

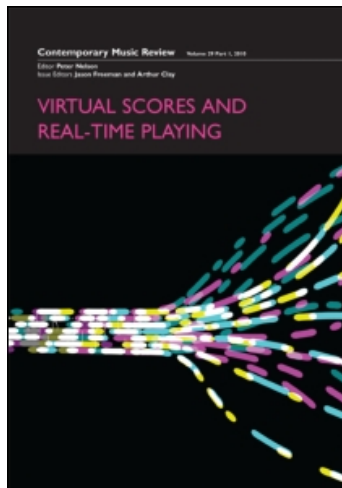
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Alternative Tunings, Alternative Tonalities

Daniel James Wolf

Several approaches to the design and use of alternative tuning systems (defined as systems other than 12-tone equal temperament) are described. Each of these tuning systems has been created in order to preserve or extend qualities associated with tonal musics. This is by no means a comprehensive catalogue of such approaches, but rather a selection among those speculative theories that appear to offer greater potential as resources for a wide community of composers. Atonal or purely timbral uses of alternative tuning systems are not examined in this article nor are historical tunings or schemes for retuning existing repertoire.

KEYWORDS: tuning systems

For Ervin M. Wilson, in advance of his 75th birthday.

This article describes several approaches to the design and use of alternative tuning systems in order to preserve or extend qualities associated with tonal musics. "Alternative tuning systems" are here defined as systems other than 12-tone equal temperament ("12tet"). A definition of "tonal musics" is not ventured here, but the qualities associated with such might include a collection of pitches the relationships among which are defined by tonal functions (e.g. a fixed "tonic" pitch; position of individual pitches in a sequence of perfect fifths or other intervals, i.e. mediant or leading tone relationships, etc.), an intervallic repertoire corresponding to the intervals between partials in a harmonic series, and a recognized continuum from consonant to dissonant configurations of pitches. This is by no means a comprehensive catalogue of such approaches, but rather a selection among those speculative theories that appear to offer greater potential as resources for a wide community of composers. It is no coincidence that I have selected the work of historical tuning theorists who have formed the background to the work of the leading contemporary tuning theorist Ervin M. Wilson. Atonal or purely timbral uses of alternative tuning systems are not examined in this article, nor are historical tunings or schemes for retuning existing repertoire.¹

I. Bosanquet, Tanaka: Keyboards without Masters

The nineteenth-century English scholar R. H. M. Bosanquet worked both in the then new field of psychophysics, with a particular interest in the phenomenon of beats of mistuned consonances, and in music theory, with an interest in the theory of temperaments and, in particular, non-12tet divisions of the octave. Although he

came to advocate a 53-tone equal division of the octave (“53tet”) as offering an optimal and practical approximation of 5-limit² just intonation, Bosanquet analyzed a broad spectrum of systems and categorized them by the size of the perfect fifth in each tuning relative to the size of the interval in 12tet, so that systems with fifths narrower than the 12tet fifths were identified as “negative”, while “positive” systems had fifths greater than the 12tet species. To support the practical introduction of a 53tet system, Bosanquet designed an ingenious keyboard based upon extending the traditional seven-plus-five (Halberstadt) keyboard with the addition of further, alternating ranks of five and seven digitals (Bosanquet 1876).

Although a harmonium following Bosanquet’s design was produced commercially, there is no evidence that music was especially composed for the instrument, nor is there any indication that Bosanquet himself found his approach to be speculative. Instead, Bosanquet was proposing a means for improved intonation for existing music. As an instrument for extending tonality, Bosanquet’s keyboard was clearly a design before its time, and would wait almost a century for the modifications of Wilson (see below), which would demonstrate that the design was adaptable to a wide variety of tuning systems.

A similar argument in favor of 53tet was made by Tanaka Shohé (Tanaka 1890), who also designed, patented and built a keyboard that is apparently still used for demonstration purposes in Tokyo (Kondo Jo, personal communication). Tanaka’s dissertation is also interesting for its pioneering use of lattices to illustrate tonal relationships belonging to the Riemannian functional model, with fifth-relationships notated on a horizontal axis and mediant relationships assigned to an intersecting axis. Like Bosanquet, Tanaka’s intentions were to improve the intonation of existing repertoire, not to identify new tonal materials.

