L'intonation juste, un renouveau esthétique et théorique

(Just intonation: an aesthetic and theoretical renewal)

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Introduction: Precursors of extended just intonation

Of all the neglected chapters of music theory, just intonation seems at first glance the least likely to resurface in the century of serialism and chance composition. The essential principle of just intonation is that simple ratios between numbers produce the ideal, pure tunings for intervals in tonal music; with the standardization of equal temperament and the advent of atonality, just intonation seemed fated to obscurity and irrelevance. What use could composers have for an ancient tuning system, designed for simple tonal structures, that lacked the ease and flexibility of equal temperament?

For Harry Partch (1901-1974), the most influential figure in the just intonation revival in America, the appeal of this antiquated practice was thoroughly modern: the desire to build a rational musical world from strict rational principles, rather than simply accepting an established practice now alienated from its origins. For Partch, the music theory of a thriving culture would be built on a solid bedrock of "Archean granite" instead of on the inherited instruments, forms, and tunings of eighteenth-century Europe.[1] Just intonation reflected the natural preferences of the listening ear: on this solid foundation, he saw the possibility for an controlled, logical expansion of musical resources, eventually breaking free of the "prison bars" of the twelve-tone keyboard.[2]

By turning to ratios as the basis for his theories, Partch placed himself in a tradition with a long history, which we can divide into three basic stages: Pythagorean, just, and extended just intonation. *Pythagorean intonation* refers to the tuning theory of the ancient Greeks, invented (as tradition has it) by Pythagoras in the sixth century B. C. and devoutly defended until the fifteenth century (and even beyond). The Greeks recognized that whole number ratios between string lengths on their monochords, based only on the multiples of 2 and 3, could define all of the intervals of their music. Simple ratios provided the foundational 2/1 (octave), 3/2 (fifth), and 4/3 (fourth). The simplicity of an interval's ratio is directly related to its consonance: the less concordant intervals of Greek theory, like the 9/8 whole tone or the 256/243 *limma,* were formed by more complex ratios including higher multiples of 2 and 3.

Pythagorean intonation dominated European music theory until the Renaissance, when theory came into serious conflict with the practice of composers and performers. The problem was this: musicians had begun to treat thirds and sixths as consonances, but in Pythagorean tuning, these

[1] Harry Partch, *Genesis of a Music: An Account of a Creative Work, Its Roots, and Its Fulfillments,* second edition (New York: Da Capo Press, 1974): xvii.

[2] Harry Partch, "Bitter Music," in *Bitter Music: Collected Journals, Essays, Introductions, and Librettos* (Urbana, Illinois: University of Illinois Press, 1991): 12.

intervals are only available as complex, harsh-sounding ratios. The Pythagorean version of the major third, for example, is the thorny ratio 81:64, which is harmonically unstable and difficult to sing. The solution proposed by Bartolomeus Ramis de Pareia in 1482 was simple but had far-reaching effects: by admitting the number 5 and its multiples into the tuning system, the jarring ratio 81:64 could be replaced by the smooth, mellow 5/4. The use of 5 also provided simple ratios for the minor third (6/5), minor sixth (8/5), and major sixth (5/3).

This tuning system, based on ratios containing multiples of 2, 3, and 5, is known as *just intonation.* Despite its admirable acoustic purity, just intonation leads to some practical difficulties, especially when implemented on keyboard instruments, which are incapable of making the fine continuous adjustments of a violin or trombone. It is impossible to tune the twelve-note-per-octave keyboard of European tradition in just intonation in one key without creating drastically out-of-tune notes in other keys. To cope with this problem, various *temperaments* have been applied to the keyboard. The current standard is equal temperament—every step on the keyboard is made the same size, exactly one-twelfth of an octave. Instead of simple integer ratios, each interval except the octave is based on an irrational number, the twelfth root of 2. Compared to the ideal just intervals, each interval is slightly out-of-tune, but still usable. In the nineteenth and twentieth centuries, the harmonic flexibility of equal temperament outweighed the pure sound of just intonation, and equal temperament became the standard tuning for all keyboard instruments.

While Partch found the natural acoustical and physiological roots of just intonation a suitably "natural" foundation for his musical system, the intervallic palette of just intonation (established in the fifteenth and sixteenth centuries) was not sufficient for the complexities of his flowing, speech-like music. Partch found it necessary to go beyond the intervals based on 2, 3, and 5 to higher prime numbers including 7 and 11: this modification introduced *extended just intonation.* To describe the prime numbers included in a tuning system, Partch introduced the useful idea of "limits"—the limit of an intonational system is the highest prime number which is a factor in its interval ratios. Thus Pythagorean intonation has a "three-limit" and Renaissance just intonation a "five-limit": Partch's own tuning system reached the eleven-limit. For Partch, the extension of just intonation to the higher prime numbers was part of a historical imperative: from earliest times, he writes, the use of musical materials has "progressed from the unison in the direction of the great infinitude of dissonance."[3] The sense of historical destiny in Partch's thought is shared by other extended just intonation advocates: James Tenney argues that due to the limitations of equal temperament, the development of harmony in Western music came to a halt in the early twentieth-century, and can only be restarted through the exploration of the intervallic possibilities of the higher primes.[4]

We can identify two main tendencies in the twentieth-century just intonation revival: one is the rejection of equal temperament and its tuning compromises for a purer, more authentic tuning system; the second is the exploitation of new, more complex harmonies based on the higher

[3] Partch, *Genesis of a Music,* 94.

[4] James Tenney, "Reflections after Bridge," liner notes to *James Tenney: Bridge & Flocking.* Hat Hut, ART CD 6193, 1996.

prime numbers of extended just intonation—that is, the intervals to be found between the upper overtones of the harmonic series. These two tendencies are not always present at the same time: for example, many composers of the early twentieth century showed eager interest in the new harmonies implied by the higher overtones, but were still unwilling to give up the advantages of equal temperament. Similarly, we find just intonation composers happily working within the Renaissance five-limit for its clarity of intonation and purity of sound, and eschewing the complexities of the higher primes.

Figure 1: Excerpt from Claude Debussy's *Canope,* Preludes, Book II (1912-1913), m. 20

The music of Claude Debussy provides many instances of equal-tempered harmonies which evoke the upper overtones: Figure 1 analyzes an excerpt from the prelude *Canope* (1912-13). Here, the whole-tone scale G A B D-flat E-flat F is arranged to imply an overtone series on a G an octave below the actual bass (below the range of a standard piano): the sounding bass is the second partial of this low G, B the fifth partial, F the seventh, A the ninth, D-flat the eleventh, and E-flat the thirteenth—all approximated as closely as possible in equal temperament. Such overtone structures can also be found in music by Ravel, Scriabin, and even Webern and Berg, as demonstrated in recent research by Olli Väisälä and Gary Don.[5] In theoretical writings by authors as diverse as Arnold Schoenberg, Paul Hindemith, and Henry Cowell, the overtone series is invoked as an explanation for the newly dissonant harmonies of early twentieth-century music.

Harry Partch: Corporealism and Monophony

While Debussy, Schoenberg, Hindemith, and Cowell all draw expanded harmonic resources from the unexplored portions of the overtone series, these composers never cut their ties to the established twelve-tone system. This step fell to Harry Partch, whose music and theoretical treatise *Genesis of a Music* are the departure points for extended just intonation in the twentieth century. Partch's book is unique in its development of a new musical language—including harmony, notation, and even instruments—from just a few basic principles.

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[5] Olli Väisälä, "Prolongation of Harmonies Related to the Overtone Series in Early-Post-Tonal Music," *Journal of Music Theory* 46/1-2 (2002): 207-283 and Gary Don, "Brilliant Colors Provocatively Mixed: Overtone Structures in the Music of Debussy," *Music Theory Spectrum* 23/1 (2001): 61-73. See also Célestin Deliège, "L'harmonie atonale: de l'ensemble à l'echelle," in *Sources et ressources d'analyses musicales: journal d'une démarche* (Sprimont: Mardaga, 2005): 387-411.

Partch's *Genesis of Music* is organized around two concepts. One is Corporealism—"the essentially vocal and verbal music of the individual" (8); Partch opposes Corporeal music, linked to poetry and dance, to the abstraction of Western instrumental art music. Partch's second guiding concept is Monophony: "an organization of musical materials based upon the faculty of the human ear to perceive all intervals and to deduce all principles of musical relationship as an expansion from unity" (71). In the Monophonic organization of pitch, Partch is always conscious of a single pitch—or to be more accurate, pitch class—representing "unity"; in more traditional musical terms, we would term this a tonic, though Partch prefers to designate it by the ratio 1/1. Thus far, we've discussed ratios as representing *intervals,* not *pitch classes.* The ratio 9/8, for example, specifies the interval of the major whole tone (about 204 cents) between two pitches, but not precisely what those pitches are. In Partch's Monophonic pitch world, ratios represent a pitch's relationship to the central pitch 1/1. Thus, 9/8 means the pitch a whole tone above 1/1, 4/3 represents the pitch a fourth above, and 3/2 the pitch a perfect fifth above. In theory, the inverse of these ratios would refer to pitches the same distance *below* 1/1: thus 8/9 would be a whole tone below 1/1, 3/4 a fourth below, and 2/3 a fifth below. For a simplified set of pitch class names, though, Partch prefers to transpose these pitches by octave into the octave from 1/1 to 2/1: instead of 8/9, a tone below 1/1, he typically uses 16/9, a minor seventh above 1/1; instead of 3/4 (a fourth below), he uses 3/2, and instead of 2/3, 4/3. Monophony and Corporealism were once found together, Partch suggests, in ancient Greece where "ratio-idea and music-enhanced word vitality" were present in the same musical culture (60). The restoration and reunion of these concepts in the modern world are the goal of Partch's theorizing and composition.

In *Genesis of a Music,* Partch describes a 43-tone scale chosen from intervals within the 11-limit within a single octave (from 1/1 to 2/1).[6] Figure 2 shows the diagram of this scale that Partch nicknamed the "One-Footed Bride". The diagram is to be read beginning with the ratio 1/1 at the lower left; pitch ascends up the left column of ratios from 1/1 to the tritone exactly half an octave above (not expressible as an integer ratio). The note names from G to C# mark this ascent. We must read the right side of the diagram from the top down—this reversal puts each interval directly opposite its inverse, with which it shares a similarity in sound and relative consonance. The fourth 4/3 is opposite the fifth 3/2, the major third 5/4 opposite the minor sixth 8/5, and so on.

