Steven Rings, *Tonality and Transformation* (Oxford: Oxford Studies in Music Theory, 2011)

CHAPTER One

# Intervals, Transformations, and Tonal Analysis

## Introduction

Though transformational theory is by now a familiar presence on the musicological landscape, ubiquitous in conference programs and theoretical journals, it remains a specialist subdiscipline within a specialist field, largely the province of initiates. Most music theorists have at least a casual acquaintance with transformational ideas, but only a handful actively pursue research in the area; for other music scholars (historians, for example), the theory is surely a closed book. As Ramon Satyendra has noted, this is due at least in part to the mathematical aspects of the approach, which he calls a "language barrier" that has inhibited "broad-based critique and commentary" (2004, 99). While that broad-based discussion has yet to emerge, the theory's reception among specialists has moved into a new critical phase, with certain of the method's foundational assumptions being held up to scrutiny on both technological and conceptual grounds—a sign of the theory's continuing vitality. But such revisions also raise a worry: as refinements to transformational methodologies become ever more recherché, the theory threatens to leave behind a host of scholars who never had a chance to come to terms with it in its most basic guise. This would be unfortunate, for transformational methods, even in their simplest applications, represent a style of music-theoretic thought of considerable power and richness, and one that is in principle accessible to a wide range of analytically minded musicians.

Thus, while this book is primarily about the application of transformational ideas to tonal phenomena, I hope it can also serve as an accessible general introduction to transformational theory. The present chapter presents an overview of the theory for the reader new to the approach (or for those who would like a refresher).[1] After a capsule summary in section 1.1, sections 1.2 and 1.3 serve as primers on the two main branches of transformational thought: generalized intervals and transformational networks. These sections introduce what we might call "classical" Lewinian intervals and transformations, as formulated in David Lewin's *Generalized Musical Intervals and Transformations* (hereafter *GMIT*), the foundational text in the field. They also survey recent criticisms of and revisions to Lewin's ideas. Each section includes a little model analysis of a tonal passage, the first by Bach, the second by Schubert. The analyses are meant to display the theory in action and to demonstrate its efficacy in illuminating aspects of tonal works, even before introducing the new technologies of this book. The analyses are again in a rather "classical" transformational idiom, adopting modes of interpretation common in the literature. This will allow us, in section 1.4, to contrast such transformational approaches with Schenkerian analysis.

## 1.1 Transformational Theory in Nuce

Transformational theory is a branch of systematic music theory that seeks to model relational and dynamic aspects of musical experience. The theory explores the manifold ways in which we as musical actants—listeners, performers, composers, interpreters—can experience and construe relationships among a wide range of musical entities (not only pitches). The formal apparatus of the theory allows the analyst to develop, pursue, and extend diverse relational hearings of musical phenomena. The theory articulates into two broad perspectives. One is intervallic, in which the subject "measures" the relationship between two musical objects, as a passive observer. The other is transformational, in which the subject actively seeks to recreate a given relationship in his or her hearing, traversing the space in question through an imaginative gesture.[2] The conceptual difference between intervals and transformations is subtle, and some recent theorists have sought to downplay it.[3] We will explore such matters in more detail later. For now we can simply note that the emphasis in both modalities is on the *relationships* between musical entities, not on the entities as isolated monads. Transformational theory thematizes such relationships and seeks to sensitize the analyst to them.

In both the intervallic and transformational perspectives, musical entities are members of sets, while the intervals or transformations that join them are members of *groups* or *semigroups*. We will discuss the italicized terms in the following section (definitions may also be found in the Glossary); readers need not worry about their formal meaning for the moment. Intervallic structures are modeled via *Generalized Interval Systems*, or GISes, which comprise a set of elements, a group of intervals, and a function that maps the former into the latter. Transformational relationships are modeled by *transformational networks*: configurations of nodes and arrows, with arrows labeled by transformations (drawn from some semigroup) and nodes filled with musical entities (drawn from some set).4 Any GIS statement may be converted into a transformational statement, a technological conversion that also implies (in Lewin's thought) a conceptual conversion from (passive) intervallic thinking to (active) transformational thinking.[5] The converse, however, is not true: there exist transformational statements that cannot be rendered in GIS terms. The transformational perspective is thus broader than the GIS perspective. For this reason, the term *transformational theory* is often used, as here, to encompass both modes of thought.

## 1.2 Intervals

### *1.2.1 GISes*

GIS statements take the form int(s, t) = i. This is a mathematical expression with formal content, which we will unpack in a moment. I would like first, however, simply to note that its arrangement on the page mimics a plain English sentence: it can be read from left to right as a formal rendering of the statement "The interval from *s* to *t* is i." We can understand GIS technology as an attempt to render explicit the conceptual structure underlying such everyday statements about musical intervals.[6]

Figure 1.1 will help us begin to explore that underlying conceptual structure. The figure shows our GIS formula again, now with its various components labeled. The italicized words indicate mathematical concepts. Here I will present informal definitions of these words, offering just enough information so that the reader understands their overall structure and can begin to appreciate their suggestiveness—both singly and in combination—as models for intervallic concepts. More detailed discussions of each term may be found in the Glossary.

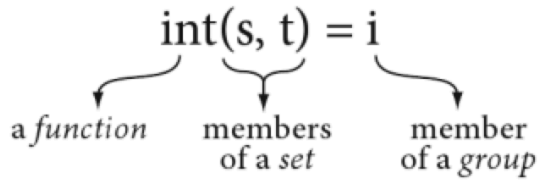


*Figure 1.1* A GIS statement with components labeled.

As the figure shows, the elements s and t are both members of a mathematical *set*. For present purposes, a set may simply be understood as a collection of distinct elements, finite or infinite. The elements are *distinct* in that none of them occurs more than once in the set.[7] Lewin calls the set that contains s and t the *space* of the GIS, which he labels S. The space S may consist of pitches, or pitch classes, or harmonies of a particular kind, or time points, or contrapuntal configurations, or timbral spectra—and so on. GISes thus extend the idea of interval to a whole host of musical phenomena, not just pitches; this is one of the senses in which they are "generalized." Note that the elements s and t are given in parentheses in the formula, separated by a comma. This indicates that they form an *ordered pair*: (s, t) means "s then t." The ordered pair (s, t) is distinct from (t, s). GISes thus measure *directed* intervals—the interval *from* s to t, not simply the undirected interval *between* s and t. For example, measuring in diatonic steps, the interval from C4 to D4 is different from the interval from D4 to C4: int(C4, D4) = +1, while int(D4, C4) = −1. This differs from some everyday uses of the word interval, in which we might say, for example, "The interval between C4 and D4 is a diatonic step." GISes do not model such statements, but instead statements of the form "The interval from C4 to D4 is one diatonic step up (i.e., +1 in diatonic space)" or "The interval from D4 to C4 is one diatonic step down (i.e., −1 in diatonic space)."

The element i to the right of the equals sign is a member of a *group*. Lewin calls the group of intervals for a given GIS IVLS. A group is a *set* (that is, a collection of distinct elements, finite or infinite) plus an additional structuring feature: an inner law or rule of composition that states how any two elements in the set can be combined to yield another element in the set. Lewin calls this inner rule a "binary composition," and we will follow that usage here.[8] Groups underlie a great many familiar conceptual structures. For example, take the set of all integers, positive, negative, and zero. As a set, this is simply an infinite collection of distinct entities: {…, −3, −2, −1, 0, 1, 2, 3, …}. But once we introduce the concept of *addition* as our binary composition, the set of integers coheres into a group, which we call "the integers under addition." Addition, as a binary composition, offers one way in which we can combine any two integers to yield another integer: given any two integers $x$ and $y$, $x + y$ will always yield another integer $z$. This is called the group property of *closure*: the composition of any two elements under the binary composition always yields another element in the same set.

Groups have three other properties. First, they contain an *identity element*, labeled $e$ (for the German word *Einheit*). The composition of $e$ with any other group element $g$ yields $g$ itself. In our group of integers under addition, the identity element is 0: 0 added to any integer $x$ yields $x$ itself. Further, for every element $g$ in a group there also exists an element $g^{-1}$ in the group such that when $g$ and $g^{-1}$ are combined $e$ is the result. The element $g^{-1}$ is called the *inverse* of $g$ ($g^{-1}$ is read "g-inverse"). In the group of integers under addition, the inverse of any integer $x$ is $-x$ (e.g., the inverse of 3 is −3, as the two of them added together yield 0, the identity element). Finally, composition within any group is *associative*. That is, given three group elements $f$, $g$, and $h$, then $(f \bullet g) \bullet h = f \bullet (g \bullet h)$.[9] To return once again to our example, addition of integers is clearly associative: for any three integers $x$, $y$, and $z$, $(x + y) + z = x + (y + z)$.

Mathematicians study groups primarily for their abstract structure, a structure that is suggested in its most basic terms by the four conditions outlined above (closure, existence of an identity, existence of inverses, associativity). The GIS formulation rests on the idea that intervals, at a very general level, have this same abstract structure—they are group-like. That is, the combination of any two intervals will yield another interval (closure). Any musical element lies the identity interval from itself (existence of identities). Given an interval i from s to t, there exists an interval from the to s that is the "reverse" of i—that is, $\mathrm{i}^{-1}$ (existence of inverses). Finally, we recognize that intervals combine associatively: given intervals i, j, and k, $(\mathrm{i} \bullet \mathrm{j}) \bullet \mathrm{k} = \mathrm{i} \bullet (\mathrm{j} \bullet \mathrm{k})$.[10]

Note that these abstract, group-like characteristics do not encompass certain common ideas about intervals. For instance, there is nothing in the four group conditions that says anything about *direction* or *distance*—two attributes often attributed to intervals. This is one area in which the GIS concept has recently been criticized.[11] Though it is tempting to interpret the numbers that we use to label group elements—like the integers +1, −5, and so on in a diatonic or chromatic pitch space—as representative of distances and directions (treating +1 as "one step up," and −5 as "five steps down," for example), those interpretations are not inherent in the abstract structure of the group. That is, the group itself, qua abstract algebraic structure, knows nothing of "one step up" or "five steps down." Instead, it knows only about the ways in which its elements combine with one another according to the properties of closure, existence of an identity and inverses, and associativity. We can conclude two things from this: (1) GISes are formally quite abstract, and may not capture everything we might mean by *interval* in a given context; and (2) not all of the intervals modeled by GISes need to be bound up with the metaphor of distance.[12] While the distance metaphor will likely be quite comfortable for most readers in discussions of pitch intervals, it nevertheless will feel inappropriate in other GIS contexts—for example, when one is measuring intervals between timbral spectra, or between contrapuntal configurations in triple counterpoint (à la Harrison 1988).[13] Indeed, the metaphor of distance will not always feel apt in the primary GIS in this book, introduced in

Chapter 2.[14] The GIS concept thus abstracts away from notions of distance, generalizing the idea of interval to relational phenomena in which the distance metaphor might not be appropriate.

