in levels, rather like exponents or dimensions. Let's say 'man' meaning some particular man is Level One. 'Man' meaning the species is Level Two. Something like 'humanity' or 'humanness' is Level Three; now we're talking about the abstract criteria for something qualifying as human.

And so forth. Thinking this way can be dangerous, weird. Thinking abstractly enough about anything . . . surely we've all had the experience of thinking about a word—'pen,' say—and of sort of saying the word over and over to ourselves until it ceases to denote; the very strangeness of calling something a pen begins to obtrude on the consciousness in a creepy way, like an epileptic aura.

As you probably know, much of what we now call analytic philosophy is concerned with Level Three– or even Four–grade questions like this. As in epistemology = 'What exactly is knowledge?'; metaphysics = 'What exactly are the relations between mental constructs and real-world objects?'; etc.[6] It might be that philosophers and mathematicians, who spend a lot of time thinking (a) abstractly or (b) about abstractions or (c) both, are *eo ipso* rendered prone to mental illness. Or it might just be that people who are susceptible to mental illness are more prone to think about these sorts of things. It's a chicken-and-egg question. One thing is certain, though. It is a total myth that man is by nature curious and truth-hungry and wants, above all things, *to know.*[7] Given certain recognized senses of 'to know,' there is in fact a great deal of stuff we do *not* want to know. Evidence for this is the enormous number of very basic questions and issues we do not like to think about abstractly.

[6] **IYI** According to most sources, G. F. L. P. Cantor was not just a mathematician—he had an actual Philosophy of the Infinite. It was weird and quasi-religious and, not surprisingly, abstract. At one point Cantor tried to switch his U. Halle job from the math dept. to philosophy; the request was turned down. Admittedly, this was not one of his stabler periods.

[7] **IYI** The source of this pernicious myth is Aristotle, who is in certain respects the villain of our whole Story—q.v. §2 sub.