[6] Partch, *Genesis of a Music,* 133. As Partch scholar Bob Gilmore has documented, Partch experimented with scales of different sizes throughout his career, all selected from the boundless Monophonic "fabric" of intervals: see "Changing the Metaphor: Ratio Models of Musical Pitch in the Work of Harry Partch, Ben Johnston, and James Tenney." *Perspectives of New Music* 33/1-2 (Winter-Summer 1995), 458-503.

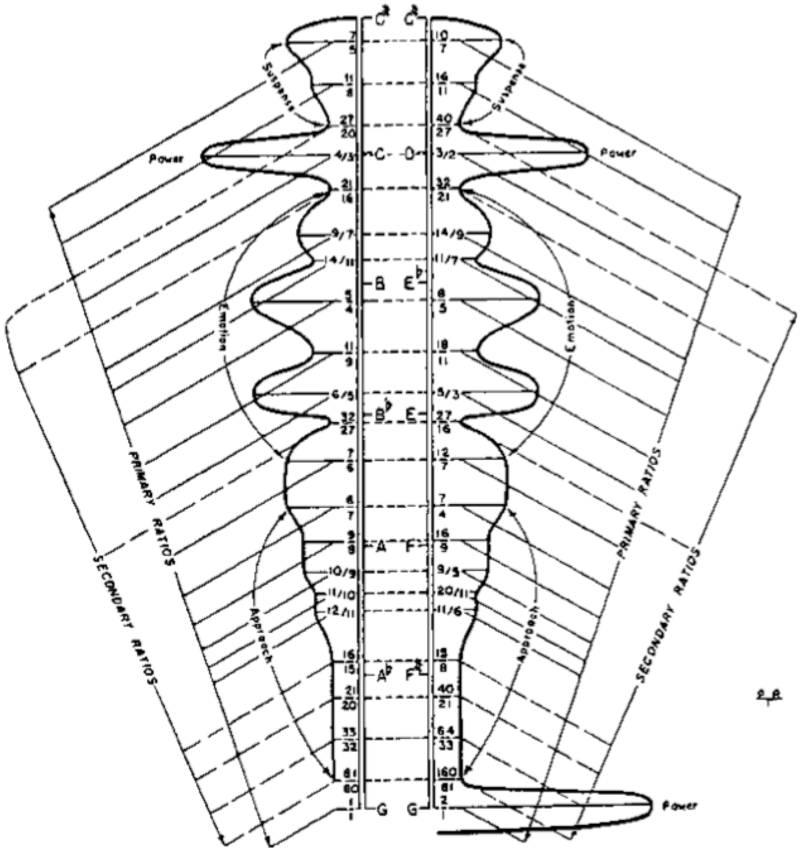

Figure 2: "The One-Footed Bride: A Graph of Comparative Consonance"[7]

Note that while all the pitches in the scale are related by just intervals to 1/1, not all are expressible as *overtones* of 1/1. For example, the minor third of the scale, 6/5, would only fit into the overtone series of a fundamental a just major third below 1/1; similarly, the fourth 4/3 would require a fundamental a fifth below 1/1. When pitches can be found in the overtone series of 1/1, they are called "Identities"—thus 3/2 is an example of the 3-Identity, and 5/4 is the 5-Identity. Most of the intervals are "primary ratios" built from the integers 1 to 11; dotted lines point out "secondary ratios" built from multiples of these primary integers including 15, 21, 27, 33, and 81.

Partch draws a curve alongside the columns of intervals to show each one's relative consonance; this curve gives the "bride" her distinctive profile. A point on the curve far from the central column represents a high degree of consonance (peaking at the octave, the bride's "foot"); a point on the curve close to the central column indicates a more dissonant interval; for example, the intervals smaller than a semitone at the bottom of the column or the complex 27/20 near the top.

The One-Footed Bride also categorizes the pitches into four types based on the interval they form with 1/1: intervals of approach (the 231 cent 8/7 septimal whole tone or smaller), intervals of emotion from the small 7/6 minor third (267 cents) to the flat fourth 21/16 (471 cents), intervals

[7] Partch, *Genesis of a Music,* 155.

of power (the perfect fourth and fifth), and intervals of suspense (diminished fifths/augmented fourths).

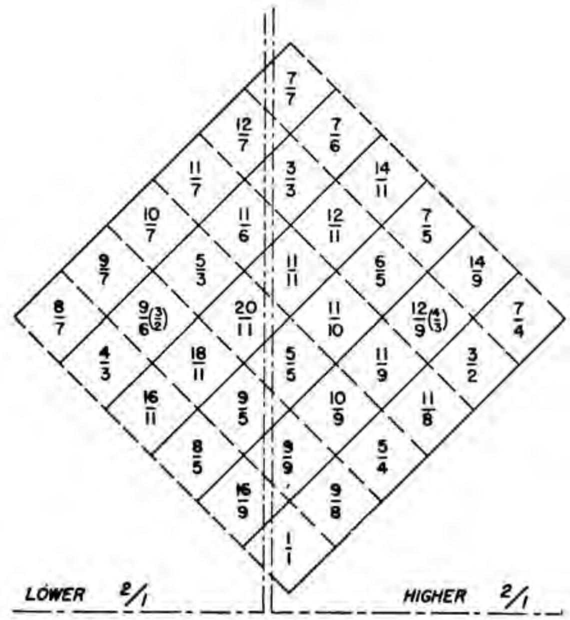

Figure 3: "The Expanded Tonality Diamond"[8]

The melodic scale diagrammed in the "One-Footed Bride" takes on a harmonic dimension in Partch's Tonality Diamond (Figure 3). The tonality diamond is a matrix of overlapping "Otonalities" (six note sets based on the odd overtones 1 through 11) and their inversions, "Utonalities" (based on the odd *undertones* 1 through 11). Figure 4 shows how the members of each Otonality/Utonality fall in ascending/descending order within an octave (cent values show the pitch class relative to 1/1 = 0):

Otonality:	1/1	9/8	5/4	11/8	3/2	7/4
cents:	*0*	*204*	*386*	*551*	*702*	*969*
Utonality:	1/1	16/9	8/5	16/11	4/3	8/7
cents:	*0*	*996*	*814*	*649*	*498*	*231*

Figure 4: Pitch content of Otonalities and Utonalities

The Tonality Diamond lays out the twelve harmonic areas—six Otonalities and six Utonalities—which include 1/1 as a member. 1/1 is alternately given 12 meanings, as a member of each Otonality and Utonality. In the Tonality Diamond, Otonalities are read from lower left to top right (within the solid lines), and Utonalities are read from lower right to top left (within the dotted lines). Thus, the Otonality on 8/7 starts at the leftmost corner, and continues to 7/7 (equal to 1/1) at the top of the diamond; the Utonality on 7/4 reads from the rightmost corner to the top of the diamond in the opposite direction. Up the center of the diagram runs a series of ratios, all

[8] Partch, *Genesis of a Music,* 159.

equivalent to 1/1: pitches to the left of this spine are in a lower octave (or 2/1, in Partch's preferred terminology) and pitches to the left are in the higher octave above 1/1. The resemblance of this diamond to the matrices of prime and inverted forms in serial music is notable, though not specifically acknowledged by Partch. Every pitch in the matrix is at the intersection between a Utonality and Otonality, allowing facile shifts between the two tonal types. Partch sees the unifying relation of all twelve harmonies to 1/1 as one of the great strengths of Monophony, even though this unity comes at the cost of free modulation as in equal temperament.

To complete the 43-tone scale of Figure 2, Partch adds 14 new pitches to the 29 distinct pitches of the tonality diamond, so that steps between adjacent scale degrees vary from between 14.4 and 38.9 cents. The inclusion of these added intervals makes possible a number of "secondary tonalities," which fall outside the Diamond because they do not include 1/1. Many of these secondary tonalities are deemed incomplete because they call for intervals falling outside the 43-tone scale; but nonetheless, they offer some expanded tonal resources beyond the twelve tonalities of the diamond.

Beginning in the 1920s, Partch created new instruments to play the extended just intervals unavailable in the traditional orchestra. His first constructions were an adapted viola, with an elongated fingerboard marked with positions for extended just intervals, and a retuned reed organ, which he named the "Ptolemy" (later to be rechristened the "Chromelodeon" in a rebuilt version). Part of the pleasure of reading *Genesis of a Music* (apart from Partch's opinionated and vigorous writing style) are the illustrations and names of his instruments, including the Cloud Chamber Bowls, the Spoils of War, the Bloboy, and the Eucal Blossom. Figure 5 is a photograph of Partch's Diamond Marimba, which he describes as "the theoretical Tonality Diamond brought to practical tonal life."[9] As in the Tonality Diamond, the Diamond Marimba combines Otonalities and Utonalities so that each bar is a member of both an Otonality and Utonality. However, the bars of the Diamond Marimba are arranged slightly differently than in the theoretical Tonality Diamond: the members of each Otonality/Utonality are ordered as stacked thirds (4:5:6:7:9:11) instead of scalar order (8:9:10:11:12:14); This modification seems to have been made in order to spread the pitches of each Tonality over a wider range, expanding the tessitura of the instrument.

[9] Partch, *Genesis of a Music,* 259.