We are nevertheless free to add notions of distance and direction to our interpretations of GIS statements, if so desired. Dmitri Tymoczko (2009), Lewin's main critic on this front, has indicated how distance may be reintroduced into a GIS by adding a metric that formally ranks the distances between all pairs of elements in the space of the GIS. In practice, this usually amounts to reading numeric GIS intervals—like +1, –5, and so on—as indicators of distance and direction, in the usual arithmetic sense (with –5 larger than +1, and proceeding in the opposite direction). We will not employ Tymoczko's distance metric explicitly in our formal work in this study, but we will often rely on the idea implicitly, whenever we wish to interpret intervals as representing various distances.[15]

Groups, for all of their abstraction, nevertheless remain suggestive as a model for generalized intervals. This is because each group has an underlying abstract structure—or, we might say figuratively, a certain "shape." This shape is determined by the number of elements in the group and the various ways they combine with one another (and with themselves). A group, for example, may be finite or infinite. It may contain certain patterns of smaller groups (called *subgroups*) that articulate its structure in various ways. A group may be commutative or noncommutative: two group elements $f$ and $g$ commute if $f \bullet g = g \bullet f$; in a noncommutative group this property does not always hold.[16] If two groups are *isomorphic* they have the same abstract structure. A GIS inherits the particular structural characteristics of its group IVLS. One way to think of this is that a given intervallic statement in Lewin's model inhabits a certain conceptual topography—a sort of landscape of intervallic relationships given shape by the structure of the group IVLS. Different types of interval may thus inhabit considerably different conceptual topographies, based on the structure of their respective groups (e.g., whether the groups are finite or infinite, commutative or noncommutative, articulated into subgroups, and so forth). This suggests that the intervallic experiences corresponding to such intervals have certain crucial differences in structure, differences embodied in the structures of their respective groups. Such differences are often interpretively productive—the formalism encourages us to attend to them carefully, as we pursue and extend any given intervallic statement within a particular analytical context.

Thus far in our survey of GIS structure, we have two separate collections that are as yet entirely independent: the space S of musical elements and the group IVLS of intervals. We have not yet shown how various intervals in IVLS can be understood to span pairs of elements in S. The leftmost element in the GIS formula, int, provides that connection. As indicated in Figure 1.1, int is a *function* or *mapping* (the two words are synonymous for our purposes). A function from a set $X$ to a set $Y$ sends each element $x$ in $X$ to some element $y$ in $Y$. Drawing on familiar schoolbook notation, we write $\mathrm{f}(x) = y$ to refer to the action of function f sending element $x$ to element $y$. Note how the schoolbook orthography exactly matches the layout of our GIS statement: compare $\mathrm{f}(x) = y$ and int(s, t) = i. The element $x$ in the statement $\mathrm{f}(x) = y$ is called the *argument* of the function, and the element $y$ is the *image* or *value* of the argument $x$ under f. The set $X$ of all arguments is called the *domain* of the function, while the set of all images in $Y$ is called the *range*.

The domain for our function int in a GIS is not simply the space of musical elements S itself, but the set of all *ordered pairs* of elements from S. Our arguments are thus not single elements from S, but ordered pairs of the form (s, t). We can see this by comparing again our two statements $f(x) = y$ and int(s, t) = i; the ordered pair (s, t) is "in the role of $x$" in our GIS statement, not simply some single element from S. The set of all ordered pairs (s, t) is labeled S × S and is called "S cross S" or the *Cartesian product* of S with itself. The function int sends each ordered pair to an element in IVLS. So, formally speaking, int maps S × S into IVLS.

As an example of how all of this works, let us take the two GIS statements suggested above, measuring the interval from C4 to D4 (and the reverse) in diatonic steps:

$$\text{int}\,(C4, D4) = +1$$
$$\text{int}\,(D4, C4) = -1$$

In both GIS statements, the space S consists of the conceptually infinite collection of diatonic "white-note" pitches (NB, not pitch classes). The group IVLS is the integers under addition, our familiar group discussed above. The mapping int sends every ordered pair of diatonic pitches to some element in the group of integers. It sends the ordered pair (C4, D4) to the group element +1 in IVLS, modeling the statement "The interval from C4 to D4 is one diatonic step up." It then sends the ordered pair (D4, C4) to a different element in IVLS, –1, modeling the statement "The interval from D4 to C4 is one diatonic step down."[17] The two intervals, +1 and –1, are inversionally related, indicating that int(C4, D4) followed by int(D4, C4) will leave us back where we started, with an overall interval of 0, as intuition dictates. This relates to a general condition for a GIS, Condition (A): given any three musical elements r, s, and t in S, int(r, s)int(s, t) = int(r, t). That is, the interval from r to s, plus the interval from s to t, must equal the interval from r to t. Thus, in our example int(C4, D4)int(D4, C4) = int(C4, C4) = 0. Or int(C4, D4)int(D4, E4) = int(C4, E4) = +2. A second condition, Condition (B), states that, for every musical element s in S and every interval i in IVLS, there exists exactly one element the in S such that int(s, t) = i.[18] Again, a musical context makes the condition clear: let the element s be the note C4 and the interval i be "one diatonic step up." Within the set of all diatonic "white-note" pitches, there is of course only one pitch that lies "one diatonic step up" from C4, that is, D4. These two conditions lend a certain logical tightness to GIS structure, providing only one interval between any two musical elements within a GIS.[19] This property is called *simple transitivity*. As a result of the two conditions, in any GIS there will always be exactly as many elements in S as there are intervals in IVLS. For example, in the GIS corresponding to pitch classes in 12-tone equal temperament, there are 12 elements in S (the 12 pitch classes) and 12 intervals in IVLS (the integers mod 12).

We now turn to some philosophical and methodological matters raised by GISes.

### *1.2.2 GISes and Cartesian Dualism*

The cumbersome structure of GIS statements enacts aspects of Lewin's critique of Cartesian dualism. Note that the main action modeled in a GIS is the action carried out by the mapping int. It is int that carries us "across the equals sign" from the left-hand side to the right-hand side of the formula int(s, t) = i. The active nature of int is especially evident if we use an arrow notation to rewrite the function. The schoolbook function f($x$) = $y$ may also be written $x \xrightarrow{f} y$, showing that the function f takes $x$ to $y$. Similarly, we can rewrite the GIS function int(s, t) = i as (s, t)$\xrightarrow{\text{int}}$i showing that int takes (s, t) to i. This notation makes visually vivid the fact that int is the primary action involved in a GIS statement, capturing the act of pairing two musical elements with an interval. The relevant thought process might be verbalized thus: "I just heard a C4 and now I hear a D4; the interval from the former to the latter is one diatonic step up."

Lewin characterizes this attitude as Cartesian because it is the attitude of someone passively calculating relationships between entities as points in some external space. The action of passively measuring is embodied by the mapping int itself. One might think of int as analogous to pulling out some calculating device and applying it to two musical entities "out there" to discern their intervallic relationship. The action in question is *not* one of imaginatively traversing the space from C4 to D4 in time, construing and experiencing a musical relationship along the way. The GIS formula is further like the Cartesian mindset in that it exhibits a certain fracturing of experience, a conceptual split between musical elements (the space S), musical intervals (the group IVLS), and the conceptual action (int) that relates the two. The cumbersome nature of the GIS formalism—with its three components (S, IVLS, int), all of which need to be coordinated, and with the action int placing the musical "perceiver" in an explicit subject-object relationship vis-à-vis the music being "perceived"—thus encodes aspects of the Cartesian split between *res cogitans* and *res extensa*, a familiar trope in Lewin's writings.[20]

We should not conclude from this that GISes are "bad" and that we should not use them in our analytical and theoretical work. Lewin himself continued to find intervallic thinking fascinating and productive long after *GMIT*, as a historical phenomenon, a theoretical/formal problem, and a mode of generating insights into musical works.[21] In *GMIT* itself he also observes certain ways in which transformational thinking is "impoverished" in comparison to intervallic thinking (*GMIT*, 245–46). In short, despite the fact that the GIS formalism enacts the Cartesian problematic that Lewin so eloquently criticized, it is still a productive and suggestive technology in many theoretical and analytical contexts.[22] GIS models will play an important role in this book.

### *1.2.3 Intervallic Apperceptions*

Lewin often refers to *intuitions* in his writings about intervals and transformations, but he never says exactly what he means by the word. It will be valuable for us to spend a little time here thinking about the matter, as the questions that it raises bear directly on the relationship between transformational technology and musical experience.

Though Lewin gives us no clear definition of what he means by intuitions, we can infer two crucial characteristics of the term as he uses it in his writings:

(1) His intuitions are culturally conditioned.

(2) They may be sharpened, extended, or altered through analytical reflection.

Lewin states (1) explicitly: "Personally, I am convinced that our intuitions are highly conditioned by cultural factors" (*GMIT*, 17). By "cultural factors," Lewin seems to mean not only differences between various world cultures—though he certainly does mean that—but also historical cultural differences within the history of European art music. For example, a sixteenth-century musician conditioned by ideas about modes, hexachordal mutation, *mi-contra-fa* prohibitions, and so forth would have different *intuitions* about a given musical passage—say in a motet by Palestrina—than would a modern musician conditioned by ideas about keys, diatonic scales, tonal modulation, and so forth.[23] The modern listener can of course seek to develop hexachordal hearings of the music in question, but to that extent—and this leads to characteristic (2)—the listener will be modifying her or his intuitions (à la Lewin) by analytical intervention. The general pertinence of characteristic (2) to Lewin's thought is manifest throughout his writings, as theoretical structures of various kinds are brought to bear on various musical experiences, sharpening, extending, or altering those experiences in diverse ways. Indeed, Lewin's entire analytical project can be understood as a process of digging into musical experience and building it up through analytical reflection. Stanley Cavell, paraphrasing Emerson, provides a very suggestive wording that we can borrow for the idea: such work involves a reciprocal "play of intuition and tuition," or, even more suggestively, it is a project of "providing the tuition for intuition."[24]

Lewin's intuitions are special in the degree to which they reflect the influence not only of broad cultural and historical conditioning, but also of theoretical concepts and other discursive constructions.[25] I thus prefer to think of such "intuitions" as *apperceptions*: perceptions that are influenced by past experience and may involve present reflection.[26] The second clause makes clear that such experiences are responsive to current analytical contemplation: a GIS or transformational statement need not be a report on some prereflective experience, but might instead help to shape a new experience (an apperception), or alter an old one, through analytical mediation. The word intuition, by contrast, runs the risk of naturalizing GIS and transformational statements, treating them as unmediated reports on prereflective (or at least minimally reflective) experience. This risk is especially evident when a given statement is made seemingly universal by locutions such as "when hearing music *x*, we [NB] have intuition *y*"—a rhetorical device that occurs with disconcerting frequency in Lewin's writings. By hewing to the word apperception, I instead hope to make clear that the sorts of experiences explored in this book are by no means universal, and will be strongly shaped not only by one's cultural background and historical context, but also by the concrete particulars of present analytical engagement.