While 53tet does provide excellent approximations of 5-limit intervals, it is problematic in at least two ways. For one, the temperament is awkward, if not unsuitable, for the performance of existing repertoire. If western classical triadic tonality can be heard as premised upon having the best major third present in the tuning system equivalent to the (octave-equivalent) sum of four consecutive perfect fifths – a property equally present in each of the major tuning or temperament environments used in common practice counterpoint and harmony (meantone, well-temperaments, 12tet³) – then this premise is unfulfilled by 53tet. In 53tet, which might be thought of a scale of 53 modestly tempered syntonic commas, the best major third remains one scale step distant from the sum of four perfect fifths; as a consequence, realizations of existing repertoire may well tend to “drift” in pitch from an initial tonic by the approximate comma interval of a 1/53 octave. But, perhaps more critically in this speculative context, given the extravagant resources required to implement 53tet in notation or instruments, it does not offer significantly better and consistent representations of intervals beyond the 5-limit.⁴

II. Ives’s “Impressions”

Charles Edward Ives is generally seen as the founder of the feast now known as the American experimental tradition, and, although limited in quantity, his theoretical and practical work with tuning systems is far from trivial and demonstrates an approach distinctive to that of his European contemporaries working in

alternative tuning systems. Ives's (1925/1962) sole treatise on the topic, "Some 'quarter-tone' impressions", is, indeed, impressionistic. It rambles a bit, with appropriate bits of Helmholtz and a now-forgotten musicologist, more vividly recalls his own father's "weakness for quarter-tone" and experiments with tuned glasses and improvised stringed instruments, but closes in quickly on some concrete observations about quarter-tone resources and how they are heard. First, he observes that simply dividing the fifth into two equal halves (two neutral thirds, each $1/24$ octave) creates a chord that is heard as an "out of tune" major or minor triad, not as an entity in itself. Ives recommends adding a fourth tone to the triad, also in a quarter-tone relationship to the root and fifth, to create a chord that "seems to establish an identity of its own". It is interesting that Ives's intuitive approach leads to the conclusion that the addition of tones to the gamut becomes more meaningful if the harmonic texture is likewise thickened, and, in the case of quarter-tones, the more distinctive vertical structure is no longer triadic but a chord with four or more tones. Ives, in a trope unusual for the earliest microtonalists, makes specific appeals to psychoacoustic research, actually offering a roughness calculation and then proposing an important corollary, namely that roughness would vary with amplitude. For this reason, I find it necessary to distance Ives's little treatise from the more extensive efforts of both Hába (1927) and Wyschnegradsky (1932), both of whom made extensive catalogues of possible microtonal musical materials within their favored equal divisions of the octave, but neither of whom engaged in an evaluation of those materials in qualitative, perceptual terms.

Ives's use of microtones in his compositional work may usefully be characterized as falling into three categories: (1) coloristic, with unisons separated by a quarter-tone interval, or in quarter-tone *portamenti* ornamenting a melodic line or chords; (2) fully integrated quarter-tone melodic and harmonic textures, as in the *Three Quarter-Tone Pieces*; and (3) tentative experiments with a form of just intonation based upon the harmonic series, as in the sketches for a *Universe Symphony*. A highly speculative case has been made by the musician Johnny Reinhard (unpublished work) that the "default" tuning for Ives's works is a pythagorean intonation rather than 12tet.

III. Novaro

The Mexican theorist, composer and instrument builder Augusto Novaro (1891–1960) must be assessed as both one of the least recognized and most imaginative musical theorists.⁵ His major theoretical writing (Novaro 1927) is a two-part essay describing first a "natural system of music", i.e. in just intonation, and then proposing a practical approximation of that system with the 72-tone equal temperament.

The "natural system of music" is in just intonation and is formally equivalent to the "tonality diamond" that would later be developed by Harry Partch (see below). Novaro structures his exploration of just intonation by two operations, the first of which is to extract octave segments of the harmonic series, and the second to calculate the ratios found in each rotation of the given segment; rotations are equivalent to the inversions of a chord found in classical western harmonic practice. For example, the segment between the third and sixth partials of the harmonic series contains the following rotations:⁶

1/1 4/3 5/3 2/1

1/1 5/4 3/2 2/1

1/1 6/5 8/5 2/1

the union of which is the following, invertible, collection:

1/1 6/5 5/4 4/3 3/2 8/5 5/3 2/1

In principle, Novaro appears interested in all such rotations of harmonic series segments, while in practice he stops at segments 6–12, yielding an 11-limit system with membership identical to that of Partch's 11-limit tonality diamond (see below).