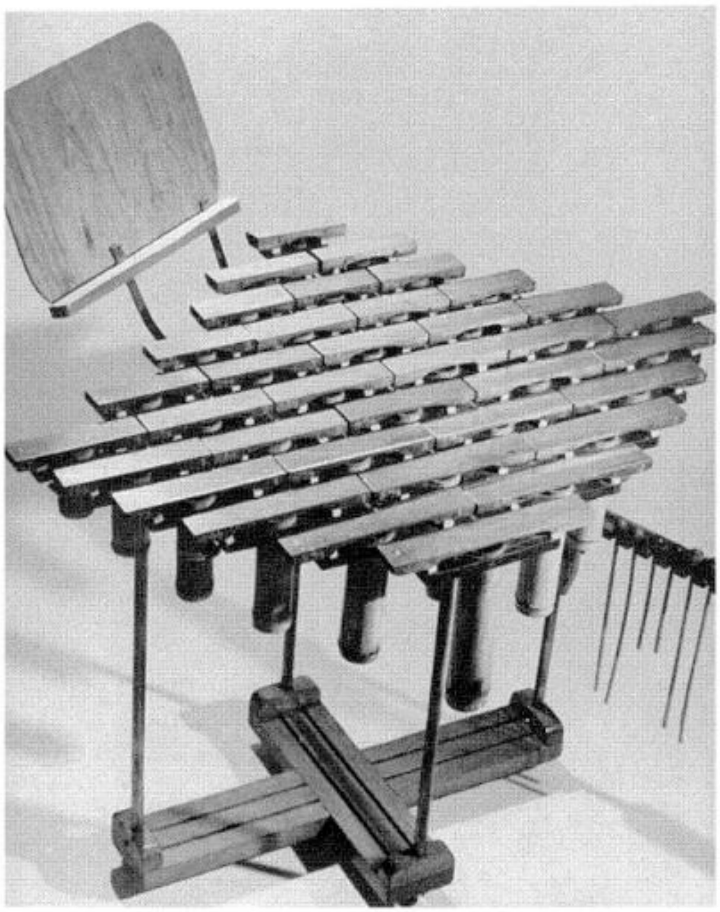

Figure 5: Partch's Diamond Marimba[10]

While Partch is extremely specific about the layout of his tuning system, he has less to say about the specific application of these concepts in composition. Numerous historical scales, including the just diatonic and Pythagorean scales, are possible in various tonalities. Partch discusses how the members (Identities) of a particular Otonality or Utonality act as the six "primary planets" of that tonality; these draw nearby pitches towards themselves with a kind of gravitational pull, with the 1-identity, like a tonal root, exerting the greatest attractional force. Another harmonic device described by Partch is "tonality flux"—the interplay between nearby pitches in different tonalities. Partch offers an example juxtaposing the 4, 5, and 6 identities of the 8/7 Otonality (231, 617, and 933 cents) and the 6, 5, and 4 identities of the 7/4 Utonality (267, 583, and 969 cents)—the pitches of the second set are all within 40 cents of a pitch in the first.

Lou Harrison: Partch's legacy and musical hybridization

Lou Harrison (1917-2003) was one of the composers most strongly influenced by Partch's *Genesis of a Music.* Harrison had studied with Henry Cowell in San Francisco and collaborat5 ed with John Cage on a number of projects in the 1930s, including several concerts of percussion music—he also studied briefly with Schoenberg in Los Angeles. Harrison was given a copy of

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[10] Partch, *Genesis of a Music,* 260.

Genesis of a Music in 1949 by the composer and music critic Virgil Thomson, and soon began to incorporate just intervals into his own compositions.

Unlike Partch, who wished to wipe away musical systems of the past to make a fresh start, Harrison has always been open to musical hybridization: he writes, "Don't underrate hybrid musics *because that's all there is.*"[11] Harrison's music borrows ideas from Western diatonic music as well as Asian musics from Korea to Indonesia; all of these borrowings, however, are inflected by his dictum that "Just intonation is the best intonation."[12]

In addition to many modes and pentatonic scales within the five-limit, Harrison has explored tunings which extend into the higher primes of extended just intonation. One such example is the piano retuning for *Incidental Music for Corneille's Cinna,* a suite composed between 1955 and 1957 for retuned "tack piano"—a piano with tacks inserted in the hammers for a harpsichord-like jangle. Harrison's tuning system is shown in Figure 6, with both ratios and values in cents. The tuning shows clear affinities to Partch's 43-tone scale, which includes all of Harrison's pitches except the 25/18 "augmented fourth." The tuning includes a variety of pure fifths and thirds, as well as septimal relationships combining B-flat (7/6) and F (7/4) with the other pitches of the scale. The harmonic writing in *Cinna* takes advantage of the ringing pure fifths of the tuning, combining them with the less familiar septimal intervals in a way reminiscent of some passages in La Monte Young's *Well-Tuned Piano* (discussed in detail later in this chapter). Harrison's *Cinna* unfolds a quasi-improvisatory texture with clear references to 17th-century ornamentation and keyboard idioms; the combination of these historical references with the unfamiliar intervals of extended just intonation creates an evocative and eerie dramatic effect.

G	A♭	A	B♭	B	C	C♯	D	E♭	E	F	F♯	G
1/1	16/15	10/9	7/6	5/4	4/3	25/18	3/2	8/5	5/3	7/4	15/8	2/1
0	*112*	*182*	*267*	*386*	*498*	*569*	*702*	*814*	*884*	*969*	*1088*	*1200*

Figure 6: Piano retuning in *Cinna*[13]

Harrison also developed a complex variant of extended just intonation he called "free style," as opposed to the "strict style" of composing with reference to a fixed scale and tonal center. In free style, melodic intervals between notes are chosen freely from a palette of just intervals: the resultant interval chains can take the intonation into distant territories very quickly.[14] Figure 7 is a brief excerpt from the *Simfony in Free Style* (1955), written for an ensemble including a variety of customized flutes and fretted string instruments. Italicized numbers show pitch class in cents, assigning the initial B the value of 1100; ratios between adjacent notes give the just interval between them. Note that the B that begins the excerpt is not the same pitch as the B beginning the subsequent phrase in the lower staff, which is 49 cents (nearly a quartertone) higher. Because

[11] Lou Harrison, *Lou Harrison's Music Primer* (New York: C.F. Peters, 1971): 45.
[12] Ibid., 4.
[13]See Leta Miller, ed., *Lou Harrison: Selected Keyboard and Chamber Music 1937-1994* (Madison, Wisconsin: A-R Editions, 1998): xl-xlv.
[14] Larry Polansky, "Item: Lou Harrison as a Speculative Theorist," in Peter Garland, ed., *A Lou Harrison Reader* (Santa Fe, New Mexico: Soundings Press, 1987).

of the enormous difficulty of accurately playing successive just intervals implying constantly shifting tonal centers, the work did not receive a live performance until 2001.[15]

Figure 7: *Simfony in Free Style,* excerpt[16]

The music of the Indonesian gamelan has long fascinated Harrison, who has been a leading figure in the development of the "American Gamelan" movement. Harrison has constructed a number of gamelan, all based on his own theories of extended just intonation. The combination of gamelan with just intonation theory creates a strange cross-cultural hybrid: just intonation has never been part of the theory behind Indonesian tunings, which are based instead on a concept of *embat,* a highly individual sense of intonation linked to the singing voice.[17] Because gamelan tunings are not traditionally fixed, many of their pitches fall close to just intervals which can be the basis for a tuning system: as Harrison's biographers Miller and Lieberman note, this imposition of just intonation on the gamelan "while not culturally *characteristic,* was culturally *possible."*[18] Figure 8 illustrates a *sléndro* tuning from the gamelan Harrison built at Mills College.

octave

perfect fourth   perfect fourth

intervals between pitches: 8:7 7:6 9:8 8:7 7:6

cents: *0 231 498 702 933 1200*

Figure 8: Harrison's tuning for the Mills gamelan[19]

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[15] Other free style works include *At the Tomb of Charles Ives* and *Phrase for Arion's Leap.* See Leta Miller and Fredric Lieberman, *Lou Harrison: Composing a World* (New York: Oxford University Press, 1998): 116-121.

[16] Excerpt reprinted in Harrison, *Lou Harrison's Music Primer,* 6-7.

[17] The conflict between *embat* and Western "intonational naturalism" is explored by Marc Perlman in "American Gamelan in the Garden of Eden: Intonation in a Cross-Cultural Encounter," *The Musical Quarterly* 78/3 (Autumn 1994): 510-555.

[18] Leta Miller and Fredric Lieberman, "Lou Harrison and the American Gamelan," *American Music* 17/2 (Summer 1999): 146-178.

[19] Miller and Lieberman, *Composing a World,* 113.

Drawing on gamelan theory in his music for Western instruments, Harrison often borrows terms for different scale types: *sléndro* for anhemitonic pentatonic scales and *pélog* for hemitonic pentatonics. In Harrison's characterization, *sléndro* scales have wide seconds and narrow thirds, while *pélog* scales have wide thirds and narrow seconds.[20] For Harrison, gamelan music offered inspiration for textures as well as pitch material—many of his works evoke the repeating ostinati and heterophonic melodic elaborations of the traditional gamelan repertoire.

Essential to all of Harrison's music is the idea of scale or mode: the wide currency of these (or closely related) concepts has made his cross-cultural borrowings compatible with one another, often to a surprising degree. With the exception of the diabolically complex "free style" works, Harrison's use of just intonation is far less dense that Partch's: as the musicologist Bob Gilmore has pointed out, Harrison seems to use just intonation more for the uniquely limpid and transparent sound of its intervals than to add more pitches to his palette.[21]

Ben Johnston: Just intonation and harmonic comprehensibility

Ben Johnston's association with Partch came not only through *Genesis of a Music,* but in the course of a six-month apprenticeship with Partch in 1950 and 1951. Johnston (b. 1926) and his wife lived in Partch's primitive studio in Gualala, California, tuning Partch's instruments and learning to play them.[22] It was, however, not until the end of the decade that Johnston began to write his own compositions in just intonation. While Partch sought to build his system from scratch, Johnston attempted to reconcile extended just intonation with the western tradition—he composed for standard Western instruments, and was even comfortable using serial procedures in combination with just intervals. In his compositions, Johnston calls for precise tuning of complex just intervals, using a detailed system of new accidentals. Among the extended just intonation composers, Johnston is one of the most articulate theorists: his thoughts on just intonation appeared in a series of influential academic articles.[23]

At the root of Johnston's theorizing is the argument that just intervals are more *intelligible* than tempered ones: thus just intonation is a relatively more *efficient* way of conveying a complex musical relationships than tempered tuning. In particular, the hierarchical implications of just intonation lend a cognitive transparency lacking in symmetrical divisions of the octave like equal temperament:

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[20] Miller and Lieberman, *Composing a World,* 110. According to Perlman, *pélog* contains "seven tones that are treated less as a single scale than as a source of pentatonic scales" (op. cit., 535).
[21] Bob Gilmore, "The Climate Since Harry Partch," *Contemporary Music Review* 22/1-2 (March/June 2003): 21.
[22] Heidi van Gunden, *The Music of Ben Johnston* (Metuchen, New Jersey: Scarecrow Press, 1986): 11-13.
[23] See especially "Scalar Order as a Compositional Resource," *Perspectives of New Music* 2/2 (1964), 56-76 and "Rational Structure in Music," *American Society of University Composers Proceedings* 11/12 (1976-77), 102-108. These and other theoretical writings are reprinted in *"Maximum Clarity" and Other Writings on Music* (Urbana, Illinois: University of Illinois Press, 2006) a volume of Johnston's collected writings edited by Bob Gilmore. Page numbers below refer to the versions in *"Maximum Clarity"*

> Interval scale thinking emphasizes symmetry of design. The harmonic and tonal meaning of symmetrical pitch structures is *ambiguity.* Chordally they produce either a sense of multiple root possibilities or of no satisfactory root possibility. Tonally they cause either a sense of several possible tonics or of no adequate tonic. Ratio-scale thinking, on the contrary, emphasizes a hierarchical subordination of details to the whole or to common reference points. The harmonic and tonal meaning of proportional pitch structures is clarity and a sense of direction.[24]

In the remainder of the article, Johnston derives a 53-tone scale based on five-limit just intonation, combined with a systematic use of accidentals which is the basis for his later notational practice (discussed in more detail below). This scale (with its careful accounting for syntonic commas using plus or minus signs before accidentals) was used compositionally in Johnston's String Quartet No. 2 (1964).[25]

Beginning with the solo trombone piece *One Man* (1967), Johnston began to extend his pitch language beyond 5-limit just intonation to embrace higher primes. *One Man* combined the septimal ratios of extended just intonation with the five-limit scales of his earlier work. The new resources of the seven-limit were explored in more detail in Johnston's String Quartet No. 4 (1973). This quartet, a variation form based on the American spiritual "Amazing Grace," progresses from Pythagorean intonation through five-limit just intonation to septimal extended just intonation, as if recapitulating the history of tuning culminating with Johnston's system.[26]

These new musical resources were accompanied by a new theoretical tool, the harmonic lattice. Johnston credits this innovation to his acquaintance with writings by the Dutch theorist Adriaan Fokker (who advocated a 31-tone equal temperament to closely approximate just intonation including septimal intervals).[27] Lattices arrange pitches along axes corresponding to each of the prime numbers of a just intonational system; the number of axes could, in theory, be expanded indefinitely. Figure 9 reproduces Johnston's 2, 3, 5, 7 lattice from his 1976 article "Rational Structure in Music."

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[24] Johnston, "Scalar Order as a Compositional Resource," 28.

[25] Van Gunden's monograph on Johnston's music includes a detailed analysis of this work (76-85). One of the most notable features is the structure of the first movement: each measure represents a step upward in Johnston's 53-tone scale. The cycle closes in the 54th measure, with a return to the original pitch level.

[26] See Randall Shinn, "Ben Johnston's Fourth String Quartet." Perspectives of New Music 15/2 (Spring/Summer 1977), 145-73.

[27] Gilmore, Introduction to *Maximum Clarity,* xviii. For an introduction to Fokker's theories see Fokker, A. D. "Equal temperament and the thirty-one-keyed organ," *Scientific Monthly* 81 (1955): 161-166.

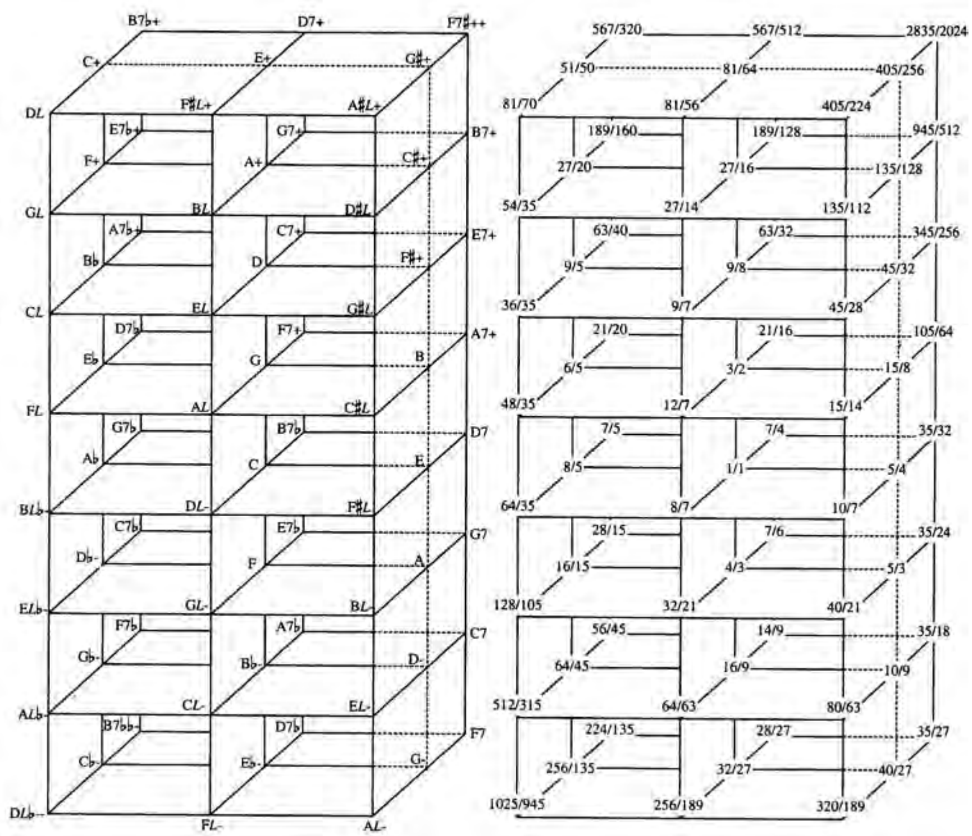

Figure 9: Johnston's 3, 5, 7 lattice[28]

The lattice appears in two different notations in this figure: the version on the left shows the note names of each pitch, while the version on the right shows the associated ratios. Like Partch, Johnston identifies each pitch by its relation to a central "tonic," 1/1. The left-right axis shows ascending just major thirds and the vertical axis ascending perfect fifths; Johnston's lattice adds a third dimension for natural sevenths (the ratio 7/4). The advantage of the lattice in conceptualizing and displaying intervals is that it concisely expresses our intuitions about distances in harmonic space: simply related pitches are near one another, while more complex ones are separated by a longer path through the nodes of the lattice. Johnston explains how scales in extended just intonation systems can be derived from a lattice: each scale begins with the "basic chord" of the system, comprised of 1/1 plus the nearest pitch on each axis, either in the "overtone" direction for a "major" system or the "undertone" direction for "minor." (This closely parallels Partch's Otonalities and Utonalities, and reflects Johnston's persistent interest in harmonic dualism and invertibility.) The scale of a given system expands on this "basic chord", subdividing each of its intervals (starting with the largest) by the addition of a new pitch that is *adjacent* in the lattice to the pitches already chosen.[29] A few additional rules ensure the

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[28] Johnston, "Rational Structure in Music," 72.

[29] This process is closely related to James Tenney's theory of "crystal growth" in harmonic space, which is based on minimizing distances in harmonic space; this theory is discussed in detail in the section below on Tenney.

compactness and regularity of scales based on any extended just intonation lattice. Writing in favor of lattices and ratio scales, Johnston again cites their comprehensibility: scales derived from ratios are "an effective aid in designing melodic and harmonic audible structure even with unfamiliar pitch materials."[30]

The note names in Figure 9 use some unfamiliar symbols from Johnston's microtonal notation system. Unlike Partch, who abandoned Western notation in favor of interval ratios treated as note names, Johnston works within the traditional system, but supplements it with new accidental symbols. His first step is to precisely define the tuning of each pitch on the staff when uninflected by an accidental. This white note scale is tuned to allow pure triads on F, G, and C, as in Figure 10.

C	D	E	F	G	A	B
1/1	9/8	5/4	4/3	3/2	5/3	15/8

Figure 10: Johnston's tuning of the "white-note collection"

This traditional tuning for the just major scale includes two different sized whole tones (9/8 between C and D and 10/9 between D and E), two different minor thirds (6/5 between E and G and 32/27 between D and F), and both perfect and out-of-tune fifths (3/2 between C and G, 40/27 between D and A). The difference between each of these pairs of intervals is the syntonic comma, 81/80. Johnston uses the symbols + and – to indicate alterations of one syntonic comma: one multiplies by 81/80 to raise the pitch and by 80/81 to lower the pitch. Thus, the 9/8 ratio from C to D becomes a 10/9 ratio from C+ to D, and the out-of-tune fifth D-A can be made perfect by lowering D to D-. The adjustment by syntonic comma can be combined with other microtonal accidentals to describe any pitch on Johnston's lattices: Figure 11 lists Johnston's accidentals through the thirteen-limit.[31]

symbols		*ratio*	*cents*	*primary usage*
+	–	× 81/80	± 22 cents	raises 10/9 to 9/8, 32/27 to 6/5, 40/27 to 3/2
♯	♭	× 25/24	± 71 cents	raises 6/5 to 5/4
L	7	× 36/35	± 49 cents	lowers 9/5 to 7/4
↑	↓	× 33/32	± 53 cents	raises 4/3 to 11/8
I3	εI	× 65/64	± 27 cents	raises 8/5 to 13/8

Figure 11: Johnston's accidentals, through the thirteen-limit

Normal chromatic adjustments are indicated by flats and sharps, expressly defined as multiplication or division by 25/24—the difference between the major third 5/4 and the minor third 6/5. For example, to lower B to B-flat, one multiplies the ratio of B, 15/8, by 24/25 with the result 9/5. For ratios in extended just intonation, Johnston uses accidentals which inflect pitches

[30] Johnston, "Rational Structure in Music," 68.
[31] This figure is based on the table in Johnston's 2003 "A Notation System for Extended Just Intonation," in *Maximum Clarity,* 87. See also John Fonville's "Ben Johnston's Extended Just Intonation: A Guide for Interpreters," *Perspectives of New Music* 29/2 (1991): 106-137.

from within the five-limit. The “7” symbol can be combined with B-flat to lower the pitch from 9/5 to 7/4, the equivalent of multiplying by 35/36. To preserve the symmetry of the system, Johnston uses an inverted 7 to show a raise in pitch by the same interval. The arrows raise/lower the pitch by 53 cents, or 33/32: this is the difference between 4/3 (F) and the eleventh partial of C (11/4). The 13 and inverted 13 symbols work on the minor sixth—A-flat (8/5) is raised to 13/8 through multiplication by 65/64. Johnston uses the primes above 13 much less often, usually only in passages which treat the upper reaches of the overtone series as a sort of chromatic scale. In some sections of his String Quartet No. 9 (1987-88), the primes of 17, 19, 23, 29, and 31 appear in this context.