### *1.2.4 GISes: Formal Limitations*

As noted above, the abstract nature of GISes allows them to model a wide array of intervallic phenomena via algebraic groups. Yet, despite this abstraction, GISes are not as general as they might at first appear, nor are they applicable to all musical situations. GISes, for example, cannot model intervals in musical spaces that have a boundary or limit. Consider an example that Lewin himself raises: S is the space of all musical durations measured by some uniform unit. This space has a natural limit: the shortest duration lasts no time at all—there is no duration shorter than it. Now imagine that we choose to measure the interval from duration s to duration t in this space by subtracting s from t (IVLS would then be the integers under addition). For example, if s is 6 units long and t is 4 units long, the interval from s to t is 4–6 = –2. Formally, int(s, t) = int(6, 4) = –2. Now recall the Condition (B) for a GIS: given any element s in S and any i in IVLS, there must exist some the in S such that int(s, t) = i. Let us now set s = 0 and i = –2. There exists no the in S such that int(0, t) = –2. Such a the would be 2 units shorter than no time at all. As Lewin himself notes, this is an absurdity (*GMIT*, 29–30). Thus, the given musical space of durations under addition, though it is musically straightforward, cannot be modeled by a GIS. Similar problems arise with any musical space that has a boundary beyond which no interval can be measured.

This relates to a more general limitation. Given Lewin's definition, any interval in a GIS must be applicable at all points in the space: if one can proceed the interval i from s, one must also be able to proceed the interval i from t, no matter what i, s, and t one selects. This limits GISes to only those spaces whose elements all have uniform intervallic environments. The vast majority of familiar musical spaces do have this property. For example, in the space of chromatic pitches, one can move up or down from any pitch by +1 semitone or –1 semitone. By extension, one can theoretically move up or down from any pitch by +*n* semitones or –*n* semitones, for any integer *n*. This is so even when the result would be too high or too low to hear—the space is still in principle unbounded.[27] Similarly, in modular spaces, such as the space of 12 pitch classes, or the space of seven scale degrees, every element inhabits an identical intervallic environment. Neo-Riemannian spaces are also uniform in this sense: one can apply any neo-Riemannian transformation to any major or minor triad. Nevertheless, there do exist spaces that do not have this uniform quality, such as the durational space outlined above, or any number of voice-leading spaces that are better modeled geometrically (as discussed in Tymoczko 2009). Thus, despite their generalized qualities, GISes are not as broad in scope as they might initially appear to be: they only apply to uniform intervallic spaces.[28]

Tymoczko (2009) raises another important criticism of GISes: they do not admit of multiple, path-like intervals between two entities. We will return to this important criticism in section 2.3, in which I will integrate Tymoczko's path-like conception into the GIS introduced in that chapter. Tymoczko also objects that GISes do not model entities such as "the interval G4→E♭4" at the opening of Beethoven's Fifth Symphony. Instead, a given GIS would model the interval from G4 to E♭4 as an instance of a more general intervallic type that applies throughout the space: for example, as a manifestation of the interval "a major third down.” Such an interval could obtain between any other pair of major-third-related elements in the space, say F4→D♭4, or B6→G6. More generally, unlike Tymoczko's "interval G4→E♭4," intervals in a GIS are not defined by their endpoints, but by the relationship the listener or analyst construes between those endpoints, a relationship that is generalizable apart from the endpoints in question. The construing of that relationship is modeled by the statement int(s, t) = i, which produces generalized interval i as output. Tymoczko's formulation provides a different and useful perspective, focusing more attention on the concrete endpoints of a specific interval (s and t), and less on the ways in which a listener or analyst might construe the relationship between those endpoints as some general interval-type i. But an attractive aspect of GIS theory is lost in the process, to which we now turn.

### *1.2.5 GIS Apperceptions and Intervallic Multiplicity*

GIS technology is responsive to the fact that one will be inclined to experience an interval from G4 to E♭4 in diverse ways based on the musical context within which one encounters those pitches. There is thus no single "interval from G4 to E♭4." Imagine the succession G4 →E♭4 in: (1) the opening of Beethoven's Fifth; (2) a serial work by Schoenberg; (3) an octatonic passage by Bartók; (4) a pentatonic passage by Debussy (or, for that matter, in a Javanese gamelan performance in slendro tuning). These diverse contexts suggest the pertinence of various GIS apperceptions for the G4→E♭4 succession. In the Beethoven, the pitch topography is diatonic, and the GIS might be any one of a number of diatonic GISes (pitch-based or pitch-class-based).[29] The interval in Schoenberg would likely suggest a chromatic GIS, while in Bartók it would evoke an octatonic GIS, and in Debussy (or the gamelan performance) a pentatonic GIS; any one of these GISes could be pc- or pitch-based. The various GISes capture the ways in which one's apperceptions of the G4→E♭4 succession might vary in response to its diverse musical/stylistic contexts.

This is a rather obvious instance of what we might call "apperceptive multiplicity "in intervallic experience. Less obvious, perhaps, is GIS theory's insistence on apperceptive multiplicity when confronting a *single* interval in a single musical passage. This suggests that the interval in question can inhabit multiple musical spaces at once. Lewin puts it somewhat more strongly than I would: "we do not really have one intuition of something called 'musical space.' Instead, we intuit several or many musical spaces at once" (*GMIT*, 250). Per the discussion in section 1.2.3 above, I would rephrase this as "we can conceive of a given interval in several different conceptual spaces when we are in the act of analytical contemplation. Those different conceptions can subtly change our experience of the interval, leading to new musical apperceptions."[30]

### *1.2.6 Vignette: Bach, Cello Suite in G, BWV 1007, Prelude, mm. 1–4*

Figure 1.2(a) shows the first two beats of the Prelude from Bach's Cello Suite in G major, BWV 1007. An arrow labeled i extends from the cello's opening G2 to the B3 at the apex of its initial arpeggio. Figures 1.2(b)–(d) model three intervallic conceptions of i, situating it in different musical spaces.

Figure 1.2(b) models i as an *ascending tenth*. This suggests the context shown on the staff: B3 is nine steps up the G-major diatonic scale from G2. The figure shows this by placing in parentheses the elements that i "skips over" in the space S of the relevant GIS. S in this example comprises the elements of the (conceptually infinite) G-major diatonic pitch gamut, and IVLS is our familiar group of integers under addition, hereafter notated $(\mathbb{Z}, +)$.[31] Given two diatonic pitches s and t in G major, int(s, t) in this GIS tells us how many steps up the diatonic G-major gamut the is from s. The figure thus models the GIS-statement int(G2, B3) = +9.[32]

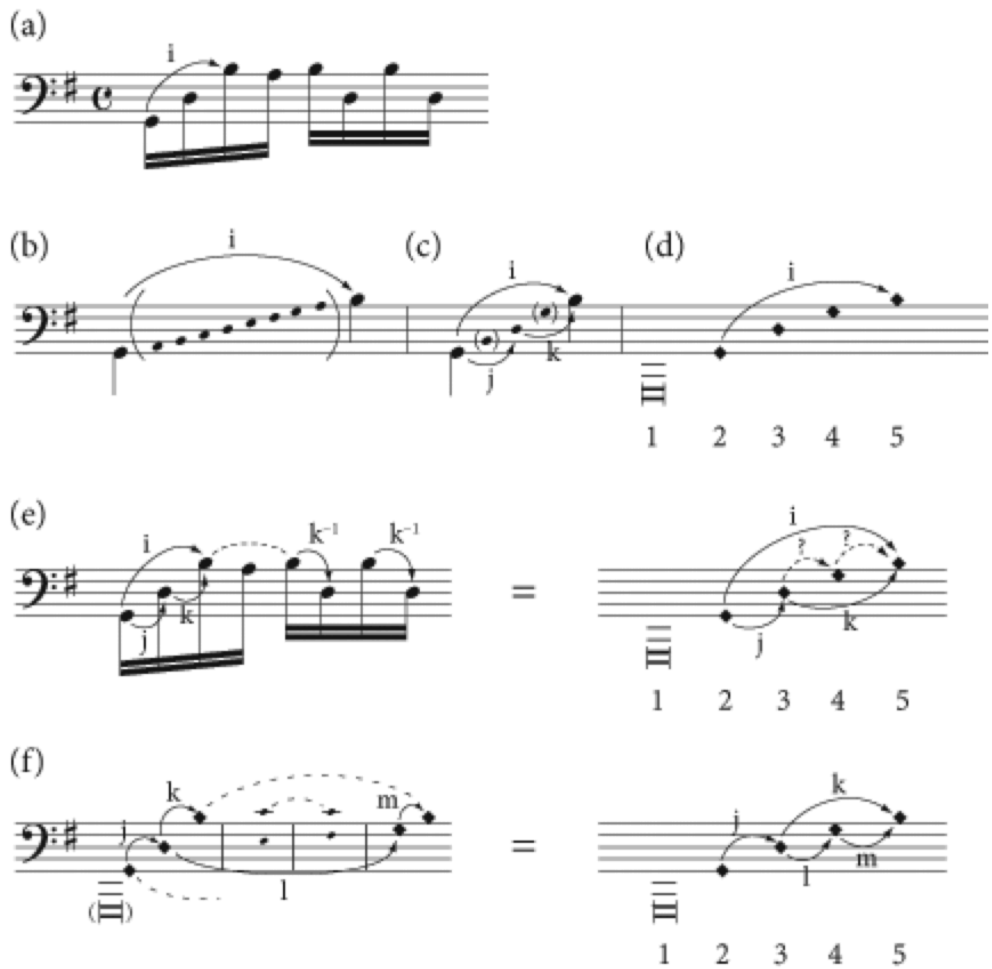


*Figure 1.2* Bach, Prelude from the first suite for solo cello, BWV 1007: (a) the music for beats one and two, with one interval labeled; (b)–(f) various GIS perspectives on that interval.

The GIS of 1.2(b) does not do full justice to the harmonic character of i. If we say that i is a *tenth*, we are likely not thinking primarily about a number of steps up a scale, but about a privileged *harmonic* interval. Figure 1.2(c) provides one harmonic context for i, depicting it as spanning elements in a *G-major arpeggio*. Our space S no longer consists of all of the elements of the G-major diatonic gamut, but just those pitches belonging to the (conceptually infinite) G-major triad, that is: {…$G_1$, $B_1$, $D_2$, $G_2$, $B_2$, $D_3$, $G_3$, $B_3$, $D_4$, $G_4$,…}. $G_2$ is *adjacent* to $B_2$ in this space, as is $B_2$ to $D_3$, and so on.[33] In this space, $B_3$ is "four triadic steps up" from $G_2$: that is, int($G_2$, $B_3$) = +4.[34]

Note that the $D_3$ in the opening gesture divides i into two smaller intervals, labeled j and k on the figure; both are "skips" of +2 in the GIS. This arpeggio space is highly relevant to the historical and stylistic context of the prelude, which imitates the French lutenists' *style brisé*. The intervals available to the *style brisé* lutenist within any given harmony are exactly those of the present GIS.