Novaro's argument for the 72-tone system to approximate and practically implement his 11-limit system uses a different formal presentation, but is similar when not equivalent to the arguments that would later be advanced by such composers as Ezra Sims (1988), Franz Richter Herf (1978), and James Tenney (1987): beyond being fully transposable and adaptable to instruments and players already oriented to 12tet (as 12tet is a subset of 72tet, each tone in 72tet can be represented by a small set of fractions-of-a-tone deviations from 12tet), 72tet offers consistent and unambiguous representations of rational intervals through the 11-limit.

IV. Yasser's "Evolution"

The Russian-American musicologist and theorist Josef Yasser (Yasser 1932) proposed that musical scales evolved historically from scales of fewer tones to scales with more tones. In Yasser's model, "tonality" evolved in the additive sequence of 2- and 3-tone scales, to five (3 + 2 tones), to seven (5 + 2), to twelve (5 + 7), and predicted that a 19-tone scale would be the next step in tonal "evolution".

In locating figures like Yasser, Partch or Novaro historically, it is useful to identify terms like "evolution" or the dualism implied by the apparent equality of harmonic and subharmonic structures in the Partch diamond (or of Novaro's equivalent construction), or even the basic stacked-thirds hexad of Partch's tonality as being part and parcel of the then contemporary intellectual and aesthetic environment. In the case of Yasser, our contemporary caution for the term "evolution" should not prevent us from reading what remains an interesting bit of speculation.

Yasser's own definitions of consonances and dissonances within the 12-out-of-19 equal "tonal" system are often contrary to what one would expect from their just approximations. In fact, 19tet is a very close approximation of 1/3-comma temperament, a tuning related to mean-tone (1/4-comma temperament) but featuring a minor third very closely approximating the just interval 6:5. Mandelbaum's (1961) dissertation is an important corrective to Yasser as well as a pioneering effort in 19tet composition.

V. Partch's *Harmonielehre*

Our principal concern here is Harry Partch's theoretical work rather than his compositions.⁷ Partch's (1974) *Genesis of a Music*, a large and idiosyncratic text, continues to have a unique hold on readers, and many younger composers have

been decisively influenced by reading *Genesis*.⁸ Partch's contributions to music theory include:

- (1) The use of a ratio notation now widespread among just intonation enthusiasts (in Partch's practice ratios notated by obliques are pitch classes without fixed octave identities).
- (2) The term "identity" to describe the function of individual tones in a chord with reference to their generating odd-factor (e.g. the tonic of a major triad is the "one identity", the fifth of the triad is the "three identity" as it is generated from the tonic via a factor of 3, and the third of the triad is the "five identity", generated from the tonic via a factor of 5).
- (3) The term "limit" (as described in note 2): in Partch's own practice, identities one, three, five, seven, nine and eleven are used; thus he worked at the "11-limit".
- (4) Partch distinguished among his harmonic materials between "otonalities" and "utonalities". Otonalities ("o" for "over") are chords built above a given tonic or one identity, in harmonic proportion; while utonalities ("u" for "under") are constructed below a given one identity, i.e. in subharmonic proportions. In Partch's ratio notation, otonalities could be identified as collections of tones sharing a single denominator while utonalities share a single numerator.
- (5) Partch gave the name "tonality diamond" to the matrix, or cross-set, of an otonal chord multiplied by its utonal inversion.⁹
- (6) Partch identified a small number of harmonic progressions in just intonation derived from very subtle, indeed microtonal, voice-leading patterns. Partch's term for such progressions was "tonal flux".

While it is perhaps easiest to identify elements of Partch's compositional output that reference musics outside of the western classical tradition (in his own words: "Christian hymns, Chinese lullabies, Yaqui Indian ritual, and Hebrew chants" [Partch 1974: viii]), it is perhaps more useful, at least in the context of this article, to identify certain elements in Partch's music and music theory that locate him quite accurately with some major trends in the classical tradition. Partch's harmonic language is well within the late romantic in that his chordal vocabulary is based upon stacked thirds, emulating an extended harmonic series (and indeed, in Partch's just intonation the emulation was even closer than that made by composers working with temperaments), with frequent use of non-harmonic suspensions. Furthermore, Partch, like Schoenberg albeit in a very different intonational environment, allows all of his tonal materials to have exact intervallic inversion.¹⁰ Partch's diamond and the familiar 12-by-12 matrix of row forms associated with Schoenberg and his school are functionally identical, although the "row box" is a closed and redundant structure with each line a complete aggregate of the 12tet set, while the diamond is constructed in the open space of just intonation with no necessary closure. Partch's music was extended to a basic hexad of identities 1-3-5-7-9-11 (Partch's *Genesis* includes a description and diagram of a diamond extended to a 13-limit; sketches exist showing Partch experimented with a 17-limit diamond [Blackburn 1997: 123–126]).