Figure 12: Opening of String Quartet No. 9, IV

Figure 12 adds just intonation ratios and pitches in cents to the first two melodic phrases of Johnston’s String Quartet No. 9. The first phrase is based (with the exception of the final E-flat) on pitch classes drawn from the overtone series of C: this is recognizable from the presence of a power of 2 in each denominator. The second phrase is a precise inversion of the first, around the axis E/E-flat. As a result, the passage is built on “undertones” of G instead of overtones of C. The treatment of pitch material in these two phrases reflects Johnston’s long-standing fascination with invertibility and harmonic dualism. The clear melodic profile and simple, metrical rhythms of each phrase are typical of Johnston’s later works, which seek to situate complex pitch relationships within easily comprehensible musical textures.

Ezra Sims: Expanded equal temperament

The music of Ezra Sims (b. 1928) lies on the border between extended just intonation and the equal divisions of the octave proposed by microtonalists like Alois Hába and Julian Carrillo. Sims notates his work in a system dividing each semitone into six equal parts of 16 2/3 cents each; this results in a total of 72 pitches per octave.[32] His notation system is shown in Figure 13. The close spacing of pitches in 72-tone equal temperament makes it possible to closely approach all pitches within 8 1/3 cents; most of the partials from 1 to 33 can be approximated much more closely (see Figure 14).

[32] As discussed later in this essay, 72-tone equal temperament has also been used by Georg Friedrich Haas and European composers; in the United States, saxophonist Joseph Maneri has used the 72-tone scale in both compositions and improvisations.

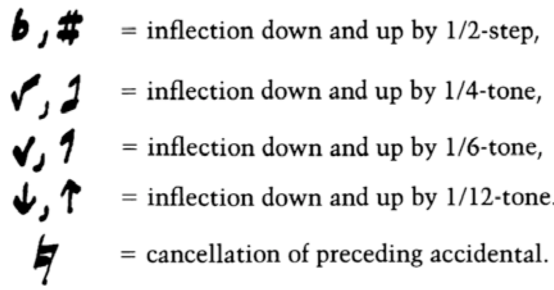

Figure 13: Sims's microtonal notation[33]

partial	*cents*	*nearest 72-tone approximation`*	*error*
33	53	50	–3
31	1145	1150	+5
29	1030	1033	+3
27	906	900	–6
25	773	767	–6
23	628	633	+5
21	471	467	–4
19	298	300	+2
17	105	100	–5
15	1088	1083	–5
13	841	833	–8
11	551	550	–1
9	204	200	–4
7	969	967	–2
5	386	383	–3
3	702	700	–2
1	0	0	0

Figure 14: approximations of partials 1 to 31 in 72-tone equal temperament

Though Sims notates his music in equal temperament, he expects that it will be heard as a close approximation of extended just intonation; ideally, players will also adjust pitches slightly to improve the fit with just intervals, much as they would when playing tonal music.[34] Much like Johnston, Sims describes the mind's preference for just ratios in terms of a minimization of computational effort: "The mind will try to understand what it perceives according to its inbuilt biases—the preferences for verticals and horizontals, harmonic ratios, perhaps the Golden Section, and so on, that are apparently hardwired into it. If it can't, and is too long frustrated in the attempt, it's very likely to just say The Hell With It and go off and play with something else."[35]

[33] Ezra Sims, "Yet Another 72-Noter," *Computer Music Journal* 12/4 (1988): 28.
[34] Ibid., 31
[35] Ezra Sims, "Long Enough to Reach the Ground or How Long Should a Man's Legs Be?" *Perspectives of New Music* 32/1 (Winter 1994): 208-213: 211.

The mind's "hardwired" cognitive preference for rational intervals allows Sims's extended equal temperament to be heard as extended just intonation.

In his music, Sims uses an eighteen-note scale which can be transposed to various pitch levels within the 72-tone-per-octave tuning, much as the seven-note diatonic scale can appear at any transposition within the tempered twelve-tone chromatic scale. The equal temperament makes it possible to modulate from key to key without leaving the 72-note pitch world; modulation is even possible to such distant areas as the 11/8 augmented fourth or 13/8 "semimajor sixth."[36] Sims describes his scale (see Figure 15, which gives a just tuning for each pitch in relation to 1/1) as "an expanded diatonic": gaps in the standard diatonic scale are subdivided so that intervals between adjacent pitches range from quarter tones to third tones.

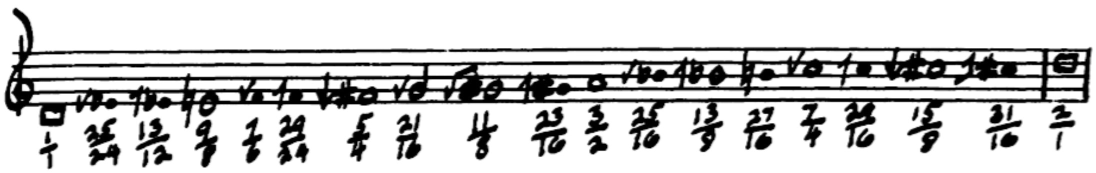

Figure 15: Sims's basic scale in the key of D[37]

For Sims, the scale and key relationships precede any particular harmonic technique: within the framework of his scale, Sims has used a variety of harmonic devices. One approach Sims describes is treating the scale degrees corresponding to the eighth to fifteenth partials as points of relative stability, which can be combined in various types of stable chords: "triadic, quartal, secundal, depending on the requirements of the piece."[38] Sims notes that pitches arranged to reflect difference and summation tone phenomena seem to reinforce one another particularly well: for instance, in the proportion 6/16/22, where 6 is the difference of 22 and 16, 16 is the difference of 22 and 6, and 22 is the sum of 6 and 16. He speculates that such formations might "represent a sort of 'lowest energy state' requiring less effort of larynx and mind than would any collection of nearby but inharmonic pitches."[39] The combination of an interval with its sum and difference tones is closely related to the electronic techniques of ring and frequency modulation; applying these concepts to instrumental writing was common in the early works of the "spectralist" composers Grisey and Murail (and later Claude Vivier). Austrian microtonalist Franz Richter Herf frequently used arithmetic series (e.g. 1, 4, 7, 10...) with similar results.[40]

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[36] Sims, "Yet Another 72-Noter," 31.

[37] Sims, "Reflections on This and That (Perhaps a Polemic)." Perspectives of New Music 29/1 (Winter 1991), 236-257: 241.

[38] Sims, "Yet Another 72-Noter," 39.

[39] Ibid., 40-41. Murail describes the use of combination tones in his *13 Couleurs du Soleil Couchant* in "Target Practice," *Contemporary Music Review* 24/2-3 (2005): 165-166. Translation of "Questions de cible," *Revue Entretemps* 8 (1989).

[40] Horst-Peter Hesse, "Breaking into a New World of Sound: Reflections on the Ekmelic Music of the Austrian Composer Franz Richter Herf," *Perspectives of New Music* 29/1 (1991): 212-235. Horatiu Radulescu describes his own use of sum and difference tones in Bob Gilmore, "'Wild

Sims argues that extended just intonation can also be found in jazz and blues music: he cites examples in specific performances by Odetta and Louis Armstrong. For Sims, these performances have essential microtonal components that are lost if the tune is transcribed into standard notation. In a transcription of Armstrong's "St. James Infirmary," Sims notes two different minor thirds, one a third tone smaller than the tempered minor third (7/6) and one a third tone larger (perhaps, Sims suggests, a 16/13 interval below 3/2: 39/32). In addition, Sims points out a melodic emphasis on two slightly different augmented fourths, approximating the ratios 11/8 and 23/16.[41] Sims uses this transcription as a source of melodic material in his *Sextet* (1983).

Sims's music is highly contrapuntal, with frequent polyrhythms and flowing textures reminiscent of the metrical complexities of Brahms. Like Harrison and Johnston, Sims's aesthetic is closely tied to the musical textures and theoretical procedures of the classical tradition. Music by all three composers has recognizable scales, melodies, and counterpoint; the use of extended just intonation is considered an expansion, rather than a renunciation of the principles of traditional tonality. In their theoretical work, they build new musical languages by analogy to the languages of the past, although with a far richer range of intervallic and harmonic possibilities. A more radical branch of extended just intonation—influenced by Indian music, John Cage, and the avant-garde movements of the 1960s—can be seen in the work of La Monte Young and James Tenney.

La Monte Young: Minimalism, Indian music, and extended just intonation
La Monte Young (b. 1935) found his way to extended just intonation independently of the research of Partch and his followers. For Young, the gateway to just intonation was his experience with Indian music, particularly as a disciple of the Indian vocalist Pandit Pran Nath. Young has compared the precise intonation required in singing Indian *ragas* to the tuning of complex ratios in extended just intonation.

Among the best-known of Young's works is the massive, improvised piano work *The Well-Tuned Piano*.[42] The *Well-Tuned Piano* tuning takes as its theoretical departure point the tuning lattice of Figure 16: the numbers beneath the notes indicate the pitch class in cents (based on A 440Hz = 900 cents). The up and down arrows (🡅 and 🡇) show a deviation from the equal-tempered version of the pitch class by approximately 31 cents (the difference between an equal-tempered and septimal minor seventh)—thus, the interval D to C🡇 is 1000 cents (D to C) minus 31 cents, or 969 cents. The relationship of all the pitches by intervals of 3/2 or 7/4 means that each pitch can be understood as an overtone, however distant, of the pitch D🡅; numbers in parentheses

Ocean': An Interview with Horatiu Radulescu," *Contemporary Music Review* 22/1-2 (2003): 105-122. See also the discussion of Hans Zender's theories below.

[41] "Yet Another 72-Noter," 33-34.