Figure 1.2(d) invokes a different harmonic space, one of *just* intervals in which i is the ratio 5:2. This model is suggestive, given the spacing of Bach's opening arpeggio: the $G_2$–$D_3$–$B_3$ succession corresponds to partials 2, 3, and 5 in the overtone series of G1. (In a more historical-theoretic vein, we might say that the notes project elements 2, 3, and 5 of a Zarlinian *senario*.) The group IVLS here differs in algebraic structure from those in Figures 1.2(b) and (c): it is the positive rational numbers under multiplication, not the integers under addition. This suggests that the harmonic interval of 5:2 inhabits a considerably different "conceptual topography" than do our stepwise (and additive) intervals of +9 and +4 in 1.2(b) and (c). We can sense that difference in topography when we recognize that, in the arpeggio GIS of 1.2(c), the interval from $G_2$ to $D_3$ is the same as the interval from $D_3$ to $B_3$: that is, both represent an interval of +2. In the just ratio GIS of 1.2(d), however, the intervals are different: int($G_2$, $D_3$) = 3:2 while int($D_3$, $B_3$) = 5:3. The difference registers the acoustic distinction between a just perfect fifth and a just major sixth. The resonant, partial-rich open strings of $G_2$ and $D_3$ with which the arpeggio begins strengthen the relevance of the just-ratio GIS here.[35]

Figure 1.2(e) shows the articulation of i into its two subintervals, again labeled j and k, as in 1.2(c). While both j and k were "skips" in 1.2(c), in 1.2(d) only k represents a "skip" in the overtone series above $G_1$; j connects two adjacent elements in the series.[36] Bach emphasizes the "gapped" interval k, repeating it twice, as $k^{-1}$, in the second half of the bar. This calls attention to the "missing" $G_3$ partial 4 in the overtone series (note the question-marked dotted arrows on the example's right side). As Figure 1.2(f) shows, this $G_3$ does eventually arrive in m. 4, at the close of the movement's opening harmonic progression. The $G_3$ bears a considerable tonal accent as a pitch that completes several processes set in motion in the work's opening measures. Note that the interval from $D_3$ to $G_3$—labeled l in the example—is filled in by step. This stepwise motion is the first concrete manifestation of the scalar GIS-space from Figure 1.2(b), now explicitly coordinating that scalar space with an interval from the harmonic spaces of 1.2(c)–(e). In fact, by the end of m. 3, $G_3$ is the only note that has not been heard in the diatonic G-major gamut from $D_3$ to $C_4$—it has thus been "missing" in both the scalar and harmonic conceptual spaces; its arrival fills a notable gap.

One could invoke other GIS contexts for i as well. One could model i as spanning the interval from ˆ1 to ˆ3 in an abstract scale-degree space, or joining root and third of the tonic harmony (the ideas are related, but not identical). Many other intervallic contexts for i are possible as well, but not all of them are relevant to the opening bar of Bach's prelude. For example, one could conceive i to extend up 16 semitones in a chromatic pitch gamut. This is a somewhat strained understanding within the context of m. 1, which as yet explicitly invokes no such chromatic division of pitch space. Such a space *is* invoked, however, at the work's climax in mm. 37–39, via the cello's chromatic ascent to $G_4$, the work's apex. Here it is very easy to hear the interval spanned from $D_3$ in m. 37 to $G_4$ in m. 39 in terms of steps in a chromatic gamut, and to coordinate the steps in this chromatic GIS with those in other diatonic and harmonic GISes relevant to the music in these bars.

Such an analysis could continue, modeling other notable intervallic phenomena in the prelude and exploring their interactions. For present purposes, it is important merely to note the style of the analysis, particularly its focus on multiple intervallic interpretations of single musical gesture.

## 1.3 Transformations

### *1.3.1 The "Transformational Attitude"*

As already noted, the transformational model represents a shift in perspective from the GIS view of the passive, outside observer "measuring intervals" to that of an active participant in the musical process. As Lewin puts it in one of his most frequently quoted passages,

> instead of regarding the i-arrow on figure 0.1 [an arrow labeled i extending from a point s to a point t] as a measurement of extension between points s and the observed passively "out there" in a Cartesian *res extensa*, one can regard the situation actively, like a singer, player, or composer, thinking: "I am at s; what characteristic transformation do I perform to arrive at t?" (GMIT, xxxi)

Lewin elsewhere dubs this the "transformational attitude," and it has become a familiar part of the interpretive tradition of transformational theory. It is a subtle and somewhat elusive concept; I will offer my own gloss on the idea and its relevance to certain acts of tonal hearing in section 3.2.1. For now, the reader may simply conceive of transformational arrows as goads to a first-person experience of various gestural "actions" in a musical passage, actions that move musical entities or configurations along, or that transform them into other, related entities or configurations.

While formal statements in GIS theory take the form of int(s, t) = i, formal statements in transformational theory are expressed using *transformational graphs and networks*. A transformational network is a configuration of nodes and arrows whose nodes contain elements from some set S of musical elements (analogous to the set S of elements in a GIS) and whose arrows are labeled with various transformations on S. A transformational graph resembles a transformational network in all respects but one: its nodes are empty.

### *1.3.2 Transformations and Operations*

A transformation on S is a *function* from S to S itself: that is, a mapping that sends each element in S to some element in S itself. We have already encountered functions in the GIS discussion above, with the function int. The transformations and operations in a transformational graph or network are also functions, but rather than mapping pairs of elements to intervals (as int does in a GIS), they act directly on single musical entities, transforming them into each other. Before exploring how this works in practice, it will be valuable to distinguish between a *transformation* and an *operation*.

Let us define S as the seven diatonic pitch classes in C major, that is, S = {C, D, E, F, G, A, B}. We now define a transformation on S that we will call "resolve to C," abbreviated ResC. This transformation sends every element in S to the element C. ResC is indeed a function from S to S itself: it takes as input each element of S, and returns as output an element of S. We can represent it by a mapping table, like that shown in Figure 1.3(a).[37] Figure 1.3(b) shows the mapping table for another transformation on S, which we will call Step: it moves each element in S up one diatonic step.

Both of these transformations can be conceived as idealized musical actions. But it is only when we consider the entire mapping table that we get a full sense of just what these actions *are*. To see this, consider the fact that both transformations have the same effect on the note B: they both map it to C. At this local level, the transformations appear to be indistinguishable. But if we perform the same actions elsewhere in the space, their differences emerge. For example, Step maps D to E, but ResC maps D to C; and Step maps E to F, while ResC maps E to C; and so on. It is only in this broader context that we can see that Step raises pitches by one step, while ResC resolves notes to C. These two actions have the same *effect* when applied to B, but the specific kinetics they imply are different—ResC suggests a gravitational centering on C, or an action that yields to such gravitation, while Step suggests a more neutral, uniform motion of single-step ascent anywhere in the space.

Step also differs from ResC in a more formal way. Every element from S appears on the right-hand side of the table for Step (Fig. 1.3(b)), while only the element C appears on the right-hand side of the table for ResC (Fig. 1.3(a)). While both ResC and Step are transformations, Step is a special kind of transformation that we will call (after Lewin) an *operation*: an operation is a transformation that is *one-to-one* and *onto*.[38] If a transformation is one-to-one and onto, every element in the set appears once and only once as the "target" for an arrow in the relevant mapping table—in other words, on the right-hand side of Figure 1.3(b). Operations thus have inverses: one can "undo" any operation simply by reversing the arrows in its mapping table. Thus, we can define $\text{Step}^{-1}$, as shown in Figure 1.3(c); as the table indicates, $\text{Step}^{-1}$ moves each element in S one diatonic step down. We cannot, however, define an inverse function $\text{ResC}^{-1}$. As shown in Figure 1.3(d), if we reverse the arrows in the mapping table for ResC, only the note C appears "at the beginning of the arrows" in the table (now on the right-hand side). Functions must be defined on all elements of their domain, but $\text{ResC}^{-1}$ is not defined on all of the notes in S; for example, it is not defined on D, as D does not appear anywhere at the beginning of an arrow on the table for $\text{ResC}^{-1}$. Furthermore, $\text{ResC}^{-1}$ is not even well defined on C, as it seems to send that note to seven different places; a function must send each element it acts on to only *one* element. Thus, ResC has no inverse—it is a transformation, but not an operation.[39]

| (a) | (b) | (c) | (d) |
|---|---|---|---|
| ResC | Step | Step$^{-1}$ | ResC$^{-1}$? |
| C⟶C | C⟶D | C⟵D | C⟵C |
| D⟶C | D⟶E | D⟵E | D⟵C |
| E⟶C | E⟶F | E⟵F | E⟵C |
| F⟶C | F⟶G | F⟵G | F⟵C |
| G⟶C | G⟶A | G⟵A | G⟵C |
| A⟶C | A⟶B | A⟵B | A⟵C |
| B⟶C | B⟶C | B⟵C | B⟵C |

| (e) | | (f) |
|---|---|---|
| Step | Step | Step$^2$ |
| C⟶D⟶E | | C⟶E |
| D⟶E⟶F | | D⟶F |
| E⟶F⟶G | | E⟶G |
| F⟶G⟶A | = | F⟶A |
| G⟶A⟶B | | G⟶B |
| A⟶B⟶C | | A⟶C |
| B⟶C⟶D | | B⟶D |

*Figure 1.3* Mapping tables involving two transformations: ResC, a transformation that is not an operation (i.e., that is not one-to-one and onto), and Step, a transformation that *is* an operation.

Transformations and operations can combine with one another through a process called *composition of mappings*. This process is illustrated in Figures 1.3(e) and (f), which show how Step followed by Step yields a new operation, Step$^2$. The new operation is defined by combining the mapping tables in 1.3(e); removing the middle column yields the table in 1.3(f), which shows the action of Step$^2$. Through processes like this, sets of transformations and operations can be combined to yield groups, or group-like entities called *semigroups*, in which composition of mappings serves as the inner law, or binary composition. Since operations have inverses, they can combine into groups. (We remember that each element in a group must have an inverse.) We speak in this case about a "group of operations." Transformations that are not operations, however, *cannot* combine into groups, as they do not have inverses. They can instead combine into a more general structure called a *semigroup*. A semigroup is a set of elements with a binary composition like a group, but one that needs only to satisfy two of the four group properties: closure and associativity. A semigroup need not contain an identity element, nor does each element in a semigroup need to have an inverse. We thus speak of a "semigroup of transformations."

The structure underlying transformational systems is thus what mathematicians would refer to as a "semigroup action on a set" (or a "group action on a set" if a group of operations is involved). The set in question is a set S of musical elements (notes, harmonies, rhythmic configurations, etc.), representatives of which occupy the nodes of a transformational network. The semigroup of transformations then "acts" on this set, modeling certain musical behaviors that are performed directly on entities in S, transforming them one into another along the arrows of the network. The action modeled by our formalism has thus changed in a subtle way from the GIS perspective. There, the action was one of calculating, modeled by the function int, which matched an ordered pair of elements with an intervallic distance. The action enshrined by a transformational arrow, by contrast, is the active performance of some characteristic musical gesture, which transforms one musical element into another. There is no equivalent to int here: semigroup (or group) elements act directly on the musical entities themselves.