VI. Fokker: Mirrors and Periodicity Blocks

The Dutch physicist, music theorist, and composer A. D. Fokker made a serious study of the speculative resources of a just intonation extended through the 7-limit, and the practical realization of such resources with 31tet. Fokker discovered 31tet in the musical writings of Christiaan Huygens, and for all practical purposes 31tet can be heard as a standard, or quarter-comma, mean-tone temperament carried out to 31 places. In addition to the excellent major thirds of mean-tone, the extension to 31 tones offers augmented sixths that are close approximations of the interval 7:4. Fokker appears to be the first theorist to represent a 7-limit tuning system graphically with three implied axes or dimensions of tonal space, the horizontal axis is assigned to fifths (3:2s), the vertical axis to major thirds (5:4s), and an oblique axis, implying a third dimension, to tones generated by the 7:4 relationship.

Rasch (1987) provides a clear introduction to a variety of techniques developed by Fokker for extending and varying tonal materials. Fokker's premise is always the representation of a configuration of pitches in terms of just intonation, and his techniques involve the direct manipulation of the lattice or harmonic series representations of pitch collections. These techniques usually follow elementary arithmetic operations, including inversions or "mirrorings" in all orientations of a given latticed tone configuration (and, much like Perle's "12-tone tonality", using each tone of the initial configuration as the axis of the transformation), as well as addition to, subtraction from and multiplication of a given tone sequence or configuration.¹¹

Central to Fokker's conception of scale construction is the Periodicity Block (PB). In brief, a PB is located in a segment of tonal space on a lattice where the outer terms are bounded by a small interval. Spreading or "tempering out" this interval throughout the lattice results in a tempered approximation of the tonal space that repeats (hence the "periodicity") indefinitely. For example, 12tet can be heard as a 12-tone segment of the tonal space created by perfect fifths, with the pythagorean comma tempered out over the set of fifths, such that the tone system repeats itself exactly every 12 fifths. Or, alternatively, 12tet can be heard as a variety of segments of a tonal space constructed from a lattice of fifths and major thirds, with, for examples, the syntonic comma or the lesser diesis tempered out.

Rasch (1987) and Erlich (1999) provide useful introductions to PBs and scale construction techniques based upon PB formalism. The internet-based "Alternate Tuning List" has further developed and formalized the PB concept. Among the developments are the generalization of the technique, through matrix addition, to n -dimensions of tonal space; the identification of the major intervals of equivalence or matrix vectors represented by equal tempered tunings; and subsequently, the identification of a new series of non-equal tuning systems, mappable to subsets of equal temperaments, featuring maximal harmonic variety under specific conditions.

VII. Wilson's Theoretical Revolution

The Mexican-American dual national Ervin M. Wilson (1928–) may, without exaggeration, be characterized as the most productive speculative music theorist since Ptolemy. He has been engaged for the past half-century in a comprehensive

project of identifying and analyzing existing scalar traditions and creating new scales and tuning systems with unified schemes of classification, notation and keyboard designs for each of these. A gifted draughtsman, he has developed elegant means for graphically representing tuning systems implying numerous dimensions of tonal space within the two-dimensional confines of paper-and-pencil illustrations.

Wilson's initial investigations with quarter-tones were unsatisfactory and it was through an expanded version of Yasser's evolutionary pattern, which Wilson has dubbed the "scale tree", that he was able to locate familial relationships among equal tempered divisions of the octave. Wilson realized quickly that Yasser's particular "evolutionary" pathway (from two or three tones, to five, then seven, twelve, and nineteen tones) was but a single branch of the scale tree among all the possible summation series in the scale tree (this particular branch was bound by converging sizes of fifths, in fractions of an octave: $2/3$, $3/5$, $5/12$, $12/19$). These familial relationships were translated directly by Wilson into expandable notations and generalized keyboard layouts, developed by subtle rotations of the keyboard designed by Bosanquet. Wilson has built numerous instruments, including re-fretted guitars, keyboard percussion instruments of wood, bamboo, brass and aluminum, and has collaborated in the design of the Hackleman-Wilson 19-tone clavichord and, with Harvey Starr, a large electronic keyboard based on an extended Bosanquet pattern.