[42] My comments on *The Well Tuned Piano* in this section draw extensively on Kyle Gann's 1993 article, "La Monte Young's *The Well-Tuned Piano," Perspectives of New Music* 31/1 (Winter, 1993), 134-162. Gann was the first musicologist to reconstruct the details of Young's tuning, which the composer kept secret since the 1960s.

show the partial number of each pitch, based on D↑=1.[43] Young professes a dislike for the intervals based on the integer five, and thus bases his tuning solely on the numbers 3 and 7: the 5:4 major third essential to Renaissance just intonation is replaced by the 7:4 septimal minor seventh.[44] This replacement gives the tuning an exotic quality, as the unusual intervals whose ratios include the integer 7 (7:4, 7:6, 9:7, etc.) mingle with the clearly ringing perfect fifths—which are subtly different than the slightly smaller fifths of standard twelve-tone equal temperament.

B-flat↓	**F↓**	**C↓**	**G↓**	
963	**465**	**1167**	**669**	
(49)	***(147)***	***(441)***	***(1323)***	
C	**G**	**D**	**A**	**E**
1194	**696**	**198**	**900**	**402**
(7)	***(21)***	***(63)***	***(189)***	***(567)***
D↑	***A↑***	***E↑***		
225	***927***	***429***		
(1)	***(3)***	***(9)***		

Figure 16: La Monte Young's *Well-Tuned Piano* pitch lattice[45]

In Young's 1981 Gramavision recording of *The Well-Tuned Piano,*[46] the opening chord D↑ A↑ C E↑ is the subject of an extended improvisation lasting nearly ten minutes—the transcription in Figure 17 shows the first two minutes of this section. This transcription includes the beginning of what Young calls the "Theme of the Dawn of Eternal Time"—all of the major sections and themes of the piece receive titles, ranging from the mundane "Opening Chord" to the poetic "The Goddess of the Caverns Under the Pools." This section strongly projects the tonal center D↑, which (as noted above) is the theoretical fundamental for the entire tuning system. Its root status is strongly supported by the AÝ a fifth above and the tonic-dominant effect of the alternating D↑ and A↑ in the lower voice. In this context, C is associated strongly with D↑ as a natural seventh—an apparently consonant tone that does not call for a resolution. Large 9/7 major thirds and 8/7 "whole steps" appear melodically.

[43] Though Young's approach (like all jut intonation) can be understood as relating to an overtone series structure, his reliance on octave equivalence separates his theory from any "spectral" tendencies. Young is less interested in replicating the overtones of natural sounds than creating a grid of audible just interval relationships.

[44] Gann, op. cit., 135.

[45] Based on a figure in Gann, op. cit., 136.

[46] Gramavision 18-8701-1.

Figure 17: Opening of *The Well-Tuned Piano*, 0'00"-1'50"

Other of Young's works include just ratios with higher prime limits. *The Melodic Version (1984) of The Second Dream of The High-Tension Line Stepdown Transformer from The Four Dreams of China* is a seventy-seven minute work for eight trumpets based on only four pitches, in the ratios 12:16:17:18; the sound is based on Young's childhood memories of the hum of electrical equipment near his Idaho home. The extraordinary length of these pieces is taken still further in his sine-wave installations, which sustain pure tones in complex just ratios for days at a time.[47]

Young is best known as one of the founders, along with Terry Riley, of musical minimalism, a term which seems rather misleading when applied to complex large-scale works like *The Well-Tuned Piano.* The description is more apt for Young's static, drone-based works, though these are still far removed from the repetitive minimalism of Philip Glass or Steve Reich, both members of a movement that can be traced back to Riley's *In C.* Like Young, Riley was a student of Pandit Pran Nath; the two also studied composition at the same time together at the University of California in Berkeley. Riley has also worked in just intonation—his works include pieces for retuned organs and pianos—*Shri Camel* (1980) and *Harp of New Albion* (1986) both use a five-limit just intonation.[48] Riley is less interested than Young in the new intervals of extended just intonation, and when retuning of instruments is impractical, Riley often plays on equal-tempered instruments.[49]

James Tenney: Harmonic space and intonational tolerance

James Tenney (1934-2006) combined a strong grounding in physics and mathematics with an adventurous compositional spirit. Early in his career (in the mid-1960s), he worked on pioneering computer music projects at Bell Labs, while at the same time participating in New

[47] Kyle Gann, "The Tingle of $p \times m^n$ -1" in *Music Downtown: Writings from the Village Voice* (Berkeley: University of California Press): 269-271. See also Gann, "The Outer Edge of Consonance: Snapshots from the Evolution of La Monte Young's Tuning Installations" in *Sound and Light: La Monte Young and Marion Zazeela* (Lewisburg, Pennsylvania: Bucknell University Press, 1996): 152-190.

[48] See Kevin Holm-Hudson on the 5-limit just intonation tuning of Riley's 1984 piano cycle *The Harp of New Albion:* "Just Intonation and Indian Aesthetic in Terry Riley's *The Harp of New Albion," Ex Tempore: A Journal of Compositional and Theoretical Research in Music,* http://www.ex-tempore.org/Volx1/hudson/hudson.htm (accessed April 15, 2008).

[49] See Riley's interview with Frank Oteri on the website *New Music Box*, "Terry Riley: Obsessed and Passionate About All Music," *New Music Box,* http://www.newmusicbox.org/article.nmbx?id=1288 (accessed April 15, 2008).

York's experimental and avant-garde art scene. With his interest in a broad range of disciplines, Tenney was able to bring a greater engagement with acoustics and psychoacoustics to bear on his theorizing than many of his compositional colleagues. The scientific side of Tenney's thought is clearest in the pragmatism of his musical theorizing—Tenney approaches the numerological abstractions of extended just intonation with a skeptical eye, always aware of how musical structures are actually perceived.

Perhaps the most far-reaching of Tenney's contributions to just intonation theory is the idea of tolerance. This is implicit in the work of other composers (Ezra Sims's tempered just intonation, for example) but Tenney provides a more thoroughly considered treatment of the subject. For Tenney, tolerance is "the idea that there is a certain finite region around a point on the pitch height axis within which some slight mistuning is possible without altering the harmonic identity of an interval."[50] A just interval with its particular harmonic quality can tolerate a degree of mistuning before that quality is lost. The degree of tolerance is variable, depending on many contextual factors, but in general varies "inversely with the ratio complexity of the interval." Simple intervals like octaves and fifths would retain their identity under greater mistuning than complex intervals like 9/8 or 17/16. A result of tolerance is that there is a practical limit on the complexity of a just intonation system; very complex ratios are likely to be heard as out-of-tune variants of simpler ratios. With the addition of tolerance, the abstractions of just intonation become applicable to a wide range of pitch phenomena—just intonation plus tolerance can explain tempered and inharmonic sonorities as well as those based on pure harmonic structures.

Tenney's idea of harmonic simplicity is formalized in a notion of *harmonic space.*[51] As in Ben Johnston's harmonic lattices, Tenney's harmonic space consists of a number of discrete points joined by axes representing prime integers. The number of axes can be theoretically infinite, but Tenney's concept of tolerance constrains this proliferation as intervals with very complex ratios are heard as mistuned versions of simpler intervals. Tenney's innovation is the addition of a weighted metric for harmonic distance. The distance between any two points on the lattice is calculated by the sum of all the steps in between the points, but steps along the low prime-number axes are considered *shorter* than those along the axes of the higher primes. The axes are weighted by their logarithms base 2: thus, a step on the 2 axis is a harmonic distance of 1, a step on the 3 axis is a harmonic distance of 1.58, and so on. Steps along each axis can be summed for composite intervals: thus the perfect fifth 3/2 can be seen as a combination of one step on the 3 axis and one on the 2 axis, for a harmonic distance of 1 + 1.58 = 2.58. The formula for calculating such distances can be expressed as $HD(F_a,F_b)=\log_2(F_a)+\log_2(F_b)$, where F_a and F_b are the frequencies of the two tones reduced to the simplest possible (relatively prime) ratio. A table of some common intervals within a single octave appears in Figure 18.

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[50] James Tenney, "The Several Dimensions of Pitch." In *The Ratio Book: A Documentation of the Ratio Symposium, Royal Conservatory, The Hague, 14–16 December 1992,* ed. Clarence Barlow (Cologne: Feedback Studio Verlag. 102-115): 109.
[51] James Tenney, "John Cage and the Theory of Harmony." In *Soundings 13: The Music of James Tenney.* Santa Fe, New Mexico: Soundings Press, 1984. 55–83. Also in *Musicworks* 27 (1984), 13–17. Reprinted in *Writings about John Cage.* Ed. Richard Kostelanetz. Ann Arbor, Michigan: University of Michigan Press, 1993. 136–61.

interval	1/1	10/9	9/8	8/7	7/6	6/5	5/4	9/7	21/16	4/3	11/8
cents	0	182	204	231	269	316	386	435	471	498	551
harmonic distance	0	6.49	6.17	5.80	5.39	4.90	4.32	5.98	8.39	3.58	6.46
interval	7/5	45/32	3/2	25/16	8/5	13/8	5/3	27/16	7/4	15/8	2/1
cents	583	590	702	773	814	841	884	906	969	1088	1200
harmonic distance	5.13	10.49	2.58	8.64	5.32	6.70	3.90	8.75	4.81	6.90	1

Figure 18: Tenney's harmonic distance measurements for common just intervals

These distance measurements allow Tenney to be very specific about how pitch sets are perceived harmonically: we understand the set as the closest possible arrangement in harmonic space. In his article "On 'Crystal Growth' in Harmonic Space," Tenney explores how pitch sets might develop through the addition of pitches one by one, always adding the point in pitch space that minimizes the sum of harmonic distances between all pitches in the set. This approach gradually builds pitch-space crystals growing from Pythagorean sets based only on 2 and 3 to five-limit just intonation sets and eventually extended just intonation structures like the crystal in Figure 19.[52]