### *1.3.3 Between GISes and Transformation Networks*

There is nevertheless a "communication channel" between GISes and certain kinds of transformation networks. Specifically, any GIS statement can be refashioned into a transformational statement; such transformational statements can also be turned back into GISes. This process of translation, taking one from an intervallic perspective to a transformational perspective, is a central theme in GMIT, as Klumpenhouwer (2006) persuasively argues. A striking aspect of Lewin's project is the way in which the conceptual transition from intervallic to transformational thinking is mediated by the technology of his theory—the technological transformation from a GIS perspective to a transformational perspective enacts formally the conceptual transformation that Lewin wishes us to undergo as we switch from an intervallic (Cartesian, observational) mode of thought to a transformational (first-person, active) one. In the process, the action of measuring (int) disappears and is replaced by an imaginative musical gesture, which the analyst is urged to perform in his or her re-creative hearing.

While any GIS statement can become a transformational statement by the appropriate formal and conceptual translation, the reverse is not true: not just any transformational system can be refashioned back into a GIS. Only certain kinds of transformation statements are "GIS-able," or conceivable in intervallic terms. There are two requirements for such a conceptual shift from transformations back to intervals. First, the transformations in question must be operations. To see this, let us return to Figures 1.3(a) and (b). Note that we can conceive of the operation Step as "interval-like." First, there is an interval that we can associate with the distance traversed by Step, namely "up one step" in diatonic pc space. Second, Step is has an inverse, and is thus reversible, as we expect all intervals to be. We can thus reframe any transformational statement that we make using Step as a GIS statement, and vice versa. We cannot, however, develop an intervallic interpretation of ResC. First of all, it is very difficult to see how we could conceive of a *single* interval that would correspond to the action traced by all of the arrows in Figure 1.3(a). In such a case, the interval from some white note to C would be the same as the interval from any other white note to C! Even if we could wrap our heads around such a curious idea, this putative interval would lack an inverse, thus failing the basic requirement that all intervals should be reversible. In short, there are certain musical "actions" we can conceive of performing that cannot be interpreted intervallically; these are the actions modeled by transformations that are not operations.

Second, in addition to the requirement that a transformational graph or network must include only operations to be interpreted in GIS terms, that group of operations must act on the elements in the space S in a particular way, which is called *simply transitive* (an idea that already arose in our discussion of GISes). A group acts simply transitively on a set if, given any two elements *a* and *b* in the set, only one element *g* in the group takes *a* to *b*. Simple transitivity will not be a property of all transformation graphs or networks, even if they include only operations. Consider, for example, a transformation network with node contents drawn from the set of 12 chromatic pcs, and arrow labels bearing a mixture of atonal transpositions (T*n*) and inversions (I*n*). (Klumpenhouwer networks are familiar instances of this kind of network.) The T*n*/I*n* group does not act simply transitively on the 12 pcs: given any two pcs, there are always *two* operations in T*n*/I*n* that can take the first pc to the second: one transposition and one inversion. Ramon Satyendra clearly explains why such a non-simply-transitive situation conceptually resists translation into intervallic terms:

> When reckoning intervallic distances we intuitively expect unique answers. It is counterintuitive to describe the straight-line distance between the chair and the table as both two feet and three feet. By requiring that a musical system satisfy the simple transitivity condition we are assured that the interval formed between any two points in a musical space may be uniquely determined. If a system is not simply transitive, it becomes counterintuitive to shift between transformational and intervallic perspectives. For instance it is intuitive to say that both $T_3$ and $I_3$ transform C to $E^b$, but it is counterintuitive to think of the interval between C and E as *both* $T_3$ and $I_3$. (2004, 103)

Thus, GISes may be understood as the conceptual "flip side" of a particular kind of transformational system: one whose transformations are all operations that act simply transitively on the space S of the network. The translation from such a transformational system into a GIS is formally rather involved, and I will not run through the details here. But the basic idea is simple. One merely keeps the space S the same from the transformational network to the GIS, reinterprets the group of operations in the transformational network as the group IVLS in the GIS, and applies int so that pairs of elements and intervals match up in agreement with the original transformational actions.[40]

### *1.3.4 Vignette: Schubert, Piano Sonata, D. 664, mvt. ii, mm. 1–7*

Figure 1.4(a) shows the first seven measures of the slow movement from Schubert's Piano Sonata in A, D. 664. Figure 1.4(b) isolates and labels some three-note gestures of interest. X is the piece's *Hauptmotiv*—a falling figure first heard in mm. 1–2; Y is a one-bar gesture closely related to X, first heard in m. 5. Altered forms of both X and Y appear in the passage: X′ changes the intervallic structure of X slightly, and T(Y) is a transposition of Y.

At the right-hand side of the example, two cadential gestures are identified, one in the soprano (Cad) and one in the bass (BassCad). The phrase concludes in m. 7 with a quick recollection of X, marked x. We will be interested in the way these gestures are internally structured, as well as in the ways in which they are transformed into one another.

The network of Figure 1.5(a) models pitch relationships within and between X and X′. The space S from which the node contents are drawn is the set of diatonic pitches (NB) in D major; the transformations are steps up and down the diatonic pitch gamut, which we will represent by the integers: +*x* is *x* steps up the diatonic gamut; – *x* is *x* steps down.[41] X traverses three falling diatonic steps, from B4 to F#(-3), while X′ traverses four falling steps, from B4 to E4 (–4). Both X and X′ begin with B4 $\xrightarrow{-1}$ A4. Schubert's articulation makes these gestures vivid. The slurred appoggiatura from B4 to A4 underlies the –1 motion, pulling B4 forward to A4, and the three gently rebounding eighth notes that follow on A4 (staccato, and slurred together) lead forward to the motives' concluding pitches.[42] Schubert's calm repetition of the many X-related figures in the movement encourages us to attend closely to their evolving progress as the piece unfolds. (The "calm repetition" comes to seem unhealthily obsessive by the time of the climax in m. 42.)

Note that, despite the evident alteration of X's internal structure in X′, the gestural motives of –1 and –3 are retained in the latter. Yet –3 now acts as an "internal" transformation, rather than the transformation that spans the entire gesture, as in X. For its part, –1 remains in its original initiating position, linking the B–A appoggiatura that plays such a prominent role in the movement. Indeed, –1 is the most persistent melodic figure in the piece, initiating nearly every one of its thematic and motivic units. The dashed arrow in 1.5(a) shows the influence of this "step descent" on a slightly larger scale, as the agent that transforms X into X′: the bottom pitch F#4 of X is bumped down via –1 to produce the E4 that concludes X′.[43]


*Figure 1.4* Schubert, Piano Sonata in A, D. 664, mvt. ii, Andante: (a) mm. 1–7; (b) some gestures of interest in these bars.

Figure 1.5(b) shows transformational relationships within and between the two Y-forms. The initiating –1 from X and X′ remains. In Y it joins B4 and A4, as in the X-forms. In T(Y), however, it joins F♯4 and E4, the pitches connected by the dashed –1 arrow in 1.5(a), making explicit the connection between the –1 arrow linking X and X′ and the appoggiatura incipits of X and Y. The other two gestural arrows in the Y-forms reverse the remaining two gestures in X: while the latter contains –2 and –3, the Y-forms contain +2 and +3. The sense of a change of direction in the Y-forms is reflected in other parameters as well, as we will see presently.

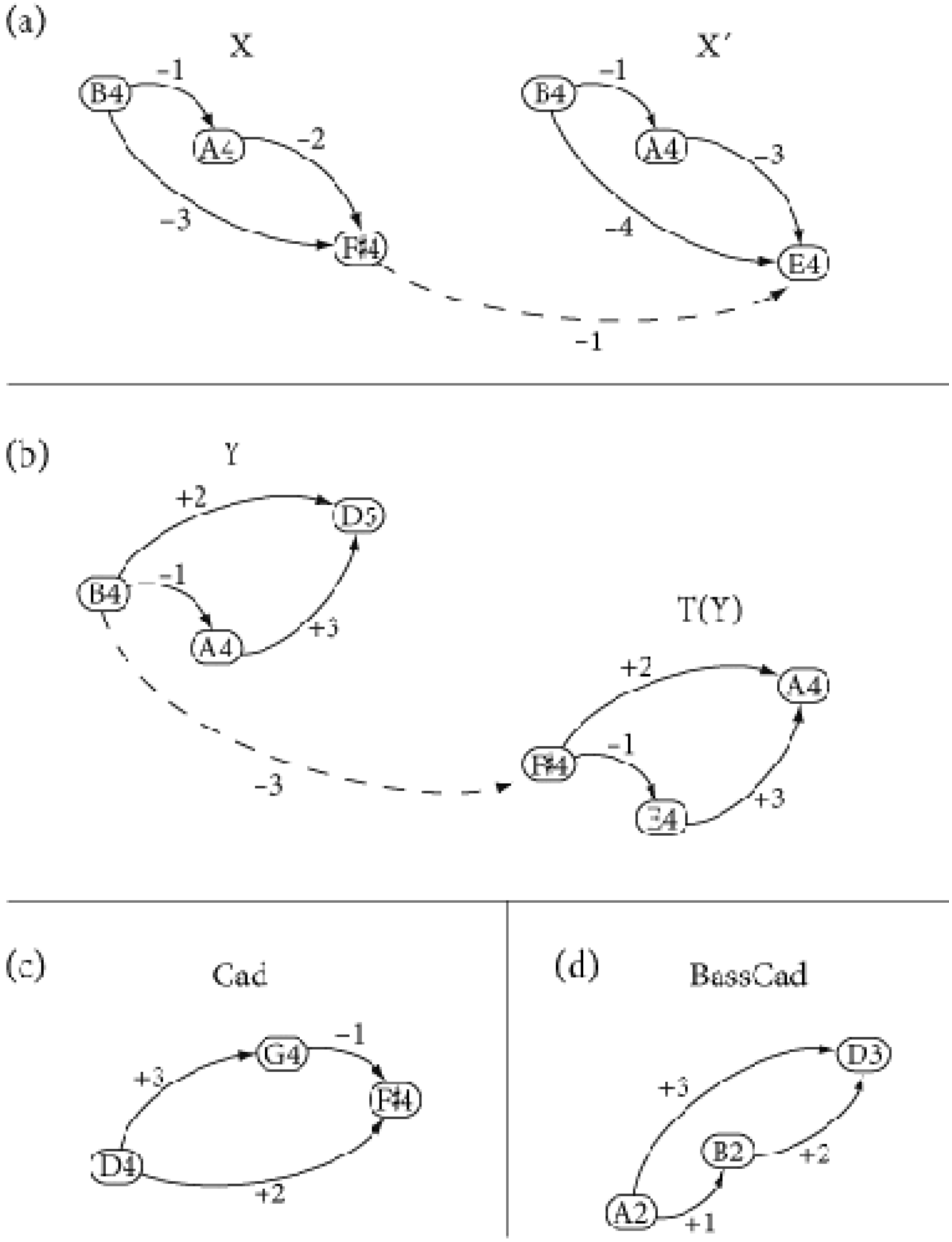


*Figure 1.5* Diatonic pitch-space networks of the gestures in Figure 1.4(b).

As shown by the dashed arrow in 1.5(b), −3 is the transformational agent that takes Y to T(Y). Like −1, which took X to X′, −3 is also present locally in both X and X′. The −3 arrow in X connects the same two elements connected by the dashed −3 arrow in 1.5(b): B4 and F♯4. The dashed arrow in 1.5(b) leads not from last-note to last-note, as in 1.5(a), but from first-note to first-note. Rather than merely affecting one note, it serves to transpose all of Y into T(Y). Thus, while −1 acts as an internal transformation in X and as a single-note transformation between X and X′, –3 acts as a spanning transformation in X and as an agent of wholesale transposition between Y and T(Y).