It should be noted that Wilson's work was done, initially, in an era when rapid number crunching was largely unavailable, especially to those working outside of an institutional setting. In the mid-1960s, Wilson gained a valuable collaborator with the geneticist and music theorist John C. Chalmers Jr, who managed to obtain mainframe computer time to generate numerous sets of tables applicable to tuning theory. The publication of a selection of these tables (the first for converting cent values to frequencies, and the second for the conversion of ratios to cents, frequencies, and string lengths) in the first issue of *Xenharmonikôn: An Informal Journal of Experimental Music* (Chalmers 1974) is often cited as a landmark in the alternative tuning community.

Wilson realized that Partch's idea of "identity" corresponded to Fokker's "factor", and, following Fokker's notion of "genus", identified by the complete set of factors generating a given block of pitches on a lattice of tonal space, Wilson developed a series of "combination-product sets" (CPS). There are tuning systems identified by the participating factors out of a given set of factors, for example, the "hexany" (a 6-toned CPS) generated by all combinations of any two factors from a set of four, notated by Wilson as 2(A,B,C,D).

CPS structures like the hexany are compact but have a maximum in harmonic variety. A widely used hexany – 2(1,3,5,7) – has the following six tones:¹²

$$1 \times 3 = 3$$

$$1 \times 5 = 5$$

$$1 \times 7 = 7$$

$$3 \times 5 = 15$$

$$3 \times 7 = 21$$

$$5 \times 7 = 35$$

yielding the following four harmonic triads:

3/5/7 (the 3,5,7 identities of tone 1)
 3/15/21 (the 1,5,7 identities of tone 3)
 5/15/35 (the 1,3,7 identities of tone 5)
 7/21/35 (the 1,3,5 identities of tone 7)

as well as the four following subharmonic triads:

15/21/35 (the /7,/5,/3 identities of tone $3 \times 5 \times 7$)
 5/7/35 (the /7,/5,/1 identities of tone 35)
 3/7/21 (the /3,/5, /1 identities of tone 21)
 3/5/15 (the /5,/3,/1 identities of tone 15)

CPS structures, like the hexany, are inversional structures, in this case at the interval $1:(3 \times 5 \times 7)$.

Whereas Partch's diamond was monotonically tonal, with all chords sharing a common tone, CPS's are locally tonal, with clear functional identities at the chord-to-chord level, but can venture, globally, far beyond familiar tonal spaces. Although most work to date has involved hexanies and the three-out-of-six-factor, 20-toned *Eikosany*, Wilson has identified and named all of the CPS structures through the four-out-of-eight-factor, 70-toned *Hebdomekontany*. He has also devised notations and techniques for mapping these scales onto keyboards of the extended Bosanquet family. These mappings are based upon identifying constant structures, where particular intervals or sets of intervals receive stable keyboard mappings: for example, the best representation of interval A/B will always be located by a (two-dimensional) keyboard distance of xy .¹³ CPS structures have been incorporated into instruments and compositions by Kraig Grady, Alison Monteith, DJW and others.

VIII. Further Modulations and Permutations

Wilson's most recent studies (Wilson, private communication) have returned to the subject of convergent-ratio number series, often series found in various diagonals of Pascal's triangle, to act as generators for scales. Such scales have the built-in property of consonant difference tones, and Wilson has been able to find sequences replicating aspects of mean-tone, pelog and slendro, as well as a variety of wholly new scale types. The relationship between Wilson's present work with convergent series and his initial insight with the scale tree is clearly not a trivial one, but the implications of the relationship remain far from completely realized.