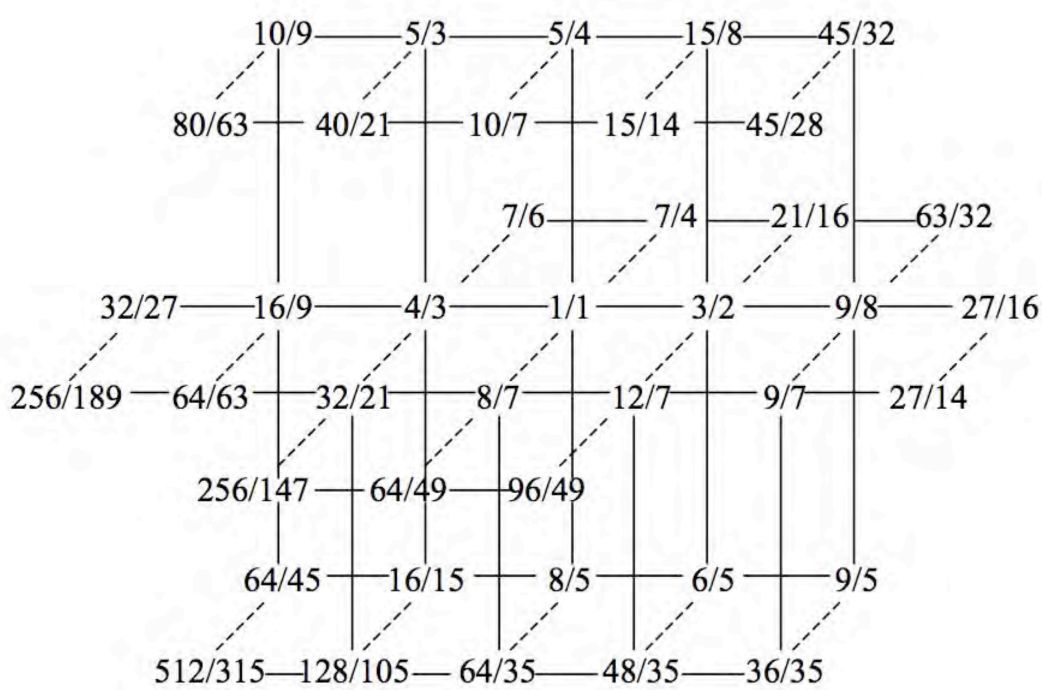

Figure 19: A "harmonic crystal" in 3,5,7-space

Throughout his compositional career, Tenney has realized just intonation in a number of different ways: unlike the "purist" system-building composers Partch and Johnston, Tenney is

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[52] "Ausweitung in eine neue Dimension: 'Kristallwachstum' im harmonischen Tonraum (1993-1998)." *MusikTexte* 112 (February 2007): 75-79. English version, "On 'Crystal Growth' in Harmonic Space (1993-1998)" in *Contemporary Music Review* 27/1 (2008): 47-56.

willing to resort to different methods of pitch organization, even including temperament. One of these methods is 72-tone equal temperament (as in the work of Ezra Sims). In *Changes: 64 Studies for Six Harps* (1985), this is realized by the retuning of six harps, each to a different pitch level; the staggered equal-tempered chromatic scales of the harps combine to produce 72 equal steps. Tenney has also used up or down arrows (or combinations of arrows) implying the division of the semitone into six parts (or even seven parts as in *Glissade* (1982), for a more precisely approximated fifth partial). In early works like *Clang* (1972), down arrows were used to (imprecisely) show the lowering of the 7th and 11th partials. Tenney's preferred notation for recent works has been "cents deviation": this shows the deviation of the desired pitch from the nearest tempered note in cents. Though he recognizes that this degree of precision may be impossible to attain, Tenney feels this notation best conveys his desired pitch relationships: "it's a small target, but it's still a target, right?"[53] This compromise with tempered tuning would have been anathema to a just intonation purist like Partch, but it offers greater accessibility for performers accustomed to tempered pitch notation.

Tenney's *Koan for String Quartet* (1984) illustrates his interest in the interaction of extended just intonation with the processes of auditory perception, and also his preference for simple, gradually unfolding processes. In a 1978 interview, Tenney explained that once the audience can predict how a piece will unfold, they can "begin to really listen to the sounds, get inside them, notice the details, and consider or meditate on the overall shape of the piece, simple as it may be."[54] In *Koan for String Quartet,* the first violin plays a tremolo between two notes, combining a gradually climbing pitch with an adjacent open string. Figure 20 is an excerpt from the last third of the piece: in the course of this excerpt, the violin changes gradually by microtonal steps from an interval of 537 cents to one of 454 cents. Tenney assigns specfic just ratios to the violin by choosing "the simplest frequency ratios within 'tolerance' of successive steps of one-sixth of a tempered semitone."[55] The proportions above each measure (and next to each pitch) describe how Tenney predicts each chord will be heard as pitches related to one another in extended just intonation. Thus, the first measure is heard as representing the relationship 7/11/15/19, while the second measure represents the relationship 13/20/27/34 (with the common tone E now understood as 27 instead of 15). In the third measure, the harmony is far simpler, with the simple proportion 2/3/4/5: a just intonation E major triad.

[53] James Tenney and Donnacha Dennehy, "Interview with James Tenney," *Contemporary Music Review* 27/1 (2008), 79-90: 80.
[54] "Interview with Gayle Young," *Only Paper Today* (June 1978): 16. Quoted in Larry Polansky, "The Early Works of James Tenney," *Soundings* 13 (1984), 114-297: 194.
[55] James Tenney, performance note in score of *Koan for String Quartet.*

Figure 20: James Tenney, *Koan for String Quartet,* mm. 183-188

Tenney specifies tuning to the nearest cent in this score, but according to his principle of perceptual tolerance, the just intervals should be recognizable even if the quartet's tuning is not exact. The varying degrees of "consonance" heard in this excerpt could be expressed in terms of different distances in harmonic space; the proportion 15/20/27/34 is made up of points relatively distant from one another in harmonic space, while the pitches of the proportion 2/3/4/5, with their small number ratios, are very close to each other.

Other American Just Intonation Composers

While the figures discussed above may be the best known American composers working in extended just intonation, many other composers have engaged aspects of just intonation theory through their compositions.

Like James Tenney, Alvin Lucier (b. 1931) often bases his compositions on acoustic phenomena. Though Lucier does not explicitly use just intonation in his works, they address many of the same psychoacoustical phenomena that are essential to just intonation theory—from beats between closely spaced tones to the spectral structure of individual instrumental tones. One of his best known pieces, *I Am Sitting in a Room,* makes audible the resonant frequencies of its performance space. The performer records a brief text ("I am sitting in a room, different from the one you are in now..."), which explains the concept of the piece in detail. The recording is then played in the same room, where it is recorded and then replayed: this process of recording and playback is repeated over and over again. As the process is repeated, certain frequencies of the recording are amplified by the room's natural acoustical resonances, while others are cancelled out. In the final cycles, only haunting whistles remain: "We discover that each room has its own sets of resonant frequencies in the same way that musical sounds have overtones."[56] Many of Lucier's works combine acoustic instruments with electronically generated sine tones: in *Music for Piano with Slow Sweep Pure Wave Oscillators* (1992), Lucier sets the piano's sounds against sine waves which change slowly in pitch. As the sine waves cross the instrumental sounds, they

[56] Alvin Lucier, "Careful listening is more important than making sounds happen," in *Reflections: Interviews, Scores, Writing* (Köln: MusikTexte, 1995), 434.

set off complex patterns of interference and beating: the sine waves set the spectral structure of the piano tones into relief, making hidden acoustic phenomena audible.

Pauline Oliveros (b. 1932) occupies an important place among composers exploring just intonation as part of a spiritual, meditative practice. Her music—often couched in verbal scores as "Sonic Meditations"—combines breathing and meditation exercises with improvisation (Oliveros performs on a justly-tuned accordion). Through these meditation, Oliveros pursues an intensely aware state of musicality she calls "deep listening."[57]

A new generation of just intonation composers has emerged, including students of Ben Johnston (Kyle Gann) and James Tenney (Larry Polansky and John Luther Adams). Just intonation has informed works in a variety of styles, ranging from Ellen Fullman's delicate works for her "Long String Instrument" (with justly tuned strings from thirteen to thirty meters long) to Glenn Branca's high-volume "symphonies" for multiple electric guitars (with other instruments including electronic keyboards and drums) tuned to pitches of the harmonic series.[58] The intricacies of tuning theory have been pursued by a number of theorists, often working outside the mainstream academic establishment: these theorists include John Chalmers, Ervin Wilson, and Ivor Darreg, as well as the members of an active online discussion group.[59]

Extended Just Intonation in European Composition

Extended just intonation has been a largely American phenomenon, due largely to the lasting influence of Partch and Johnston. In the 1950s and 60s, just tuning seemed to particularly thrive in the Midwest and West, areas less closely associated with European musical thought than the urban centers of the East Coast, where serialism prevailed in the increasingly academic world of new music. While microtonality was certainly not new in European new music, European composers tended to explore expanded equal temperaments rather than extended just intonation—see for example the music of Alois Hába or Ivan Wyschnegradsky. While the subdivision of the octave into twenty-four or more equal parts can be used to approximate extended just intervals, in most cases composers using such divisions were more interested in exploring their mathematical and combinatorial properties, using the finer division of the octave to support new patterns and scales. In his article "Mikrotonalitäten," composer Georg Friedrich Haas suggests that there are four distinct approaches to microtonal pitch organization:

> 1) tempered divisions of the octave into equal parts other than twelve; 2) tuning systems based on the proportions of the overtone series (just intonation); 3) 'tone-splitting' (Klangspaltung), that is, the use of very small intervals close to the unison: the focus of compositional interest is the beating and interference between the tones; and 4) aleatoric microtonality through particular instrumental

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[57] Heidi Von Gunden. *The Music of Pauline Oliveros* (Metuchen, New Jersey: Scarecrow Press, 1983).

[58] Ellen Fullman, "The Long String Instrument" *Musicworks* 85 (Spring 2003): 20-28. Glenn Branca's interview with William Duckworth appears in *Talking Music* (New York: Schirmer, 1995).

[59] http://groups.yahoo.com/group/tuning, accessed April 15, 2008.