Figure 1.5(c) shows the gestural kinetics of Cad, which includes the same three transformations as Y and T(Y), −1, +2, and +3, though in a different order. For the first time, –1 does not initiate the gesture, but terminates it. The reversal is appropriate for a cadence. The sense of reversal is heightened by the presence of x (the mini form of X) at the end of the phrase, turning the movement's initiating gesture into an agent of closure. Cad also reverses previous material in a more formal sense: it is a retrograde inversion of Y and T(Y). The diatonic pitch-space operation that takes T(Y) to Cad is retrograde-inversion-about-F♯4. F♯4 is both the initiating pitch in T(Y) and the cadential pitch in m. 7; the inversional balance around F♯4 strengthens its role as a point of temporary cadential repose. Cad is the first gesture to begin with an ascent, as well as the first to depart from the articulative pattern of X: it is fully legato, covered by a single slur, and linked by ornamental connectives between its nodal points.

Of the three transformations in X, only –1 does not appear in *positive*—that is, ascending—form in the melodic gestures of Figures 1.5(b) and (c). It *does* appear in ascending form in the bass, however, as shown in 1.5(d).[44] Moreover, the motivic A–B dyad is reversed here. Until this point, B has always proceeded to A via –1. BassCad now retrogrades this crucial gesture, taking A to B via +1. The sense of cadential retrograde interacts nicely with the comments just made about Cad's various reversals.[45] While Cad exhibits an inversional relationship with the Y-forms, BassCad exhibits an inversional relationship with X. Minus signs in X are replaced by pluses in BassCad, as X's falling, initiating gesture is transformed into a rising, cadential bass figure.

These pitch and contour relationships interact compellingly with durational aspects of the music. Some of these interactions are shown in Figure 1.6. The contents of the nodes in Figures 1.6(a)–(d) and (f) are ordered pairs of the form (pitch, duration), where duration is the note value corresponding to the length of time the given pitch persists (either literally or implicitly) in the music.[46] The transformational labels are also ordered pairs in which the first element is a diatonic pitch interval (as in Figure 1.5) and the second is a durational transformation. In the examples in the left column (1.6(a), (c), and (e)), the durational transformations are rational numbers indicating proportions. For example, the proportional transformation 2 in 1.6(a) takes the opening quarter note to the following half note; the proportional transformation 1/2 in 1.6(c), on the uppermost arrow, takes the opening quarter note to the concluding eighth note; and so on. In the examples in the right column (1.6(b), (d), and (f)), the durational transformations are *additive*, adding or subtracting note values in the intuitive way (e.g., half – quarter = quarter).[47] The two different methods of transforming durations offer different perspectives on the gestures. In X, for example, the successive durations increase by different sized proportions (2 and 1-1/2) as shown in 1.6(a), while the additive increases, shown in 1.6(b) are by the same amount (an added quarter in each case). The conceptual and experiential differences between the two species of rhythmic transformation are also reflected in their differing group structures: the integers under addition (in the additive rhythmic transformations) vs. the nonzero rational numbers under multiplication (in the proportional rhythmic transformations).

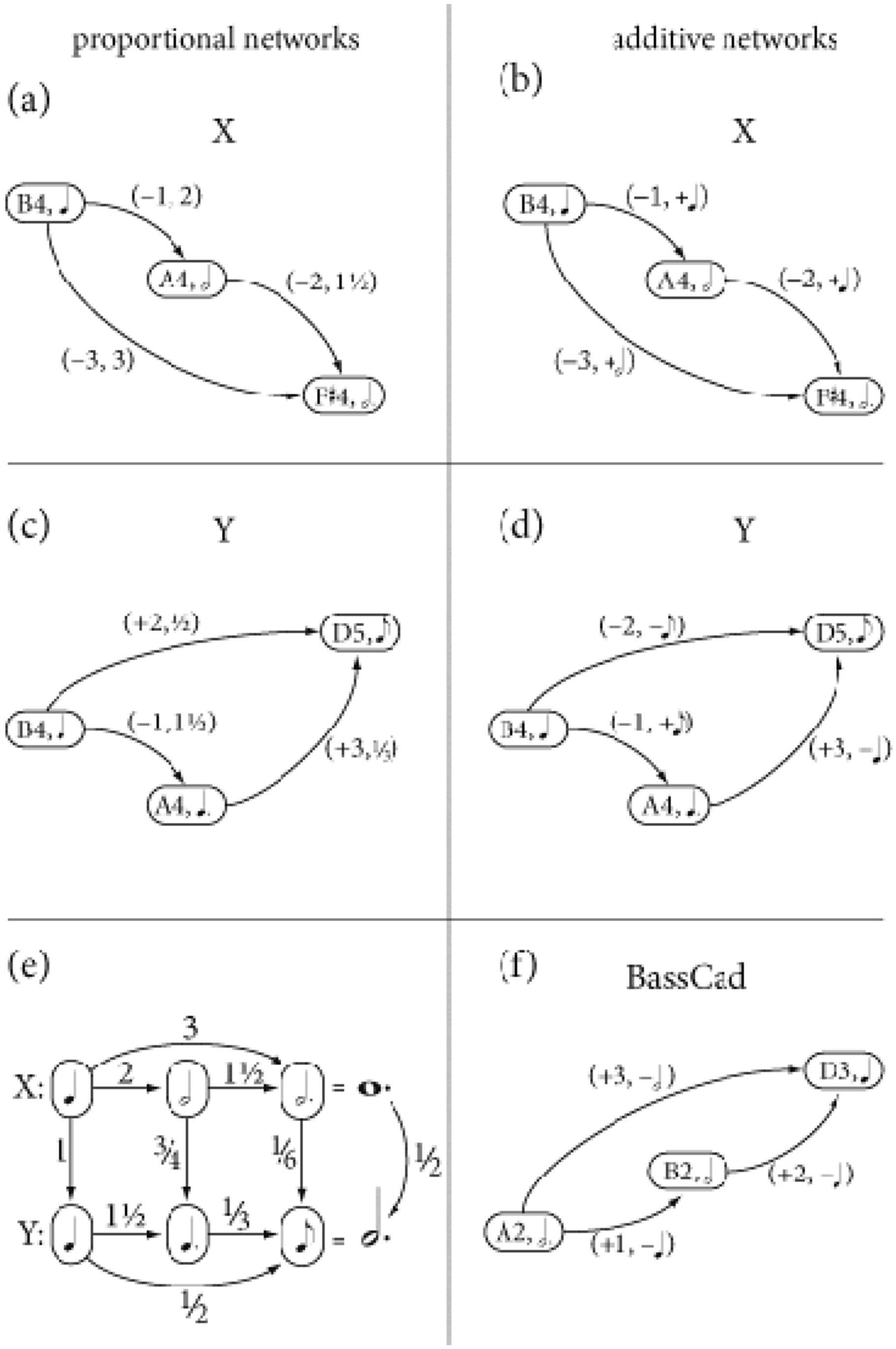


*Figure 1.6* Networks of pitches and durations for the Schubert Andante. Durational transformations in (a), (c), and (e) are proportional, while those in (b), (d), and (f) are additive.

These dual transformation systems reveal interesting correspondences between pitch and rhythm in the passage. Note first that for all descending pitch motions, durations increase; for all ascending pitch motions, durations decrease. The connection is suggestive of a metaphorical correspondence between durations and weight, with the longer, "heavier" durations at the bottoms of the gestures. The aptness of the metaphor is especially evident in the Y-forms: the gesture flicks upward at the last moment to catch the eighth note, which floats up like a helium balloon. X, by contrast, constantly sinks, as note values gradually increase in length and heft.[48] Figure 1.6(e) compares the X- and Y-forms in this regard, showing the proportional relationships between their respective elements. The vertical arrows show the alteration of each successive element in the three-note gestures. Pitch one is not altered at all; it remains a quarter in both X and Y (durational proportion 1). Pitch two is then slightly shortened by the proportion 3/4—a proportion made evident by the repeated eighth notes (four in X, three in Y)—while the third pitch is shortened drastically, by 1/6. The rightmost events in each gesture are thus at the durational extremes of the network. The result is a net decrease in duration across the span of Y, reflected by the arched 1/2 arrow along the bottom of the example, as opposed to a net increase in X, the 3 in the upper arched arrow. Figure 1.6(e) provides a rich sense of the ways in which Y "pulls up short" in comparison to X. The proportional relationship of X to Y is palpable to both performer and listener; it corresponds to the increase in harmonic rhythm in mm. 5–6 and, ultimately, to the early arrival of the cadence on beat three of m. 7.[49]

There are other compelling correspondences in the examples. For example, the –3 gesture that links B4 and F#4 in X is not only inverted in pitch space in Y's A4–D5; it is also inverted in durational-proportion space. That is (+3, 1/3) in 1.6(c) is the formal inverse of (–3, 3) in 1.6(a).[50] Note also that, just as BassCad is an inversion of X in pitch space, it is also an *exact inversion of X in additive duration space*. The arrow labels in 1.6(b) are replaced with their formal inverses in 1.6(f): to turn the transformations in 1.6(b) into those of 1.6(f), one needs merely to reverse the pluses and minuses for both pitches and durations. BassCad thus inverts X as a complete pitch/time gesture. This observation interacts suggestively with BassCad's role as a textural inverse of X (bass rather than melody) as well as a syntactic inverse (a cadential rather than initiating gesture).

### *1.3.5 Comment*

The analyses of sections 1.2.6 (Bach) and 1.3.4 (Schubert) have demonstrated that GIS and transformational methodologies, even in their current state, make available suggestive insights into tonal music—insights that differ from those generated in other analytical approaches. Further, those insights in no way call into doubt the tonal status of the music. Yet, while tonal aspects of the two works were discussed in informal ways in the analyses (through references to things like tonics, dominants, and cadences), the formal apparatus of the analyses did not model those ideas in any direct way. This was especially evident in the Schubert analysis, which made no attempt to explore the subtle interpenetration of D major and B minor that characterizes the movement's harmony. As Peter Smith (2000) has noted, the relationship between the pitches B and A is especially striking in this regard—their status relative to one another, as either stable or decorative pitches, depends heavily on tonal concepts. Consider the opening six-three sonority {D, F$^{\#}$, B}. In the context of the opening bar, it functions as a tonic D chord subjected to a contrapuntal 5–6 displacement. But, as Smith notes, it also carries hints of B-minor in first inversion—hints that connect both to the concluding moments of the previous movement, and to later events in the Andante (such as the root position B-minor chord in mm. 10–11). The subtle shift of hearing that Smith notes in regard to the opening six-three is a characteristically tonal effect, but one that our transformational methodology, in its current state, cannot capture. The development of the apparatus's tonal sensitivity is the work of Chapters 2 and 3. …

**Notes**

1. The discussion here complements the fine introductions to the theory from Satyendra (2004) and Michael Cherlin (1993).