IX. Some Futures

Several composers have elaborated speculative theories based on the use of alternative tuning systems to extend particular aspects of tonality. Barlow (1980), von Schweinitz, Tenney (1987), Sims (1988), and Zender (2003) deserve special mention, the latter three sharing advocacy of a 72tet system. (For examples of "atonal" approaches to 72tet, see Maneri and Van Duyn (1986) and Werntz (2001).) Blackwood (1985) has used the notion of a "recognizable diatonic" scale as the basis of evaluating equal tempered scales, and has composed *études* in all of

the temperaments between 13- and 24tet. Darreg (Darreg and McLaren 1993) composed a large number of works in an even larger variety of equal temperaments and with decidedly more eclectic approaches than Blackwood's diatonic model. It is more pressing here, however, to identify a series of musicians who have made rather more radical extensions of the tonal idea.

La Monte Young has composed instrumental and vocal works as well as sound environments based upon aggregates of tones related to one another as are partial tones from harmonic series. In general, the organizing principal used by Young in selecting his pitches is the avoidance of a given interval and its multiples (in Young's case, the avoided interval is usually the 5:4 major third and, by extension, all intervals involving 5 as a factor), with the simultaneous preference for other intervals, especially 3:2 fifths and 7:4 harmonic sevenths. His most recent works have included pitches drawn from the collection of tones related to one another as are the partial tones in octaves 512–1024 of a harmonic series. Precise perception of tonal relationship involving numerical relationships of such complexity is, in terms of present knowledge of psychophysics and the neuroscience of music, not possible with the perceptual *apparati* usually associated with apprehending musical pitches, but Young's works with their unique modes of presentation suggest that other mechanisms (e.g. those involving timbral and spatial perception) may be invoked in perceiving acoustic events of both great complexity and extended durations.

Chalmers (1993) has exhaustively catalogued historical and speculative tetrachordal divisions as well as a variety of techniques for using tetrachords as the basis of more complex pitch structures, in order, for example, to harmonize the essentially melodic structure of the tetrachord, or to use tetrachords as the basis for Partch-style diamonds or CPSs.

An impressive approach to the resources of 22tet from the perspective of tonality is presented by Paul Erlich (1998). In his paper, Erlich demonstrates the limits of diatonic models in the 22tet environment and proposes, instead, basing 22tet "tonality" on a 10-note basic subset. 22tet proves to be ill suited indeed to many patterns found in familiar diatonic musics, but very rich in harmonic possibilities of its own.

William Sethares (1998) has initiated discussion of the variety of possible relationships between tuning systems and timbral structures. In particular, he has devised a sophisticated and useful framework for constructing scales in which a full range of consonant and dissonant intervallic relationships are heard for a given timbre, or, conversely, for creating timbres in which the entire consonance/dissonance spectrum of a given scale is redistributed.¹⁴

Finally, the potential offered by combining tuning systems within a single work both successively and simultaneously is great and relatively unexplored. For example, following Wilson's scale tree model, a piece of music might begin in a given n -tet and modulate to another n -tet by changing the size of all instances of a given interval class. Thus, a piece in 5tet could modulate to 7tet simply by decreasing the size of fifths from 3/5 octave to 4/7 octave and further into 9tet by a decrease to 5/9 octave. Simultaneously, different tunings systems may be assigned to individual streams in a polyphonic ensemble with strategic use of common pitches or specific interval relationships between streams. DJW has composed studies in both of these directions.

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Notes

1. This should not be read to suggest that the author has any reservations against these approaches. In fact, works of Douglas Leedy, Lou Harrison and György Ligeti illustrate well the rich possibilities offered by historical tunings for new composition; while the informal uses of microtonality by