> techniques, whose pitch is not precisely specified: for example, prepared piano, some percussion instrument sounds, the ad-libitum detuning of strings, etc.[60]

Ideas from extended just intonation occasionally emerge in the work of composers using expanded equal temperament—for example, Wyschnegradsky describes the interval of five and a half semitones as related to the 11:8 ratio. But in general, such ratio considerations are subservient to an overriding model based on the abstract geometries of pitch distance.[61]

Karlheinz Stockhausen's *Stimmung* (1968) was one of the most influential works to adopt the intervals from the harmonic series associated with extended just intonation. The entire work for six vocalists is based on a single overtone series, built on a low B-flat and including the seventh and ninth partials. The singers' vowel sounds are carefully combined to emphasize different combinations of formants—the result is closely related to the Tuvan and Tibetan practices of overtone singing. The work is stepped (like La Monte Young's) in Eastern mysticism: in addition to the collaborative, ceremonial treatment of the vocal ensemble, the text of the work consists of the "magic names" from a variety of cultures around the world and short texts written by Stockhausen.

Among composers of Stockhausen's influential generation, György Ligeti has been most interested in exploring the world of extended just intonation, though he uses this material with a personal stamp. Ligeti's works have used a variety of means to create a kind of "hybrid microtonality," combining different kinds of pitch theorizing including just intonation, temperament, and uncertain or wavering tuning. These include retunings of parts of the ensemble (as in *Ramifications* and the *Violin Concerto*), microtonal inflections (the *Double Concerto*), the use of high harmonics on strings and brass (the *Cello Concerto* and *Hamburgisches Konzert*), and even historical temperaments (*Passacaglia ungherese)*. Ligeti has referred to some of the pitch adjustments in the *Double Concerto* as "Partch effects"; another link to the American just extended just intonation composers is the influence of Ligeti's pupil Manfred Stahnke, who studied with Ben Johnston in Illinois. Like Ligeti, Stahnke often combines aspects of just intonation with other intonational systems for an intentionally impure hybrid, as in his 1987 *Partch Harp* for retuned harp and synthesizer.[62]

The problems addressed by composers working in extended just intonation also fascinated composers of the spectral movement. Tristan Murail and Gérard Grisey are the best known of

[60] Georg Friedrich Haas, "Mikrotonalitäten," in *Musik der anderen Tradition: Mikrotonale Tonwelten* (Munich: Musik-Konzepte, Edition Text+Kritik, 2003), 59-65.
[61] A possible counterexample are the composers writing in 31-tone equal temperament associated with Adriaan Fokker in the Netherlands. This division of the octave allowed close apporoximations of seven-limit just intervals. Perhaps because of the difficulties of realizing this music without specially constructed keyboard instruments, the tuning has not found widespread acceptance beyond this circle.
[62] Bob Gilmore, "The Climate Since Harry Partch," 29-30. Stahnke describes how just intervals are used in several of his works in "Mein Weg zu Mikrotönen," in *Musik der anderen Tradition: Mikrotonale Tonwelten* (Munich: Musik-Konzepte, Edition Text+Kritik, 2003), 125-140.

this group, and widely regarded as the movement's founders, but spectralism has influenced many other composers including Kaija Saariaho, Magnus Lindberg, and Jonathan Harvey.[63] The differences between spectralism and just intonation are subtle but significant. While just intonation composers tend to base their theories around pure tunings and frequency ratios, spectral composers ground their work in the analysis of actual sonorities—often including inharmonic or otherwise distorted spectra. Thus, it is not uncommon to see spectral composers using pitch sets based on the stretched spectra of piano strings—sometimes this natural stretching is even exaggerated for a stronger effect. The just intonation composers tend to refer to the pure harmonic spectrum as a kind of Platonic ideal form for harmonic relationships—those who are more scientifically inclined locate the organizing power of the ideal overtone series in the mechanisms of aural perception. Spectral composers, on the other hand, locate the "natural" in the acoustical structure of actual sounds: they are thus much more aware of distortions away from ideal harmonicity as well as the actual amplitude of each spectral component (and its place in the sound's temporal envelope). Spectralists and extended just intonation composers tend to call for very different degrees of intonational precision: while extended just intonation calls for precisely defined pitches (or at most, no more than about 8 cents of error, as in Ezra Sims's 72-tone temperament), spectralism tends to approximate pitch information to a quartertone or eighth-tone grid. Perhaps most importantly, spectralism carries a different aesthetic history—one far closer to European serialist and post-serialist music. The different aesthetics are clear if one compares the traditionalist textures of Harrison or Johnson with spectral music—the Indian-based music of Young or Tenney's minimalist structures are also distant from spectralism's more active and dramatic musical discourse.[64]

Tempered forms of extended just intonation (in the manner of Ezra Sims) have been explored independently by a number of European composers. The Austrian composer Franz Richter Herf (1920-1989) designed his own system of microtonal notation dividing the octave into 72 equal parts; his system was inspired by the microtones of Croatian folk music and the theories and music of Alois Hába. Herf was particularly interested in collections of pitches whose partial numbers form an arithmetic series: that is, a series in the form a, a+b, a+2b, a+3b, etc. A combination of such series provide the pitch organization for his *Ekmelischer Gesang* (1975) for solo violin.[65] Georg Friedrich Haas (b. 1953) has also adopted 72-tone equal temperament: his works in this tuning system include the seventy-five-minute orchestral work *in vain* (2000), which combines overtone-based harmonies with twelve-tone equal temperament tritones, fourths, and fifths. His First String Quartet (1991) retunes each string of the quartet to allow complex overtone-based combinations of natural harmonics, much in the manner of La Monte

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[63] One could argue that Olivier Messiaen anticipated many of the concerns of the spectralists; see Julian Anderson's "A Provisional History of Spectral Music," *Contemporary Music Review* 19/2 (2000): 7-23.

[64] Among spectral composers, Horatiu Radulescu's largely static and precisely tuned works (for example, *Inner Time II* (1993) for 7 clarinets) show the greatest similarity to the music of Young and Tenney. See Gilmore, "'Wild Ocean': An Interview with Horatiu Radulescu."

[65] Horst Peter Hesse, *Grundlagen der Harmonik in mikrotonaler Musik.* (Innsbruck: Edition Helbling, 1989) and "Breaking into a New World of Sound," *Perspectives of New Music* 29/1 (Winter 1991): 212-235.

Young's *Chronos Kristalla* (1990).[66] Hans Zender has also turned to 72-tone equal temperament in several works, drawn by its combination of close approximations of the upper partials and a capacity for lightning-quick modulations and harmonic multivalence. Zender is particularly interested in the harmonic effects of sum and difference tones (as created by the electronic technique of ring modulation—see Stockhausen's *Mantra* among many other works). Among the complex structures he derives by this procedure is the elegant Fibonacci-series harmony formed by the pitches approximating overtones 1, 2, 3, 5, 8, 13, 21, 34, 55, etc., where each new upper pitch is the sum of the harmonic numbers of the two pitches below. In such a series, the intervals between adjacent pitches gradually converge on the same size, about 833 cents.[67]

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The many facets of extended just intonation discussed here underscore the powerful appeal of this approach to composers of the mid- to late-twentieth century. By engaging the emerging scientific discipline of psychoacoustics, extended just intonation has become a promising, scientifically supported theory, not just an archaic remnant of a tonal past. As Harry Partch declared, "It need hardly be labored that music is a physical art, and that a periodic groping into the physical, a reaching for an understanding of the physical, is the only basic procedure, the only way a music era will attain any significance."[68] In its promise of a consistent system built on acoustic facts, just intonation is able to fulfill many of the desiderata of a modernist music theory; in particular, an independence from the languages of the past (as represented by equal temperament) and the possibility for limitless expansion into the outer reaches of the overtone series. Since the early experiments of Harry Partch, extended just intonation has developed from a fringe phenomenon to a more central position in the discourse around pitch structure in new music.

Historically, the emergence of just intonation must be viewed in part as a reaction to other musical developments. James Tenney has described the appeal of extended just intonation as a way of expanding beyond the "exhausted harmonic resources" of the standard twelve-tone collection. Extended just intonation allowed the construction of complex harmonies and musical surfaces without abandoning tonality—it suggests an expanded tonality rather than atonality. For other composers, just intonation was a backlash against the abstractions or serial music. Like spectral music, music in extended just intonation could abandon the motivic and geometric play of serial structure in favor of simple, transparent forms—forms built explicitly to engage the mechanisms of our aural perception.

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[66] Georg Friedrich Haas, "Mikrotonalitäten," 64-65.
[67] Hans Zender, "Gegenstrebige Harmonik" in *Musik der anderen Tradition: Mikrotonale Tonwelten* (Munich: Musik-Konzepte, Edition Text+Kritik, 2003), 167-208. Also in *Die Sinne Denken: Texte zur Musik 1975-2003*, ed. Jörn Peter Hiekel (Wiesbaden: Breitkopf & Härtel, 2004). Péter Eötvös also explored the Fibonacci series intervals in his *Intervalles-Intérieurs* (1974): see his liner notes to *Intervalles-Intérieurs, Windsquenzen,* Budapest Music Center Records, BMC CD 092, 2003.
[68] Harry Partch, "Show Horses in the Concert Ring," in *Bitter Music,* 174-180.

Extended just intonation represents a turn away from the Western tradition of structural listening—listening syntactically to the development of themes, motives, and architectonic forms—and towards a meditative, in-the-moment mode of listening. The appeal of just harmonies over tempered harmonies is that the just harmonies are defined *in their own terms,* not designed as part of a system of syntactic relationships. That is, our appreciation of just harmonies focuses on their nature, not their place in a system of relationships. As Stockhausen said of *Stimmung:* "Time is suspended. One listens to the interior of the sound, to the interior of the *harmonic spectrum,* to the interior of a vowel: TO THE INTERIOR"[69] This turn from music based on narrative forms combining many sounds to the contemplation of a single sound (with, of course, its own complex internal overtone structure) appears in many guises: Pauline Oliveros's "deep listening," Lucier and Young's sound installations, Tenney's meditative *koans* and swells, and the "one note" orchestra works of Giacinto Scelsi. The psychedelic aesthetic of the 1960s and the simultaneous explosion of interest in Eastern spirituality was an important influence on the spread of "deep listening," as were John Cage's exhortations to explore the nature of the sounds that surround us. As long as composers continue to be inspired by physical and sensual properties of musical tones and their combinations, the theories of extended just intonation will play an important role in compositional theory.

[69] Karlheinz Stockhausen, liner notes to *Stimmung: Pariser Version.* Stockhausen Edition, CD 12, 1993: 72.

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