2. As Henry Klumpenhouwer puts it, transformations model "moments of action carried out by and within the analyst" (2006, 278).

3. See, for example, Hook 2007b, 172–77. The distinction between the intervallic and transformational perspectives was of central importance to Lewin, forming part of a general critique of Cartesian views of musical experience, as discussed in section 1.2.2 (see also Klumpenhouwer 2006).

4. Given the theory's emphasis on relationships over isolated musical elements, the technical emphasis in the discourse is typically on groups and semigroups, basic concepts from abstract algebra. Transformational theory is thus an algebraic music theory. Recent developments in geometrical music theory—see, for example, Callender, Quinn, and Tymoczko 2008—represent a departure from this algebraic foundation. Though such geometrical approaches are sometimes considered subsets of transformational theory writ large, I will limit the term *transformational* here to algebraic approaches.

5. Klumpenhouwer (2006) describes the conceptual transition from intervallic to transformational thinking as the general theme of *GMIT*.

6. Whether GISes succeed fully in this regard is a question to which we will return.

7. A set in which elements appear more than once is called a *multiset*. Multisets have music-theoretical applications, but we will not explore them in this study.

8. Most mathematicians call Lewin's "binary composition" a *group operation* or *binary operation*. Lewin, however, somewhat idiosyncratically reserves the word *operation* for a different formal concept, as we will see, thus making *binary composition* preferable in this context.

9. The symbol • here is a generic symbol for the binary composition in any group. When the idea of group composition is understood, such symbols are sometimes eliminated. In that case, our associativity notation would look like this: $(fg)h = f(gh)$.

10. See *GMIT*, 25–26.

11. Tymoczko 2008 and 2009.

12. As Rachel Hall puts it, "GISes *can* express notions about distance, but are not forced to do so" (2009, 209).

13. Cf. Hall 2009, 208–9.

14. That GIS, which calculates intervals between qualitative tonal scale degrees, involves a group of intervals that might better be understood as comprising familiar intervallic *qualities* rather than distances, such as the quality of a minor third, as opposed to that of an augmented second. The distance metaphor is especially inapt in connection with certain exotic interval types that we will explore in sections 2.5 and 2.6.

15. Edward Gollin (2000) explores another model for distances in a GIS, measuring *word lengths* in the elements of the intervallic group. In the group of neo-Riemannian operations, for example, the word PLP, of length 3, is longer than the word RL, of length 2. A given group admits of multiple distance-based interpretations, based on which group elements are chosen as *unitary* (words of length 1 ) via the formalism of group presentation, as Gollin demonstrates.

16. Two familiar noncommutative groups in music theory are the group of transpositions and inversion from atonal theory, and the group of neo-Riemannian operations. In the former group, it is not generally true that $T_m$ followed by $I_n$ is the same as $I_n$ followed by $T_m$. For example $T_3$ –then $I_2$ equals $I_{11}$, while $I_2$-then-$T_3$ equals $I_5$. In the neo-Riemannian group, given operations X and Y, it is not generally true that XY = YX. for example, PL ≠ LP, RL ≠ LR, PR ≠ RP, and so on.

17.  The locations "one diatonic step up" and "one diatonic step down" evoke ideas of distance and direction, suggesting the pertinence of Tymoczko's distance metric to this particular GIS.

18 Conditions (A) and (B) appear in the formal definition of a GIS in *GMIT*, 26.

19. To be clear, there is only one interval between two musical entities *within a single GIS*. As discussed in section 1.2.5, GIS methodology rests on the idea that there is in fact an indeterminate *multiplicity* of possible intervals between two musical entities. That multiplicity arises not within a single GIS, but via the multiple potential GIS structures that may embed the two elements in question.

20. The relevant philosophical matters are penetratingly treated in Klumpenhouwer 2006. On the problematics of the subject-object relationship in passive musical *perception*, see Lewin's well-know phenomenology essay (Lewin 2006, Chapter 4).

21. Lewin published three extensive articles specifically on GISes, not transformational systems, after *GMIT*: Lewin 1995, 1997, and 2000–2001.

22. My ideas on these matters were clarified through conversation with Henry Klumpenhouwer. My view differs slightly from Klumpenhouwer's published comments, in which he states that Lewin wants us to "replace intervallic thinking with transformational thinking" (2006, 277). I feel that Lewin's ethical directive in *GMIT* is not quite this strong– that he wants us not to *replace* intervallic thinking but to become more aware of its Cartesian bias, and to be self-conscious about that bias whenever "thinking intervallically" in some analytical context.

23. Cf. the discussions of hexachordal versus tonal perceptions in Lewin 1993, 48n15 and Lewin 1998b.

24. Cavell 1999, 236 ("a play of intuition and tuition") and Cavell 2002, section 4 ("providing the tuition for intuition"). For Emerson's original quote see Emerson 1993, 27.

25. As Henry Klumpenhouwer notes, "In distinction to other uses of the term, Lewin's intuitions have some conceptual content" (2006, 278n3).

26. In this book I will understand apperceptions loosely in William James's sense, as experience colored by "the previous contents of the mind" (1939, 158). Such apperceptions may involve conscious reflection, or they may not. For example, one's past experiences with a certain musical idiom will strongly color one's current and future musical experiences with music in that idiom, whether one has consciously reflected on the idiom or not. Apperceptions, thus conceived, are simply current experiences under the influence of past experience, and open to present reflection. This departs from certain philosophical understandings, in which conscious reflection is a necessary component of all apperceptions.

27. To bound it, one would need to assert a specific high pitch beyond which one cannot progress up by one semitone, and/or a specific low pitch beyond which one cannot progress downward by one semitone. On theoretically unbounded spaces that can be perceived only in part, see *GMIT*, 27.

28. What I have been calling *uniform*, Tymoczko (2009) calls *homogenous* and *parallelized*. The latter term means that one can move a given interval from point to point, applying it anywhere in the space.

29. The GIS introduced in Chapter 2 would be especially well suited to modeling the interval in question. It would further distinguish the $G_4 \longrightarrow E^{b}_4$ at the outset of the Fifth from the same pitch succession in $E^{b}$ major (say, in the primary theme area of the *Eroica*), or from an enharmonically equivalent succession in E minor (say, in the "new theme" in the *Eroica* development [e.g., cello, downbeat of m. 285 to that of m. 286]).

30. While my rewording focuses on listening experiences stimulated by analytical reflection, it is not clear from his comment that sorts of listening contexts Lewin has in mind. He may indeed have meant that multiplicity is a fact of everyday musical experience: when we hear music in any context, we "intuit" multiple musical spaces at once and thus hear intervallic relationships in manifold ways—even when we are not in an analytically reflective mode. This may be true, but I am not sure how one could test the idea. nor do I know what exactly is meant by "intuit" and "intuition" in Lewin's passage. Is the listener consciously aware of these manifold "intuitions"? Or are they perhaps instead a congeries of more or less inchoate sensations that one has when listening, which can be brought into focused through analytical reflection? I am more comfortable with the latter position, which moves toward my rewording.

31. $\mathbb{Z}$ is a common label for the set of integers, taken from the German *Zahlen* (numerals).

32. Lewin (*GMIT*, 16–17) discusses the discrepancy between the familiar ordinal intervallic names of tonal theory (tenths, fifths, thirds, etc.), which indicate number of scale steps *spanned* between two pitches, and the intervals in a scalar diatonic GIS, which indicate the number of scale steps *up* from one pitch to another (negative steps are steps down). The GIS introduced in Chapter 2 employs the familiar ordinal names for intervals between scale degrees.

33. Adjacent pitches here correspond to the "steps" in Fred Lerdahl's "triadic space" (2001) or to "steps" in the "chordal scale" of William Rothstein's imaginary continuo (1991, 296).

34. IVLS is once again ($\mathbb{Z}$, +). Note, however, that the integers now represent acoustically larger intervals than did the same group elements in the GIS of Figure 1.2(b). For example, in the GIS of 1.2(b) int($G_2$, $B_2$) = +2, while in the GIS of 1.2(c), int($G_2$, $B_2$) = +1. Hook 2007a offers relevant comments on relating two GISes that have the same abstract group of intervals (like ($\mathbb{Z}$, +) here), through the group elements in the two different GISes may represent intervals of different acoustic size. See also Tymoczko 2008 and 2009.

35. The cellist can emphasize the partial series by placing a slight agogic accent on the opening $G_2$, a gesture that makes good musical sense anyway, given the work's upcoming stream of constant sixteenth notes. The partials activated by the $G_2$ are an octave higher than those in Figure 1.2(d), but they nevertheless still suggest the pertinence of the just-ratio GIS in this resonant opening.

36. I have worded this carefully: interval k in 1.2(d) is a skip in the overtone series above G1. It is not, however, a "skip" *in the GIS*, in the same sense that j and k are "skips" in the GIS of 1.2(c)–further evidence of a shift in conceptual space. In 1.2(c) the space S of the GIS consists of the pitches of the G-major arpeggio, which are spanned by "steps" (modeled by additive integers). In the GIS underlying 1.2(d), the space S in fact consists of an infinitely dense set of pitches, which are spanned by frequency ratios (modeled by multiplicative rational numbers). The group IVLS in this GIS consists of *all* of the positive rational numbers–not just the low-integer ratios explored in the figure (3:2, 5:3, and so on), but also higher integer ratios like 16:15 (a "major semitone" in Pythagorean theory), and even

enormous integer ratios such as 531,441:524,288 (the acoustically tiny Pythagorean comma). In conformance with GIS Condition (B), the space S of the GIS thus includes infinitely many pitches in the gap between, say, $G_2$ and $D_3$. (For example, it includes the pitch residing a Pythagorean comma above $G_2$.) This makes clear that the GIS structuring Figure 1.2(d) is not a linearly plotted space of "steps" and "skips" as in 1.2(b) and (c)–it is a space of frequency ratios, which has a considerably different shape.

37. Note that we could also use the functional notation from the discussion of GISes above as a replacement for any one of the arrows in this table: for example we could write ResC(D) = C.

38. Functions that are one-to-one and onto are also called *bijections*. For further discussion, see the entry for *Function* in the Glossary.

39. Most of the familiar transformations in transformational theory are operations (that is, they are one-to-one and onto, and thus have inverses). The neo-Riemannian transformations for example, are all operations, as are the familiar $T_n$ and $I_n$ operations on pitch classes from atonal theory.

40. Satyendra 2004 offers a lucid account of this process of translation, which Lewin defines formally at the beginning of Chapter 7 in *GMIT*.

41. We are dealing with a group of operations–the integers under addition. The operations act simply transitively on the infinite set of diatonic pitches, from which our node contents are drawn. the networks in Figure 1.5 can thus be translated into GIS terms if we so desire.

42. Peter Smith (2000, 6) make suggestive observations along these lines, especially involving the way $A_4$'s evident structural status is undercut by the articulation, which causes it to "lead ahead to $F^{\#}$."

43. Other transformations could take X to X', such as Jonathan Bernard's "unfolding" (1987, 74–75), or Lewin's related FLIPSTART (*GMIT*, 189). The resulting configuration would then need to be transposed by -1 to produce X', once again demonstrating the thematic role of -1 in the music.