- a timbrally focused composer like Robert Erickson, members of the “New Complexity School”, the numerous microtonal works among the late “number” pieces of John Cage or in those scores of Alvin Lucier based upon audible beats between mistuned unisons show how even a naïve approach to pitch relationships can have interesting and wide-reaching tonal implications.
2. “Limit”, a term devised by Harry Partch, indicates the highest prime (prime limit) or odd (odd or Partch limit) factor participating in the ratios of a given tuning system.
 3. Note that pythagorean tuning is here excluded. In pythagorean tuning, a better approximation of the 5:4 major third is found as the octave-reduced sum of eight descending perfect fifths rather than the octave-reduced sum of four ascending perfect fifths. Thus, in pythagorean tuning, C–F₇–G is a more consonant triad than C–E–G, but this represents a functional convention that is clearly alien to common practice counterpoint and harmony. An extended pythagorean tuning, using the “schismatic” diminished fourth as a basic consonance in a polyphonic texture, is surprisingly under-explored and potentially a rich environment for new composition.
 4. By contrast, 31tet, described in the section on A. D. Fokker, offers excellent approximations through the 7-limit, and does so with significantly more conservative resources. A precise formalization of the consistency of a given scale was one of the initial achievements of the internet-based “Alternate Tuning List”. The Alternate Tuning List was originally hosted by the Center for Contemporary Music at Mills College. It is presently at: <http://groups.yahoo.com/group/tuning>
 5. It is unfortunate to learn, from Estrada (n.d), that: “Los archivos de don Augusto Novaro, creador de instrumentales y de teorías musicales, han quedado en manos de familiares nada empeñosos en propiciar la investigación.” Surviving in the public record are the cited theoretical text (Novaro 1927) and but a few fragments of composed music, including a small study on the syntonic comma, composed in 53tet.
 6. Ratios are here notated following the convention adopted by members of the internet-based “Alternate Tuning List”: rational intervals are indicated by a colon (1:1, 3:2, 5:4, etc.) while pitches fixed by a rational relationship to a given tonic are indicated by an oblique (1/1, 3/2, 5/4, etc.). In Partch’s usage, the “oblique” ratios are pitch classes and are always notated as improper fractions.
 7. The relationship between Partch’s theoretical and compositional projects is a complex one. It is important here to note only that his earliest compositions, while in an embryonic form of the just intonation he would eventually adapt, were rather more focused on having a sufficiently microtonal profile to transcribe effectively the intonation of natural speech. On the other hand, Partch’s later works were so dominated by idiophones, with their irregular spectra, that his tonal practice could become quite flexible with respect to his own theoretical ideas. (Partch himself even used the term “atonal” to describe portions of his own work.) If a listener or analyst wishes to locate “Partch the theorist” in the works of “Partch the composer”, it is therefore essential to look to those works in the middle of his output: “Eleven Intrusions”, “Dark Brother”, “Oedipus”, “Even Wild Horses”, “The Bewitched”.
 8. It is interesting that many composers have since followed Partch’s pattern, by assaying a major theoretical text regarding pitch relationships in advance of composition based on the new theoretical model. A parallel between Schoenberg’s *Harmonielehre* and Partch’s *Genesis* is far from fanciful.
 9. As noted earlier, Novaro had anticipated this structure. However, it appears that Partch’s source, if any, was in the work of the psychologist Meyer (1929), who proposed a cross-set-derived system, but a cross-set of a harmonic series multiplied by itself. Interestingly, Meyer, whose approach was strictly analytical, not speculative, accepted an (inconsistent) approximation of his system by 24tet.
 10. Partch never makes a claim for the perceptual equivalence of harmonic (otonal) and subharmonic (utonal) materials and, in fact, the extended association of arpeggiation with subharmonic chords in his scores suggests that Partch was engaged, if only subconsciously, in finding compositional solutions to the perceptual difficulties associated with dense subharmonic materials.
 11. Fokker appears to have precedence for the use of “chord multiplication” as later used by Pierre Boulez. Boulez’s own connections to microtonality have been rather obscured; early on, he was an associate of Wyschnegradsky, and the earliest versions of *Polyphonie X* and the second and fifth movements of *Le Visage Nuptial* initially involved the integral, serial use of quarter-tones (see Zeller 1995).
 12. Among the community of musicians working with CPS tuning systems, the pitches in a given CPS are most often treated as pitch classes, but this is not always the case. Further, it should be noted that the factors in a CPS are not restricted to primes and can, in fact, have any numerical value. Indeed, Wilson himself has frequently included composite numbers among his sets, and several musicians have worked with irrational values.

13. A good example of a keyboard mapping that does not reflect a constant structure is the mapping used by La Monte Young for his composition *The Well Tuned Piano*. In Young's mapping, the interval 7:4 is found at keyboard distances of a major sixth and a major seventh. A constant-structure mapping would have a single keyboard distance for such an interval, but Young's tuning makes this impossible, and indeed, an ascending succession of keys on Young's instrument will not necessarily produce an ascending sequence of pitches.
14. It is appropriate here to note that there is a voluminous and lively discussion in both musical and scientific circles over the issue of whether there exists a preference for stretched (and/or narrowed) octaves, the location of such a preference in physical or biological systems, and the ramifications of such a preference for actual music making in terms of tunings and timbres.