44. Note that in Figure 1.5(d) I have drawn an arrow from $B_2$ directly to $D_3$, bypassing the $C^{\#}_3$ on beat two of m. 7, in agreement with Schubert's slurring. The reading corresponds Peter Smith's understanding of $C^{\#}$ as a passing tone (2000, 9, Ex. 4b). See also the Schenkerian sketch in Figure 1.8.

45. On the role of reversals as "closural," see Narmour 1990.

46. For example, the $F^{\#}_4$ in m. 2 is understood to have a dotted-half duration, as it is the melodic pitch that implicitly controls the entire bar.

47. This is a transformational equivalent of the problematic GIS space discussed above (and in *GMIT*, 29–30). I employ it here to show its musical intuitiveness, and to show that it *can* work as a transformational system, through it is formally awkward: one must posit an element *a* in the space S of durations that corresponds to a "duration-less instant." Any duration *x* is transformed to *a* if it is acted on by a negative duration whose absolute value is greater than *x*'s. An elegant way around this problem is to treat the system in question not as a transformational system at all, but as a system based on a "tangent space," as explored in Tymoczko 2009. Such spaces admit of bounded "dead ends" beyond which no transformations or intervals may be conceived. Our duration-less instant is one such dead end.

48. Such pitch/time transformational perspectives offer fruitful ways to think about Schubert's vividly somatic gestural language. For a compelling discussion of the thematic role of gesture in Schubert, see Hatten 2004.

49. Mm. 1–7 are a sentence in William Caplin's (1998) sense. X and X' correspond to the basic idea (b.i.) and its repetition (b.i.'). The increase in harmonic rhythm and surface activity in mm. 5ff. is typical of a sentence's continuation phrase. This increase in activity is visibly evident in the "piling up of gestures" shown on the right-hand side of Figure 1.4(b).

50. Formally, $(+3, {}^{1}/_{3})^{-1} = (-3, 3)$. In making this statement, we rely on the fact that the group of transformations in question is a *direct product group*, as discussed in the Glossary. Notice that the inversional relationship between these pitch/duration pairs does *not* hold in the additive networks of 1.6(b) and 1.6(d).

**Glossary**

**Argument**: see *function.*
**Automorphism**: an *isomorphism* from a *group* to itself.
**Bijection**: see *function.*
**Binary composition**: The "inner law" in a *group* or *semigroup* that dictates how any two elements in the group or semigroup combine to create a third element.
**Cartesian product**: The Cartesian product of two *sets A* and *B*, notated *A* x *B*, is the set of all *ordered pairs* of the form (*a, b*), such that *a* is a member of *A* and *b* is a member of *B*.
**Commutativity**: Two *group* elements *f* and *g* commute if $f \bullet g =$ . To take a music-theoretical example, transpositions $T_m$ and $T_n$ in atonal theory always commute with one antoher. By contrast, transpositions and inversions in atonal theory do *not* general commute with one another.

A group that consists entirely of elements that commute with one another is called a *commutative group* or an *Abelian group* (after Niels Abel).
**Cyclic group** ($\mathbb{Z}$, $\mathbb{Z}_n$): a *commutative group* that can be generated by repeated iterations of a single group element. This elements is called the *generator* of the group. Finite cyclic groups are very familiar in music theory. The group of twelve transpositions $T_n$ is a cyclic group of order 12, notated $\mathbb{Z}_{12}$; it can be generated by twelve iterations of $T_1$, $T_{11}$, $T_5$, or $T_7$. The infinite cyclic group, notated $\mathbb{Z}$, is isomorphic to the integers under addition.

**Function**: a function f from set $X$ to set $Y$ sends every element $x$ in $X$ to some element $y$ in Y. We can either write f($x$) = $y$ or $x \xrightarrow{f} y$ to indicate the action of function f sending $x$ to $y$. Set $X$ is called the *domain* of the function, while the set of outputs that the function produces is called the *range*. The element $x$ in the expression f($x$) = $y$ is called the *argument*, while $y$ is called the *value* or *image*.

Note that f sends each element in $X$ to only *one* element in $Y$. If f sends all of the elements in $X$ to different elements in $Y$, we say that the function is *one-eo-one*, or an *injection*. If every element in $Y$ is the target of some element in $X$ under f, we say that f is *onto*, or a *surjection*. If f is both *one-to-one* and *onto* we say that it is a *bijection*. For f to be a bijection from $X$ to $Y$ the cardinality of $X$ must equal that of $Y$.

**Directed Graph (digraph)**: A *graph* whose set $E$ of edges consists of *ordered pairs* of elements from the vertex set $V$. More formally, a directed graph is a *relation* on $V$. The ordered pairs in $E$ are often called *directed edges* and are usually drawn with arrows. Directed graphs (which Lewing calls "node/arrow systems") are central to transformational graphs and networks.

**Generalized Interval System (GIS)**: A central construct in transformational theory, used to rener intervallic statements and apperceptions formal. A Generalized Interval System, or GIS, is an *ordered triple* (S, IVLS, int), in which S is a *ste*, IVLS is a *group*, and int is a *function* from the *Cartesian product* S x S into IVLS. A GIS must satisfy two conditions (as defined in *GMIT*, Def. 2.3.1):

(A): For all r, s, and t in S, int(r, s)int(s, t)= int(r,t).

(B): For every s in S and every I in IVLS, there must exist a unique t in S such that int(s, t)= i.

**Graph**: A *set V* of vertics (or "nodes" or "dots") and a set $E$ of edges (or "lines"), which are two-element subsets of $V$.

Note that this definitions says nothing about pictures of nodes and lines. I graph is fully defined simply by enumerating the elements of its sets $V$ and $E$. For example, we can define a graph $G = (V_G, E_G)$ such that $V_G = \{1, 2, 3\}$ and $E_G = \{\{1, 2\}, \{1, 3\}\}$. We could draw a picture to depict graph $G$ containing three dots or nodes, labeled 1, 2, and 3, with lines connecting 1 and 2, and 1 and 3. But such a picture is not of the esence for the graph—it is fully defined by the enumeration of the elements of $V_G$ and $E_G$ above. ….

Not all of the elements of $V$ need to have lines adjacent to them in a graph. One can in fact define a graph with no edges at all; it would simply consist of vertices—"dots" unattached to one another by "lines." In other words, the set $E$ may be empty. The set $V$, by contrast, must be nonempty and finite.

**Group**: a basic algebraic structure that consits of a *set* along with a *binary composition* that allows one to combine any two elements from the set to generate a third element in the set. The fact that the element so generated ia a member of the set satisfies the property of *Closure*. To qualify as a group, the structure must satisfy three more properties. *Existence of an identity*: There must be one element $e$ in the group such that, when it is composed with any other element $g$ in the group (via the binary composition), $g$ is the result. (The label $e$ comes from the German *Einheit*). *Existence of inverses*: For every element $g$ in the group there exists an element $g^{-1}$ such that $g$ composed with $g^{-1}$ yields the identity element $e$. *Associativity*: Given three group elements *f, g,* and $h$, then $f \bullet (g \bullet h) = (f \bullet g) \bullet h$.

**Homomorphism**: A *function* that maps the elements from one group to those in another so as to preserve the action of the *binary composition*. The homomophism $h$ from group $G$ to

group $H$ sends the product of elements $g_1$ and $g_2$ in $G$ to the product of $h(g_1)$ and $h(g_2)$ in $H$. If we notate the binary composition in group $G$ as • and the binary composition in group $H$ as *, then we write $h(g_1 \bullet g_2) = h(g_1) * h(g_2)$.
**Involution**: A *group* element $g$ is an involution if it is its own inverse—that is, if $g \bullet g = e$.
**Isomorphism**: a *homomophism* that is one-to-one and onto (i.e., a bijection). If two groups can be mapped onto one another via isomorphism, they have the same underlying algebraic structure and are said to be *isomorphic*.
**Operation**: a *transformation* that is one-to-one and onto (i.e., bijective). For more, see *transformation*.
**Ordered pair, ordered *n*-tuple**: A pair of elements notated in parentheses and separated by a comma for which order matters. $(a, b)$ means "*a*, then *b*." $(a, b)$ is distinct from the ordered pair $(b, a)$. One may also have ordered *n*-tuples of any length.
**Relation**: Any subset of a *Cartesian product*.
**Semigroup**: A basic algebraic structure consisting of a *set* and a *binary composition* that allows any two elements of the set to combine to produce a thrid element of the set. Unlike a *group*, a semigroup only needs to satisfy two properties: closure and associativity. Put another way, a group is a special kind of semigroup, one with two additional structural propertis; existence of an identity, and existence of inverses.
**Set**: A finite or infinite collection of distint elements. The elements are distinct in that none of them occurs more than once in the set (a set that contains duplicates is called a *multiset*). If the elements of a set are considered unordered, they are enclosed in curly brackets {}. If they are considered ordered, they are enclosed in parentheses (). …

The set $B$ is a *subset* of the set $A$ if all of the elements of $B$ are also members of $A$. … We say that $B$ is a *proper subset* if it does not contain all of $A$. Given $B$ as a subset of $A$, we can say conversely that $A$ is s *superset* of $B$. $A$ is a *proper superset* of $B$ is $A$ contains elements not in $B$.

In a Generalized Interval System or GIS, a set is any finite subset of elements from the space S of the GIS.
**Simple transitivity**: A *gorup* acts on a *set* in simply transitive fashion if, for any $s$ and $t$ in the set, there is only one element $g$ in the group such that $g$ takes $s$ to $t$.
**Subgroup**: A subset $H$ of elements from a *group* $G$ that satisfies all four of the conditions for a group under the initial binary composition (closure, existence of an identity, existence of inverses, associativity). The subgroup $H$ will always contain the identity element of the group $G$.
**Transformation**: In transformational theory, a transformation is a *function* from a *set* to itself. An *operation* is a transformation that is a bijection—that is, one-to-one and onto.

Most of the familiar transformation from atonal theory and transformational theory are in fact operations. For example, the transposition and inversions of atonal theory are operations. $T_n$ maps the set of twelve pcs one-to-one and onto itself, adding $n$ to each pc inteber. $I_n$ also maps the set of twelve pcs one-to-one and onto itself, subtracting each pc from $n$.
**Transformational graphs and networks**: A central construction of transformational theory, meant to model dynamic relationships among musical entities. A transformational graph is a *digraph* whose arrows have been labeled with *transformations* from some *semigroup* (which may be a *group*). Lewin stipulates that the transformations on the arrows must "compose" in a certain way, so that the transformations on any two arrow

pathes between the same two nodes sum to the same semigroup element. Hook (2007a) has loosened this requirement (see section 3.3.4).

A transformational network is a transformational graph whose nodes have been filled with elements from some set S, in accorance with the labels on the graph's arrows.

**Transformational theory**: A branch of systematic music theory that seeks to mdel dynamic and relational aspects of musical experience via Generalized Interval Systems (or GISes) and transformational graphs and networks. The founational text of transformational theory is Lewin's 1987 treatise *Generalized Musical Intervals and Transformations* (*GMIT*).

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